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ON F–CLOSED SPACES

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Recently the class of F–closed topological spaces was defined by Chae and Lee [CL_1]. More than once they claimed that the class of F–closed spaces is contained properly between the classes of S–closed spaces and quasi–H–closed spaces. However, despite providing many examples they did not provide an example of a space which is quasi–H–closed but not F–closed. In this note we show that no such example exists, by proving that a space is F–closed if and only if it is quasi–H–closed.

By the word 'space' we mean a topological space which satisfies no addtional (separation) properties, unless explicitly stated.

If A is a subset of a topological space (X, τ) then τ int$A$ and τ cl$A$ denote the interior and closure of A with respect to τ respectively. We may denote these sets by int$A$ and cl$A$ if there is no possible confusion.

A subset A of a space (X, τ) is called

(i) semiopen if $U \subset A \subset \mathrm{cl}U$ for some open set U;

(ii) semiclosed if its complement is semiopen;

(iii) the semiclosure of B, denoted scl$B$, if it is the intersection of all semiclosed sets containing B;

(iv) feebly open if $U \subset A \subset \mathrm{scl}U$ for some open set U;

(v) an α–set if $A \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}A))$;

(vi) a regular open set if $A = \mathrm{int}(\mathrm{cl}A)$.

The collection $RO(X, \tau)$ of all the regular open subsets of (X, τ) is a base for a topology on X called the semi–regularization of τ, and denoted by τ_s. In general, $\tau_s \subset \tau$. The reader is refered to the papers of Mršević, Reilly and Vamanamurthy [MRV] and Janković [J_1] for detailed discussions of semi–regularization topologies.

Njastad [Nj] showed that the collection τ^α of all α–sets in (X, τ) is a topology on X, and that $\tau \subset \tau^\alpha$. Janković and Reilly [JR Proposition 1]

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proved the following result. Subsequently, this result has been obtained independently by Noiri [No, Lemma 3.2], and Chae and Lee [CL_2, Theorem 2.1].

Lemma 1. *A set in (X, τ) is feebly open if and only if it is an α-set.*

Since $RO(X,\tau) = RO(X,\tau^\alpha)$ we have the following result of Janković [J_2, Corollary 2.3].

Lemma 2. *For every space $(X,\tau), \tau_s = (\tau^\alpha)_s$.*

In the jargon of Cameron [C], Lemma 2 states that τ^α is ro-equivalent to τ. So our next result follows immediately from [C, Theorem 3].

Lemma 3. *For every $U \in \tau^\alpha, \tau clU = \tau^\alpha clU$.*

Definition 1. ([PT]) A space (X,τ) is quasi-H-closed (denoted QHC) if every open cover of X has a finite proximate subcover (every open cover of X has a finite subfamily whose closures cover X). A Hausdorff QHC space is H-closed.

Definition 2. ([T]) A space is S-closed if every semiopen cover of X has a finite proximate subcover.

Definition 3. ([CL]) A space is F-closed if every feebly open cover of X has a finite proximate subcover.

Cameron [C] has called a topological property R semiregular provided that a space (X,τ) has property R if and only if (X,τ_s) has property R. We state as an explicit result the remark of Cameron [C] that QHC is such a property.

Lemma 4. *(X,τ) is QHC if and only if (X,τ_s) is QHC.*

Proposition 1. *(X,τ) is F-closed if and only if (X,τ^α) is QHC.*

Proof. Immediate from Lemmas 1 and 3.

Proposition 2. *(X,τ) is QHC if and only if (X,τ^α) is QHC.*

Proof. Follows from Lemmas 2 and 4.

These two propositions provide the proof of the promised result.

Proposition 3. *(X,τ) is F-closed if and only if (X,τ) is QHC.*

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