Journal of the Korean Institute of Industrial Engineers
Vol. 16, No. 1, June, 1990

Lot Size Determination in the Kanban System †

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Kanban 시스템에서의 로트크기 결정

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Abstract

Kanban, a pull system for inventory control is in direct contrast to conventional push systems. In the pull system, the kind and quantity of items needed by the succeeding stage are withdrawn from the preceding stage, only at the rate and at the time they are consumed.

In this paper, lot size models are formulated in two special cases of practical interests and simple solution procedures are adapted to minimize the total cost of the kanban system. An numerical example is solved to illustrate the method.

1. Introduction

Recently a great deal of attention has been arised by the significant productivity improvements attributed to the Japanese production and inventory management techniques. In particular, the just-in-time(JIT) system with kanbans has received most of this interest. The kanban system is a multi-stage production and inventory control system. It harmoniously controls the production of the necessary items in the necessary quantities at the necessary time in every process of a factory. One unique feature of the kanban system is a demand-pull compared to that of the conventional push system. A push system is simply a scheduled-based system.

† Research Supported by the Korea Research Foundation in 1988.

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The amount and time of item flow at each stage are forecasted in advance. Based on this forecast value, items are pushed from an upstream stage to a downstream stage.

A pull system, on the other hand, matches production with demand. The succeeding stage pulls the kind and quantity of items from the preceding stage, when needed. This serves as a signal to the preceding stage to produce just enough units to replace those withdrawn. In the ideal pull system, in-process inventory at each stage is one unit. However, it is very difficult to realize the ideal lot size of one unit, especially at those stages where one touch setup is not feasible. Practically units are produced in small losts or containers.

The purpose of this paper is to determine economic lot sizes or container capacities for the kanban system. A standard container of component items corresponds to the quantity necessary for one lot of upstream production.

In the literature there are the results of many investigations on multi-stage manufacturing systems{Crowston et al. (1973), Schwarz and Schrage(1975), Taha and Skeith(1970), Moily(1986)}. However, little research has been done with pull systems, and especially the kanban system. In this paper we formulate mathematical models in two special cases of practical interests and propose solution procedures to minimize the total cost of the kanban system.

Notations

S_i = Setup cost at stage i ;

h_i = Unit inventory holding cost per unit of time at stage i ;

d_i = Demand rate at stage i ;

P_i = Production rate at stage i ;

Q_i = Lot size at stage i (to be determined);

$E_{i,j}$ = Number of units of component item j required as input to produce a unit of item i (denote an item by the index of the stage producing it).

2. Model Description

A multi-stage assembly production/inventory system with each stage producing one type of item is considered, as shown in Fig. 1. In a multi-stage assembly system

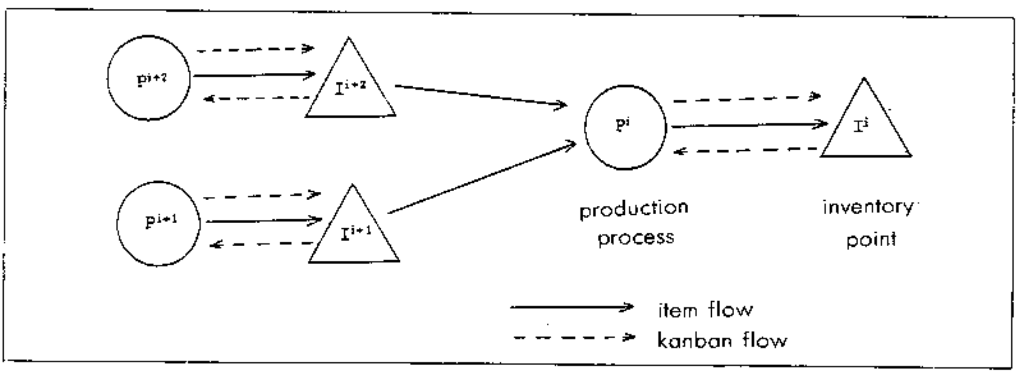


Fig. 1. Kanban System for a Multi-stage Assembly Process.

items, having at most one successor but any number of predecessors, are processed on several stages of production.

Each stage includes a production process and an inventory point. The stage of i=1 means the final stage. Whenever the first piece of a full container is used by the production process of the succeeding stage, the attached kanban to the container is removed and sent back to the preceding stage. The free kanban then triggers production of another standard lot of the same item. Once a full container is produced, the kanban which authorized production of the full container is attached to it and the container is sent to the inventory point of that stage. This procedure causes all the stages to be chained together. For convenience we assume that the value of in-process inventory at the succeeding production process is equal to that of the preceding inventory point.

3. Mathematical Models

The following assumptions are being made:

(1) Demand rate for the end item is known and it is constant over time.

(2) No two items in the system are produced by a single production stage or used the same component item as an input.

(3) Stockouts are not permitted.

(4) The production rate of each production process is greater than or equal to the rate of consuming it at the succeeding production process.

(5) Time delay in transferring a kanban and a container is assumed zero.

(6) Time horizon is infinite.

In this section, we shall consider lot sizing models to the kanban system for two particular cases.

3-1. A one-lot-for-one-lot policy

The one-lot-for-one-lot policy is a simple and convenient one. It means that one lot or one full container of each immediate predecessor of stage i is required to make one lot of stage i.

The total variable cost at the i-th stage, $Z_i(Q_i)$ consists of the following variable costs:

(1) Cost of production setups or orders

The setup/order cost is the cost for each setup/order multiplied by the number of setups: i.e., $\frac{S_i d_1}{Q_i}$

(2) Cost of carrying inventory

In the kanban system, the preceding stage's units of inventory are transferred in small losts or containers to the succeeding stage where it is processed. By assumption (1), (5), the number of kanban needed at each stage is one. Assumption (5) is supported by the fact that lead times in deterministic systems can be ignored since they do not affect lot sizes.

The inventory fluctuations at the i-th stage are illustrated by Fig. 2.

On the basis of above properties it follows immediately that the cycling period is given by Q_i/d_1.

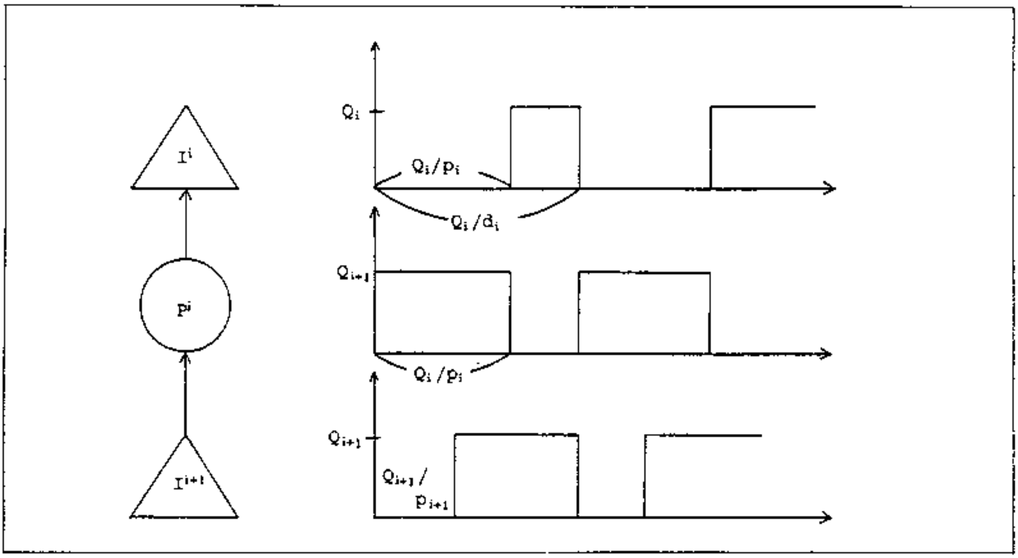


Fig. 2. Behavior of the Inventory Levels in a One-lot-for-one-lot Policy.

The average amount in inventory at a inventory point I^i is:

$$Q_i - \frac{Q_i \cdot Q_i/P_i}{Q_i/d_i} = Q_i(1-d_i/p_i)$$

The average in-process inventory at a production process p^i is:

$$\frac{Q_{i+1} \cdot Q_i/P_i}{Q_{i+1}/d_{i+1}} = Q_i \cdot \frac{d_{i+1}}{P_i}$$

Hence the cost of carrying inventory at the i-th stage is given by

$$h_i \cdot Q_i(1-d_i/P_i) + h_{i+1} \cdot Q_i \frac{d_{i+1}}{P_i}$$

(3) Cost of producing items

In the EOQ formula, only holding cost and setup cost are included. However, many authors such as Schonberger(1982), Monden(1981) make clear that cutting lot sizes tends to trigger the important benefits which is not considered in the EOQ; namely, better quality, more flexibility, less scrap and rework, and higher productivity. Probably these benefits are more significant that the direct benefit of less inventory carrying cost, occurring when lot sizes are reduced. This is why the Japanese struggle to reduce the setup time or lot sizes.

With a view to reflecting these effects of manufacturing lot sizes upon the production costs, we denote a production cost function, $P(Q_i)$ to represent the cost of producing item i. It contains all the production-related costs except the inventory holding cost and the setup cost. Thus, these production cost functions play a role to make smaller lots profitable in determining economic lot sizes. However, unfortunately it is quite difficult to show explicitly the production cost function for lot size, Q_i variable because the function varies according to the characteristics of the products, workers, and the machines. Under the specific circumstances it may be obtained from past data or simulation. In order to minimize the total cost we also assume that the production cost function is linear.

The total cost of the system, $Z_1(Q)$, is written as follows:

$$Z_1(Q)=\sum_{i=1}^{n}\left[\frac{S_i d_i}{Q_i}+H_i Q_i+(a_i Q_i+b_i)\right] \quad \cdots (1)$$

$$Q_j=E_{ij}Q_i \quad \cdots\cdots (2)$$

Where

$$H_i=h_i\left(1-\frac{d_i}{p_i}\right)+h_{i+1}\frac{d_{i+1}}{p_i} \quad \cdots\cdots (3)$$

a_i, b_i: given constants

Optiomal lot size can be obtained by equating the first derivative of $Z_1(Q)$ with respect to Q_1 to zero, which provides

$$Q_1=\left[\sum_{i=1}^{n}\frac{S_i d_i}{E_{1,i}}\Big/\sum_{i=1}^{n}(H_i+a_i)E_{1,i}\right]^{\frac{1}{2}} \quad \cdots\cdots (4)$$

$$Q_j=E_{1,j}Q_1 \quad \cdots\cdots (5)$$

3-2. A multiple-lots-for-one-lot policy

A multiple-lots-for-one-lot policy is defined such that an integer number of lots or full containers of each immediate predecessor of stage i are required to make one lot of stage i. This policy corresponds to the component lot-splitting lot sizing policy considered by Moily(1986). Bitran and Chang(1987) say that this policy is supported by the philosophy of just-in-time production because it can avoid using large containers in the upstream stages and decrease the chance of occurring partially filled containers.

Under this policy, the inventory fluctuations at the i-th stage are shown by Fig. 3.

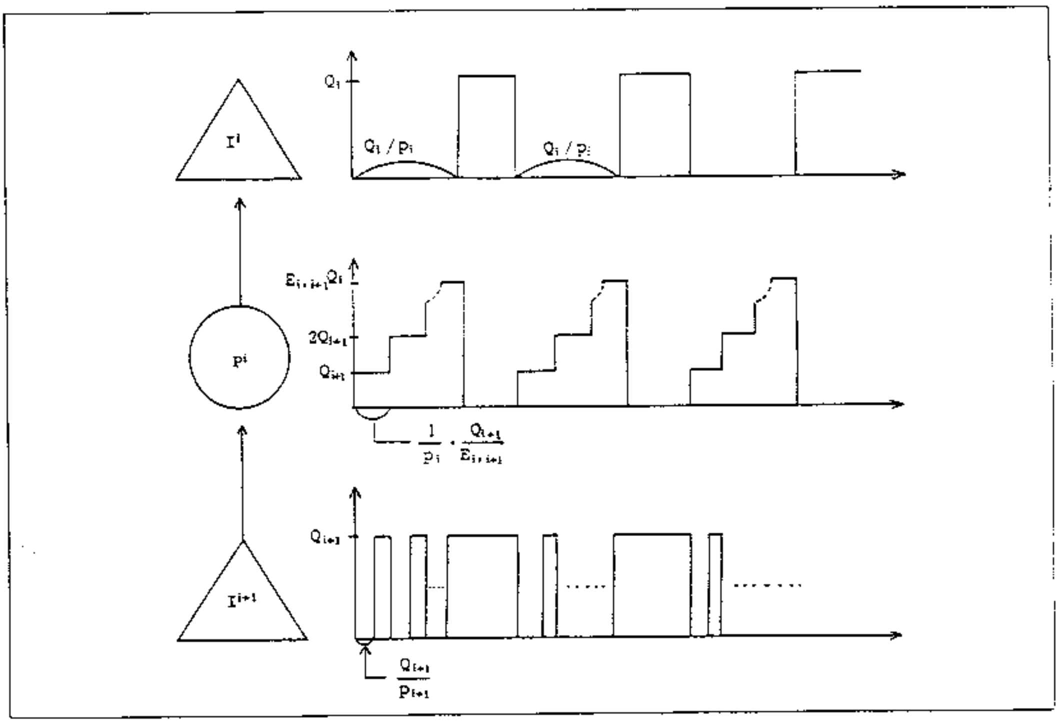


Fig. 3. Behavior of the Inventory Levels in a Multiple-lots-for-one-lot Policy.

The cycle repeats itself in every period $\frac{Q_1}{d_1}$.

The average amount in inventory waiting at a inventory point I^i is:

$$Q_i - \frac{Q_1/P_i \cdot Q_j \cdot E_{1,i}/Q_1}{Q_1/d_1} = Q_i(1 - \frac{d_1}{P_i})$$

The average in-process inventory at a production process p^i is:

$$\frac{1}{2}(Q_{i+1} + E_{i,i+1}Q_i) \cdot \frac{Q_1}{P_i} \cdot \frac{E_{1,i}Q_1}{Q_i}$$

$$= \frac{1}{2}\frac{d_1}{p_i}(E_{i,i+1} \cdot Q_i + Q_{i+1})$$

Therefore, the cost of carrying inventory at the i-th stage is given by

$$h_i Q_i (1 - \frac{d_1}{p_i}) + \frac{1}{2} h_{i+1} \frac{d_1}{p_i}(E_{i,i+1}Q_i + Q_{i+1})$$

The total variable cost of the system, Z_2 (Q) is written as follows:

$$Z_2(Q) = \sum_{i=1}^{n}[\frac{S_i d_i}{Q_i} + H_i Q_i + (a_i Q_i + b_i)] \quad \cdots\cdots (6)$$

$$Q_j = \frac{1}{R_{ij}} E_{ij} \cdot Q_i;\ R_{ij} \in (1,2,\cdots\cdots) \quad \cdots\cdots\cdots (7)$$

Where

$$H_1 = h_1(1 - \frac{d_1}{p_1}) + h_2 \frac{d_2}{2p_1} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

(for final item 1)

$$H_i = h_i(1 + \frac{d_{i-1}}{2p_{i-1}} - \frac{d_i}{p_i}) + h_{i+1}\frac{d_{i+1}}{2p_i} \quad \cdots (9)$$

(for all items of $i = 2, \cdots, n$)

$$H_n = h_n \frac{d_{n-1}}{2p_{n-1}} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (10)$$

(for raw materials n)

a_i, b_i: given constants

Owing to integer restriction of R_{ij}, there are no general methods for minimizing the total cost. $Z_2(Q)$. In this section, we use heuristic approach, which is as follows{for the optimal solution procedure, refer to Moily(1986)}

The optimal lot size Q_1^0 can be obtained from equation(6),

$$Q_1^0 = [\sum_{j=1}^{n} \frac{R_{1j}S_j d_j}{E_{1j}} / \sum_{j=1}^{n} \frac{(H_j + a_j)E_{1,j}}{R_{1,j}}]^{\frac{1}{2}} \quad \cdots\cdot (11)$$

and the corresponding minimum cost is

$$Z_2^0 = [4(\sum_{j=1}^{n} R_{1,j}S_j d_j / E_{1,j}) / (\sum_{j=1}^{n} \frac{(H_j + a_j)E_{1,j}}{R_{1,j}})]^{\frac{1}{2}}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

However, to derive the optiomal lot size Q_1, from equation (4), the optimal values of R_{1j} have to be determined in advance. Now the problem is to find the values of $R_{1,j}^0$ variables which minimize Z_2^0 in euqation(12). Using partial differentiation of Z_2^0 with respect to R_{1j}, etc., the values of $R_{1,j}^0$ are obtained as follows:

$$R_{1,j}^0 = \left[\frac{(\sum_{j=1}^{n} \frac{R_{1,j}S_j d_j}{E_{1,j}})(H_j + a_j)E_{1,j}}{\frac{S_j d_j}{E_{1,j}}(\sum_{j=1}^{n} \frac{(H_j + a_j)E_{1,j}}{R_{1,j}})}\right]^{\frac{1}{2}}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (13)$$

This equation(13) is now used to find the $R_{1,j}^0$ by an iterative process since the $R_{1,j}^0$ are on both sides of the equation. The first estimates can be obtained from

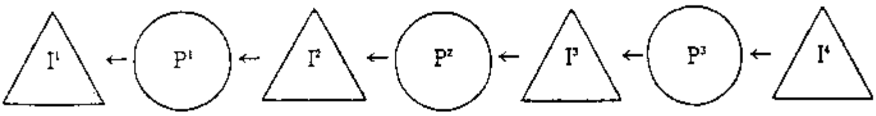
$$R_{1,j}^1 = \left[\frac{(\sum_{j=1}^{n} \frac{S_j d_j}{E_{1,j}})(H_j + a_j)E_{1,j}}{\frac{S_j d_j}{E_{1,j}}(\sum_{j=1}^{n}(H_j + a_j)E_{1,j})}\right]^{\frac{1}{2}}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

By substituing these estimates on the right-hand side of equation(13), new estimates can be given on the left. This process continues until the new estimates are equal to the previous estimates. Accordingly, the lot sizes Q_i which are variables to be determined can be obtained from equations(7), (11).

Example

We consider a 4-stage production system exhibiting the following structure. Stage 1 represents the final assembly stage and stage 4 purchase of raw materials.



At each stage the following data area available.

stage	d_i	P_i	S_i	h_i	a_i	b_i	E_{ij}
1	1000	1200	23	0.30	0.5	500	-
2	1000	1200	13	0.25	0.5	500	1
3	2000	2500	6	0.12	0.3	300	2
4	2000	2500	18	0.10	0.2	300	2

From equation (8), (9) and (10)

$$H_1 = 0.30(1-\frac{5}{6}) + 0.25\times\frac{5}{12} = 0.15$$

$$H_2 = 0.25(1+\frac{5}{12}-\frac{5}{6}) + 0.12\times\frac{5}{6} = 0.25$$

$$H_3 = 0.12(1+\frac{5}{12}-\frac{4}{5}) + 0.10\times\frac{2}{5} = 0.11$$

$$H_4 = 0.10\times\frac{2}{5} = 0.04$$

$$\sum_{j=1}^{4}\frac{S_j d_j}{E_{1,j}} = 60,000$$

$$\sum_{j=1}^{4}(H_j + a_j)E_{1,j} = 2.70$$

To obtain the first estimates we use equation (14), which gives

$$R^1_{1,2} = [\frac{60,000\times0.75}{13,000\times2.70}]^{\frac{1}{2}} = 1.13 \fallingdotseq 1$$

$$R^1_{1,3} = [\frac{60,000\times0.82}{6,000\times2.70}]^{\frac{1}{2}} = 1.74 \fallingdotseq 2$$

$$R^1_{1,4} = [\frac{60,000\times0.48}{18,000\times2.70}]^{\frac{1}{2}} = 0.77 \fallingdotseq 1$$

Substituting these values on the right-hand side of equation (13) gives the following new estimates.

$$R^0_{1,2} = [\frac{66,000\times0.75}{13,000\times2.29}]^{\frac{1}{2}} = 1.29 \fallingdotseq 1$$

$$R^0_{1,3} = [\frac{66,000\times0.82}{6,000\times2.29}]^{\frac{1}{2}} = 1.98 \fallingdotseq 2$$

$$R^0_{1,4} = [\frac{66,000\times0.48}{18,000\times2.29}]^{\frac{1}{2}} = 0.88 \fallingdotseq 1$$

Since new estimates are equal to the first estimates the process terminates. These indeed are the optimal values. Therefore, we can obtain optimal lot sizes from equation (7), (11).

$$Q^0_1 = [\frac{66,000}{2.29}]^{\frac{1}{2}} = 169.77 \fallingdotseq 170$$

$$Q^0_2 = \frac{1}{R^0_{1,2}}\cdot E_{1,2}\cdot Q^0_1 = 170$$

$$Q^0_0 = \frac{1}{R^0_{1,3}}\cdot E_{1,3}\cdot Q^0_1 = \frac{2}{2}\,Q^0_1 = 170$$

$$Q^0_4 = \frac{1}{R^0_{1,4}}\cdot E_{1,4}\cdot Q^0_1 = \frac{2}{1}\,Q^0_1 = 340$$

4. Conclusion

For the kanban system, we have developed economic lot size models in which work-in-process inventories are significantly considered and the cost of producing items is included in the formulation. Although the production cost function for lot size is not explicitly described, the lot size models discussed here can provide a framework for determining lot sizes or container capacities in the specific production system. A further development of this research would be to allow the general case of not restricting the relative lot size or container size between stages.

References

[1] Bitran, G.R. and Chang, L., 1987, "A Mathematical Programming Approach to a Deterministic Kanban System," Management Science, 33, 427-439.

[2] Crowston, W.B., Wangner, M. and Williams, J.F., 1973, "Economic Lot Size Determination in Multi-Stage Assembly Systems," Management Science, 19, 517-527.

[3] Kimura, O. and Terada, H., 1981, "Design and Analysis of Pull System, A Method of Multi-Stage Production Control," International Journal of Production Research, 19, 241-253.

[4] Moily, J.P., 1986, "Optimal and Heuristic Procedure for Component Lot-Splitting in Multi-Stage Manufacturing Systems," Management Science, 32, 113-125.

[5] Monden, Y., 1981a, "What Makes the Toyota Production System Really Tick?," Industrial Engineering, No. 1, 36-46.

[6] Monden, Y., 1981b, "Smoothed Production Lets Toyota Adapt to Demand Changes and Reduce Inventory," Industrial Engineering, No. 8, 42-51.

[7] Monden, Y., 1981c, "How Toyota Shortened Supply Lot Production Time, Waiting Time, and Conveyance Time," Industrial Engineering. No. 9, 22-30.

[8] Schonberger, R.S., 1982, Japanese Manufacturing Techniques: Nine Hidden Lessons in Simplicity. The Free Press, New York.

[9] Schwarz, L.B. and Schrage, L., 1975, "Optimal and System Myopic Policies for Multi-Echelon Production/Inventory Assembly Systems," Management Science, 21, 1285-1294.

[10] Taha, H.A. and Skeith, R.W., 1970, "The Economic Lot Sizes in Multi-Stage Production Systems," AIIE Trans., 157-162.