□ 論 文 □

행태의 다양성을 고려한 토지이용-교통모형의 개발

The Land Use-Transportation Model with Taste Heterogeneity

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요 약

1960년 초부터 토지이용과 교통의 상관관계를 계량적으로 설명하고자 하는 연구가 진행되어왔다. 이러한 연구는 경제이론을 배경으로 하여 개발된 McKinnon-Type 모형과 Mills-Type 모형, 그리고 경제학이 반영되어있지 않은 Lowry-Type 모형으로 크게 대별할 수 있다. 이제까지의 이러한 연구는 각 가정의 주거입지선정과 직장선정에 있어서 취향의 다양성(taste heterogeneity)을 고려하여 있지않고 있어 본 연구에서는 로짓모형을 이용하여 Alonso(1964) 모형을 더욱 발전시켜 토지이용-교통의 일반 균형 모형(general equilibrium model)을 개발하였다.

이 통계적 토지이용-교통모형은 완전 경쟁하의 일반균형상태에서 주택임대료, 노동력 임금, 상품가격이 내생적으로 산출되어지며, 동시에 효율적인 교통체계 하에서 일반균형상태의 생산량과 생산부지, 그리고 주거수와 주거부지가 어떻게 분배되는 지를 내생적으로 결정한다. 이 논문에서 효율적인 교통체계라함은 해당 존에서 도로에 사용된 토지의 임대료가 교통체증비용의 전 통행량에 대한 합과 동일하도록 하는 최적교통체증정도(optimal congestion level)을 유지할 수 있는 도로 체계를 뜻한다. 또한 비 효율적 교통체계하에서는 토지 이용에 왜곡이 생겨서 전체적 비용의 상승으로 각 국민 혹은 각 주민이 생활에서 얻을 수 있는 효용가치가 떨어짐을 분석할 수도 있다.

Ⅰ. Introduction

Despit major progress in urban economics since the early sixties, progress in the development of computable general equilibrium models of urban land use and transportaion has not been satisfactory. Two solution techniques have been applied to solve computable general equilibrium models. One is linear programming which Mills (1972) introduced to solve his general equilibrium model. The other is a fixed point method (Scarf Algorithm) which MacKinnon (1974, 1976) introduced into urban economics. These models need to be improved for theoretical completeness as well as for practical usefulness. In addition to such incompleteness in theory and difficulties in computation, the previous studies did not consider the heterogeneous behavior of households in choosing their residential locations and work places.

The major objective of this study is the development of a computable general equilibrium model which allows heterogeneous behavior among households in choosing their residential location and work place with efficient road allocation and optimum congestion tolls. An important feature of the model is that it does not assume any predetermined center. This non–monocentric general equilibrium model of urban land use is based on discrete choice theory. Discrete choice models have theoretical superiority over other approaches because they allow the incorporation of hetergeneity and choice dispersion into models of land use, which greatly enhances the empirical relevance and applicability of these models. It is hoped that the model may suggest new directions for empirical testing and policy oriented modeling in the near future.

In our model, all prices of commodities including land rent and wage are decided at general equilibrium and the location of firms and households is simultaneously determined by the model's non–monocentric structure. Therefore, such a model will yield location patterns for production and residential uses. The amount of land for streets will be efficiently allocated under optimal transportation congestion tolls. Because a faulty transportation system leads to a distorted land use pattern, an efficient transportation system is used in this study.

Ⅱ. Review of the relevant Literature

The linear programming and fixed point methods have been applied to computable models in order to operationalize the theory of land use and transportation. Herbert and Stevens (1960) introduced linear programming as a method for operationalizing Alonso's Partial equilibrium model (1964). Mills (1972) introduced linear programming to solve his general equilibrium model with exports and imports as proposed in his theoretical article (1967). Mills' formulation had impact and was developed more generally by Hartwick and Hartwick (1974) and by Kim (1978, 1979, 1986). Hartwick and Hartwick introduced a multinucleated city with intermediate goods into the model of Mills. Kim demonstrated the power of the approach by treating a number of policy issues : zoning, subway investment, and alternative transport modes. Kim (1986) also introduced some nonlinearities to better capture variations in transportation mode choice. Moore (1986) adapted Mills' general equilibrium approach into a perfect foresight dynamic general equilibrium model in his Ph.D. dissertation. More recently, Rho (1988) developed an algorithm to solve Kim's entropy constrained programming model (Kim, 1986) and he empirically tested the model and the algorithm with Chicago data. Despite this progress, the linear programming approach has some shortcomings theortically and empirically.

More (1986) discussed the limitations of linear programming formulations. The linear programming formulations use Leontief functions (fixed coefficient production function), which assume that output are assoclated with fixed ratios of the various inputs. Therefore, in the Leontief case, the substitution between an abundant input and a scarcer resource can not be explained. In addition, this linkage of the input–output model with land use model will bring the problems of input–output model into the Mills's grneral equilibrium model. Moses (1974) pointed out the inelasticity in demand with respect to prices and the perfect elasticity of supply functions of primary factors in input–output models. Linear programming models are also limited by the assumption of constant returns to scale in production and consumption and limited by the assumption of the divisibility of production or other activities. In contrast, continuous production functions, such as the popular Cobb–Douglas function, which is used in this study, capture substitution between the various production inputs. With a Cobb–Douglas production function, it is not necessary to assume constant returns to scale, even though this is assumed in this study for simplicity. The assumption of divisibility of production or other activities is also implied with a Cobb–Douglas production function.

Fixed point approaches have considerable advantages over linear programming approaches in that they do not require fixed–coefficient approximations of the production and consumption technologies. The introduction of the fixed point algorithm into urban economics is due to the work of MacKinnon (1974, 1976), King (1977, 1980), and Richter (1978). Applications of these techniques within urban economics were performed by Arnott and MacKinnon (1977) and by Sullivan (1983a, 1983b, 1983c, 1985, 1986). All of these applications dealt with monocentric cities, because of the difficulties in solving complex general equilibrium models by using fixed point algorithms.

No author has yet formulated a non–monocentric general equilibrium model based on discrete choices. Partial equilibrium models have been developed and solved computationoally by Anas (1987), de Palma and Papageorgiou (1988). Anas (1982) and Anas and Duann (1985) showed that a housing market model bassed on discrete choice is computationally tractable.

Ⅲ. The Land Use–Transportation Model

Our non–monocentric general equilibrium model assumes that each household maximizes his utility and each firm maximizes profit. Log–linear utility functions and Cobb–douglas production functions are used for households and firms, respectively. It will be supposed that all households are equally qualified in the labor force. A log linear utility function is defined over each commodity, land and the household's leisure. This formulation allows substitution among these components depending on their prices. Each commodity is potentially available in each zone and the same commodity available in different zones is considered to be a product variant in the sense of Dixit and Stiglitz (1977). The consumer can potentially consume different quantities of each retail commodity from each zone.

The Cobb–Douglas production function is defined over each production resource (intermediate commodity), land and labor. It is assumed that the production technology exhibits constant returns to scale. The production function allows substitution among input resources, land and labor. We will assume that all products are perfectly divisible.

We will also assume perfectly competitive input and output markets and free entry of firms. Therefore, each firm will make zero profit at equilibrium.

Because this general equilibrium model allows the heterogeneous behavior by households in choosing their residential location and work plac, the market equilibrium conditions for land and labor are stochastic. The behavior of firms, on the other hand, is modeled as deterministic. A stochastic equilibrium is defined by requiring that expected demands are equal to expected supplies or supplies (for more details, see Anas 1982, 1987).

1. The Model of the Household

Suppose that the household maximizes a log–linear utility function defined over land, the household's leisure, and each commodity (for non–retail goods we just set the appropriate utility coefficient to zero), subject to a budget constraint.

$$\max_{Z_{ij},\, q_{ij},\, L_{ij}} U_{ij} = \Sigma_r \Sigma_k \alpha_{rk} \ln Z_{ij}^{rk} + \beta \ln q_{ij} + \gamma \ln L_{ij} + \epsilon_{ij} \quad (1)$$

s. t.

$$\Sigma_r \Sigma_k Z_{ij}^{rk}(p_{rk} + 2C_{ik}) + R_i q_{ij} + 2vC_{ij} = w_j(H - T_{ij} - L_{ij})$$

Here α_{rk}, β, and γ are the utility coefficients and we assume $\Sigma_r \Sigma_k \alpha_{rk} + \beta + \gamma = 1$ (the utility function is homogeneous of degree one), ϵ_{ij} is the random part of utility, which is uncorrelated with the other variables and represents taste heterogeneity. Z_{ij}^{rk} is the quantity of commodity γ produced and sold in zone k at price p_{rk}, consumed by households residing in zone i and working in zone j. q_{ij} is the household's quantity of land at i where the househld works in zone j. p_{rk} is defined as the price of commodity r at k. R_i is the land rent in zone i, and W_j is the qage in zone j. L_{ij} are the household's leisure hours purchased by foregoing work at the wage rate W_j. H is the household's fixed total time endowmet, T_{ij} is the travel time (total shopping and commuting travel time) spent by a household who lives at i and works at j. C_{ij} is the congestion toll paid by a trip from i to j. the parameter v is the number of work days in a unit period. Thus, the household makes a round trip commuteper day but makes shopping trips everywhere to purchase all of the retall commodities in each period of time. We will assume that these shopping trips are uniformly distributed over v. We will also assume that the averge travel from a destination or an origin within a zone traverses half the zone's length. Therefore, the congestion toll, C_{ij}, and the travel time, g_{ij}, are :

$$C_{ij} = \frac{1}{2}(t_i + t_j)\Delta + \sum_{l=i,j} B_{ij}^{l} t_l \Delta$$

$$g_{ij} = \frac{1}{2}(AC_i + AC_j)\Delta + \sum_{l=i,j} B_{ij}^{l} AC_l \Delta$$

The matrix B_{ij}^{l} is an incidence martix and Δ is the length of a zone. If zone ℓ is an element of the shortest path R_{ij} of the origin destination(i, j), $B_{ij}^{l}=1$. If it is not, $B_{ij}^{l}=0$. With given transportation cost (time cost and toll) in each zone, the shortest path can be endogenously calculated by using a minimum path algorithm. t_l is the transportation congestion toll per unit distance in zone ℓ. AC_l is the average travel time per unit distance with a certain congestion level at zone ℓ. Hence, the total travel time of a household during a unit period is :

$$T_{ij} = 2vg_{ij} + \sum_{r}\sum_{k} 2g_{ik} Z_{ij}^{rk}$$

By substituting the above equation into the budget constraint, we can rewrite the model of the household as follows :

$$\max_{Z,\, q,\, L} U_{ij} = \Sigma r \Sigma r \alpha_{i}^{r} \ln Z_{ij}^{rk} + \beta \ln q_{ij} + \gamma \ln L_{ij} + \varepsilon_{ij} \qquad (2)$$

s. t.

$$\Sigma_r \Sigma_k \; Z_{ij}^{rk}(P_{rk} + 2C_{ik} + 2w_j g_{ik}) + R_i q_{ij} + 2v (C_{ij} + w_j g_{ik}) = w_j(H - L_{ij}) \qquad (3)$$

To solve this problem, a two-stage optimization is valid. First, suppose that zone pair i j is the optimal residence-workplace pair, then the household must choose the quantities of commodities, leisure and land it must consume given the optimal zone pair i j. Second, conditional on the result of this inner optimization for each zone pair i j, the household must choose the optimal residence-workplace locations, a zone pair i j. This household location pattern gives household location probabilities depending on the assumed distribution of the random utilities, and the result of the inner optimization.

The inner optimization yields the household's demands for each commodity and land, and the supply of labor for each household. These can be derived by using the Lagrangin multiplier method. The demand function for land is :

$$q_{ij} = \frac{\beta \{w_j H - 2v(C_{ij} + w_j g_{ij})\}}{R_i} \qquad (4)$$

The household's demands for each commodity is :

$$Z_{ij}^{rk} = \frac{\alpha_{ik}^{r} \{w_j H - 2v(C_{ij} + w_j g_{ij})\}}{P_{rk} + 2C_{ij} + 2w_j g_{ij}} \qquad (5)$$

Notice that $P_{rk}+2C_{ij}+2w_j g_{ij}$ is the effective price of commodity r produced in zone k for a consumer living in zone i and working in zone j. The effective price consist of P_{rk}, the mill price, $2C_{ik}$, the monetary travel cost(congestion toll), $2w_j g_{ij}$, the value of travel time. The household's supply of labor (or demand for work) at the workplace j is :

$$H - T_{ij} - L_{ij} = H - T_{ij} - \frac{\gamma \{w_j H - 2v(C_{ij} + w_j g_{ij})\}}{w_j} \qquad (6)$$

Substituting these demands (4), (5), and (6) back into the utility function U_{ij}, we will get the indirect utility function. The indirect utility function is;

$$Uij = \ln\{w_j H - 2v(C_{ij} + w_j g_{ij})\} - \Sigma_r \Sigma_k \alpha^{i}_{k} \ln (P_{rk} + 2C_{ij} + 2w_j g_{ij}) - \beta \ln R_i - \gamma \, in w_{ij} + \varepsilon_{ij}$$

Let V_{ij} be the common part of the optimized indirect utility ($V_{ij} = U_{ij} - \varepsilon_{ij}$) given choice of the zone pair i j. The outer problem can be derived as a multinomal logit, nested logit, generalized extreme value or probit model depending on the distributional assumptions concerning the ε_{ij}'s (see Anas, 1982). The logit model is the simplest and most tractable among them. We will assume that the taste heterogeneity constants (random parts of utility) are independently and identically Gumbel distributed. Then, the choice probabilities (probability of choosing zone j as a residential location and zone j as a workplace) are multinomial logit such that :

$$\Psi_{ij} = \frac{\exp(\lambda V_{ij})}{\sum_{l=1}^{I} \sum_{m=1}^{J} \exp(\lambda V_{lm})} \tag{7}$$

$1/\lambda$ is the teste heterogeneity measure, more precisely, $\lambda = \pi/(\sigma\sqrt{6})$ where σ^2 is the variance of the I. I. D. Gumbel distributed random utilities. As $\lambda \to \infty$ (no taste heterogeneity) the Ψ_{ij} which corresponds to the highest V_{ij} goes to unity and all other Ψ_{ij}'s go to zero. Ψ_{ij} is the joint probability that the household chooses residence at i and supplies labor at j. Thus, the choice probability and the quantity of demands per household are all functions of prices only.

2. The Model of the Firm

Using the same notion we developed for the household side, suppose the firm maximizes profit with a Cobb-Douglas production function which exhibits constant returns to scale.

$$\max_{X, M, Q, \bar{y}} \pi_{rj} = p_{rj} X_{rj} - M_{rj} w_j - Q_{rj} R_j - \sum_s \sum_i Y^{si}_{ry} (P_{si} + m_s C_{ij})$$

The Cobb-Douglas production function is :

$$X_{rj} = M_{rj}^{\delta_r} Q_{rj}^{\mu_r} \prod_s \prod_i {Y^{si}_{rj}}^{\phi^{si}_{rj}} \tag{8}$$

where $\delta_r + \mu_r + \Sigma_s \Sigma_i \phi^{si}_{rj} = 1$ for each r and j.

Here, M_{rj} and Q_{rj} are the firm's labor and land inputs and Y^{si}_{rj} is the quantity of the S commodity produced in location i that the firm located in j uses as an intermediate input in the production of commodity r. m_s is passenger car equivalent flow to ship unit production resource s(intermediate good) by firms. C_{ij} is the congestion toll from zone i to zone j for a passenger car flow. X_{rj} is the output of commodity r in zone j. The result of profit maximization with this Cobb-Douglas production function yields the firm's input factor demand functions. These demand function can be derived from the first order condition. The demand function for labor and land are :

$$M_{rj} = \frac{\delta_r P_{rj}}{w_j} X_{rj} \tag{9}$$

$$Q_{rj} = \frac{\mu_r P_{rj}}{R_j} X_{rj} \tag{10}$$

for the s intermediate commodity.

$$Y^{si}_{rj} = \frac{\phi^{si}_{rj} P_{rj}}{P_{si} + m_s C_{ij}} \cdot X_{rj} \tag{11}$$

Form the above equations, we can derive the function which explains the price relationships. This can be derived from the demand function, (9), (10) and (11). Or it can also be derived by using the marginal rate of technical substitution from the production function.

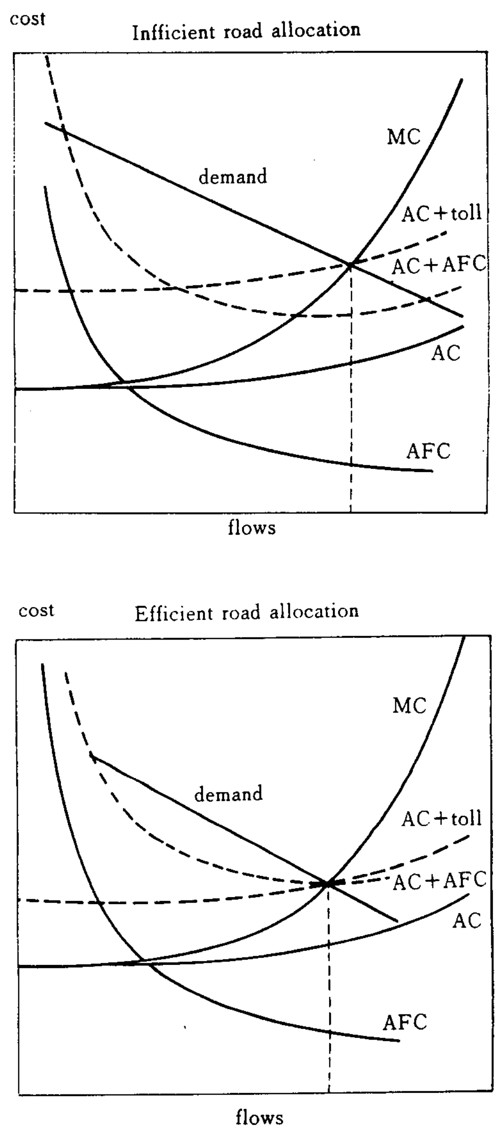
the prices are related by

$$P_{rj}=\frac{W_j^{\delta_r} R_j^{\mu_r}\ \Pi_s\ \Pi_i(P_{si}+m_sC_{ij})^{\phi_{rj}^{si}}}{\delta_r^{\delta_r}\ \mu_r^{\mu_r}\ \Pi_s\ \Pi_i\,{\phi_{rj}^{si}}^{\phi_{rj}^{sr}}} \qquad (12)$$

As we can see (12), a price is determined relative to other prices, not as an absolute value. Therefore, the commodity prices can be solved by using this equation with given wages and rents.

3. An Efficient Transportation System

If transportation cost are not correctly priced and roads are not efficiently allocated, the equilibrium rent gradient does not reflect the social value of land in different locations. Therefore, it will lead to distorted land uses. To avoid this distortion in land uses, this study will uses the optimum pricing policy in transportation congestion which levies congestion tolls to users equal to the difference of the social marginal cost and the private average cost of travel. The street will be also efficiently designed in such a way that the optimum collective toll is just sufficient to pay for the cost of transportation facilities with an efficient road capacity. The figure 1 explanis the case of efficient road allocation and the case of inefficient road allocation, separately. MC is the social marginal cost and AC is the average cost. AFC is the average fixed cost (cost of transportation facility divided by flows). The gap between the marginal cost and the average cost is the difference between social cost and average individual cost which is caused by the congestion externality. For the efficient use of streets, congestion tolls need to be collected to close the gap between the social marginal cost and the



〈Figure 1〉 Congestion Toll and Average Fixed Cost average cost. Social marginal cost pricing is efficient, because the congestion tolls discourage travelers from using congested roads. In the transportation system with an inefficient road allocation, the congestion toll is higher than the average fixed cost (AFC) as we can see AC + toll > AC + AFC at equilibrium flows in the case of inefficient road allocation of Figure 1. This difference is the profit of the authority (government). Therefore, road capacity can be expanded until the profit equal to zero (until the long-run marginal cost (MC) and average cost(AC) is reached). The difference between MC and AC, which is the optimum congestion toll, is equal to th average fixed cost (AFC) with the efficimt road allocation of Figure 1. This concept is the same as Mills (1984) who suggested to link the optimal congestion toll and the zero profit investment. He said, "If the economically efficient level of road capacity has been built, the optimum congestion toll is just sufficient to cover the cost of the (transportation) facility". In this theroetical study, we will adopt this first best congestion pricing policy.

We will use the well-known BPR congestion function as a mathematical congestion function developed by the Bureau of Public Road (BPR) in 1964. This congestion function assumes that the average travel time is a convex function of the traffic flow quantity divided by road capacity.

$$AC_i = a\{1 + b(\frac{F_i}{K_i})^c\} \tag{13}$$

$$\text{where } K_i = \zeta S_i^{\varphi} \tag{14}$$

where a, b, c, ζ and φ are parameters (b=0.15 and c=4 in BPR), F_i and K_i are the link flows and the road capacity at zone i, respectively. We will assume that road capacity a function of land inputs only by using a Cobb-Douglas production technology with diminishing returns to scale. The marginal cost with the BPR formula is as follows.

$$MC_i = \frac{\partial(AC_iF_i)}{\partial F_i} = a\{1 + b(1+c)(\frac{F_i}{K_i})^c\} \tag{15}$$

The congestion toll is the difference between MC_i and AC_i

$$\tau_i = MC_i - AC_i + abc(\frac{F_i}{K_i})^c$$

Because the units of τ_i are units of time, we need to convert them to monetary units to get the monetary congestion toll. It is assumed that the value of time is the same as the traveler's wage. Because the wages of households depend on their work places, we will use the value of time at a zone as the average value of time of all travelers using the zone. Let's denote the number of trips passing zone i made by households who work in zone j as F_i^j. Then, the monetary conjestion toll (t_i) in zone i is

$$ti = abc(\frac{F_i}{K_i})^c \frac{\Sigma_j F_i^j \ W_j}{F_i} \tag{16}$$

Even though we use the average value of time of all travelers at a zone, the t_i the average increase in the value of time arising from the addition of one more trip which traverses the zone. Therefore, this is the optimum congestion toll which the additional trip has to pay for the social marginal cost.

With efficient road capacity, the optimum congestion tolls are just sufficient to pay for the cost of the road facilities. In our case, they are just sufficient to pay for the land rent of streets. The mathematical form of this equality is as follows.

$$S_i R_i = vF_i t_i \Delta \tag{17}$$

where Δ is the length of a zone. S_i is the quantity of land used for roads in zone i. By substituting (14) and (16) into (17), we can get the efficient amount of road capacity with given flows, F_i.

$$K_i^* = \left\{ \frac{vabc\, F_i^c\, \xi^{1/\rho}\, \Delta\, (\Sigma_j F_i^c W_j)}{R_i} \right\}^{\rho/1+c\rho} \tag{18}$$

Using this K_i^*, the efficient road allocation for each zone can be calculated from (14) and the optimal congestion tolls can be obtained from (16) with given the link flows.

4. Competitive General Equilibrium Model

The models of the household and the firm with efficient road allocation can be combined to obtain a static spatial general equilibrium model which is computable. Suppose N is the total number of households in the urban area and let Ψ_{ij} be the probability that a household chooses to live in zone i and to work in zone j. This probability is given by the logit model (7).

Also, let the houshold's demands for land and the commodities be q_{ij} and Z_{ij}^{rk} which can be calculated by (4) and (5), respectively.

Let the household's supply of labor be $H - T_{ij} - L_{ij}$ which can be calculated by (6). Denoting their dependence on prices only, these functions are $q_{ij}(w_j, R_i)$, $Z_{ij}^{rk}(w_j, P_{rk})$, and $L_{ij}(w_j)$.

On the side of the industry, the demands for labor, and intermediate commodities are given by (9), (10) and (11). Using general notation, these are also denoted as functions of input and output prices faced by industry's output, X_{ij}. Thus, the demand functions are $M_{rj}(w_j, P_{rj}, X_{rj})$, $Q_{rj}(R_j, P_{rj}, X_{rj})$, and $Y_{rj}^{sj}(P_{rj}, P_{sj}, X_{rj})$. Let A_i be the quantity of land in zone i, and let S_i be the quantity of land used for roads in zone i. As we discussed before, we will assume that total transportation congestion toll spent by all households and firms will be used to pay for the land taken up by roads in the city. Let E_{ri} be the given amount of the export (if positive) or import (if negative) of commodity r in zone i. $\bar{P}$, $\bar{R}$, and $\bar{W}$ denote the zonal prices of commodities, the rents and the wages in vector form, respectively. The following set of simultaneous equations defines equilibrium in the land, labor and each commodity markets for each zone. The land market equilibrium condition in each zone i is :

$$N \sum_j \Psi_{ij}(\bar{P}, \bar{R}, \bar{W})\, q_{ij}(R_i, w_j) + \sum_r Q_{ri}(P_{ri}, R_i, X_{ri}) + S_i = A_i \tag{19}$$

The left hand side (LHS) of Equation (19) represens the total demand for land in zone i, and the right hand side (RHS) of it is the available land in zone i. The first term of LHS represents the expected amount of land demanded by the households choosing to reside in zone i. The second and thrid terms are the amount of land demanded by firms and used reserved for streets, respectively. Therefore, the above equation (19) says that the amount of land demanded by the expected number of households residing in zone i, plus by the firms located in zone i, plus the amount of land used for streets in zone i should be equal to the given available amount of land in zone i at equilibrium.

The labor market equilibrium condition in each zone i is :

$$N \sum_j \Psi_{ji}(\bar{P}, \bar{R}, \bar{W}) \{H-T_{ij}-L_{ij}(w_i)\} = \sum_r M_{ri}(P_{ri}, W_i, X_{ri}) \quad (20)$$

The LHS of the above equation (20) represents the expected sulpply of labor by the households willing to work in zone i. The RHS of it gives the quantity of labor demanded by all firms located in zone i. Therefore, the above equation means that the expected supply of labor should be equal to the labor demanded by firms in zone i for the market to clear.

The commodity market equilibrium condition in each zone i and for each commodity r is :

$$N \sum_j \sum_k \Psi_{ij}(\bar{P}, \bar{R}, \bar{W}) Z_{ij}^{rk}(P_{ri}, w_k) + \sum_s \sum_k Y_{sk}^{ri}(P_{ri}, P_{sk}, X_{sk}) + E_{ri} = X_{ri} \quad (21)$$

The LHS of above equation (21) is the consumption demand for commodity r in zone i by all households plus the production demand by all firms as an intermediate good to produce commodity s, plus the export demand. The RHS is the supply of the commodity r produced in zone i. The first term in LHS represents the expected amount of commodity r in zone i demanded by all households. The second term is the amount of commodity r in zone i used as an intermediate good to produce all other commodities by all firms. The third term on the LHS is the given amount of commodity r which is exported to the outside of the city (if positive) or imported from outside (if negative). Hence, The above equation says that the amount of commodity r in zone i demanded by households, firms and by trade should be equal to the amount of it produced in zone i at equilibrium.

The variables in this system of equations are $\bar{P}$, $\bar{R}$, $\bar{W}$, and $\bar{X}$ in vector form. These are $2 \cdot C \cdot J + 2 \cdot J$ in number if there are J zones and C commodities. The number of equations, (19), (20) and (21), are $C \cdot J + 2 \cdot J$. The additional $C \cdot J$ equations are the price relationships given by (12). Note that, in this constant returns to scale model, the number of firms is indeterminate but the total industry output is determined.

There are no fixed Leontieff coeffivients since both consumption and production vary at each location and are sensitive to all prices. Furthermore, the model produces both prices and quantities at equilibrium, unlike an input–output model, which produces only quantities and is inconsistent with respect to prices (see Moses, 1974). The excess demand system can be solved directly for the prices by using (12), (19), (20) and (21). And all quantities can be computed by using (4)–(6) and (9)–(11).

Ⅳ. Algorithm

Instead of solving simultaneously the simultaneous non–linear equations of the equilibrium conditions–(19), (20) and (21)–(12), we block them into three recursive stages–stage one, in which we solve for all of the commodity prices; stage two, in which we solve for all quantities produced and optimum congestion tolls; and stage three in which we equilibrate land and labor markets.

(STAGE 1) Solving For the Prices of Commodities

The fisrt stage is to solve the price relation equations, (12), with given rents ($\bar{R}$) and wages ($\bar{W}$). Implicitly, the price relation equations are reducible into one equation by substitution. This one equation for a commodity is only a function of wages and rents. If the wages and rents are given, we can solve for the price of the commodity. Then, all prices of the other commodities can be found by substitution. But deriving such an equation explicitly is clumsy. Therefore, instead of solving these explicitly, we will solve them interatively by using the following solution algorithm. Below, Ω^{P} is a small and positive tolerance.

(Step 1) Initialize rents ($\bar{R}^0$) and wages ($\bar{W}^0$) and prices of commodities at arbitrary values ($\bar{P}^0$) and congestion toll ($\bar{t}^0$).

(Step 2)Solve the price relation functions (12) for each commodity, plugging ($\bar{R}^0$), ($\bar{W}^0$) and ($\bar{P}^0$) into the right hand side.

Let the new prices of commodities be (P^{*}). Then,

(Step 3)Find $\varepsilon = \max\{\varepsilon_{ri} : \varepsilon_{ri} = |\overset{0}{P}ri - \dot{P}ri|, \forall r, i\}$

(Step 4) If $\varepsilon > \Omega$ then

- $\hat{\overset{0}{P}}ri = (1-\sigma^P)\overset{0}{P}ri + \sigma^P \dot{P}ri$, where $0 < \sigma_P < 1$ (we use $\sigma_P = 1$)
- Redefine $\overset{0}{P}ri = \hat{\overset{0}{P}}ri$

Go to (Step 2).

If $\varepsilon < \Omega$, then, move to the second stage

(STAGE 2) Solving For the Quantities of Production and Optimum Congestion Tolls

The second stage is to solve for the quantities of the commodities produced at each zone from the commodity market equilibrium conditions, (21). The optimal congestion toll and the efficient road allocation (amount of land for streets) will be calculated from the price vector found in STAGE 1 by using (13), (16) and (18). Given $\bar{R}^0$, $\bar{W}^0$, $\bar{P}^0$ and $\bar{t}^0$, the commodity market equilibium conditions become simultaneous linear equations. To see this, we can rewrite the commodity market equilibrium conditions, (21), as :

$$X_{rl} - \sum_{s}\sum_{k} \frac{\phi\, \xi_s\, i_k\; P_{sk}}{P_{rl} + m_r C_{lk}} X_{sk} = N \sum_{j}\sum_{k} \Psi_{jk} Z_{jk}^{rl}$$

By rearranging the above equation,

$$(1-\phi_{il})R_{rl} - \sum \frac{\phi_{il}\, p_{sk}}{p_{rl} + m_r C_{lk}} X_{sk} = N \sum\sum \Psi_{jk} Z_{jk}^{rl} \qquad (22)$$

Given the trial values for all the unknowns except the commodity outputs, the above equations (22) is a standard simultaneous linear equation matrix such as $\bar{A} \cdot \bar{B} = \bar{B}$. We can see that the coefficients on the LHS of (22) are the same as the technical coefficients of an input—output model. Therefore, the technical coefficients of I—O table can be directly used while removing the problems of the I—O model(no substitution and inelasticity with respect to prices). This linear system can be solved by Gaussian elimination. Below, Ω_t is a samll and positive tolerance.

(Step 5) Calculate the quantity of a commodity demanded by households for each product, which is the constant matrix of RHS of (22). Find the coefficient matrix from the LHS of (22).

(Step 6)Solve for $\bar{X}$ by using the Gaussian elimintation technique. Let's denote $\overset{0}{X}ri$ as quantity of a commdity r produced in zone i with given $\bar{R}^0$, $\bar{W}^0$ and $\bar{P}^0$.

(Step 7) Calculate all link flows (commuting trips, shopping trips and commodity flows). Then, solve for the efficient road capacity by using (18), and solve for the efficient road allocation and the new optimal
congestion toll(t') by using (14) and (16), respectively.

(Step 8) $\varepsilon^t = \max\{\varepsilon_i^t : \varepsilon_i^t = |t_i^0 - \dot{t}_i| \ \forall i\}$

If $\varepsilon^t \rangle \Omega^t$, then, set $\bar{t}_i^0 = \dot{t}_i$, and go to (Step 5).

If $\varepsilon^t \rangle \Omega^t$, then, move to the third stage.

(STAGE 3)Sloving For Equilibrum Rents and Wages

Finally, the third stage is to solve for the land and labor market equilibrium conditions. As we discussed in the first and second stages, the prices and quantities of production correspond to give rents and wages, the $\bar{P}^0$ and $\bar{X}^0$– which are calculated in stages 1 and 2– are the equilibrium prices and quantities of production. In the third stage, it is checked whether or not $\bar{R}^0$ and $\bar{W}^0$ satisfy the land and labor market equilibrium conditions. If they do not satisfy them, the $\bar{R}^0$ and $\bar{W}^0$ should be adjusted interatively until the equilibrium conditions are satisfied. Instead of using such an interative method for Stage 3, we can use an algorithm such as MINPACK(modified Newton–Raphson method), directly. Sometimes, the Nwton–Raphson method cannot easily find the solution when the starting points are far from the true solution. This means that MINPACK does not always guarantee a solution. Therefore, we may need an algorithm with always converges to the solution. To solve the land use model, we can use a technique which is similar to the solution algorithm for traffic network equilibrium models(see Sheffi, 1985). We can rewrite the land labor maket equilibrium conditions, (19) and (20), as folllows :

$$R_i' = \frac{N\Sigma_j\Psi_{ij}\beta\,\{W_jH - 2v(C_{ij}W_jg_{ij})\} + \Sigma_r\mu_r P_{ri}X_{ri}}{A_i - S_i} \tag{23}$$

$$W_i' = \frac{\Sigma_r\delta_r P_{ri}X_{ji} - N\Sigma_j\Psi_{ji}\gamma\,\{2v(C_{ji} + W_ig_{ji})\}}{N[\Sigma_j\Psi_{ji}\{(1-\gamma)H - T_{ji}\}]} \tag{24}$$

With given $\bar{R}^0$, $\bar{W}^0$, $\bar{P}^0$ and $\bar{X}^0$, the right hand sides of these equations (23) and (24) can be explicitly evaluated. Let's denote these new rents and wages by $\bar{R}'$ and $\bar{W}'$, respectively. These new rents and wages will be different from $\bar{R}^0$ and $\bar{W}^0$. If the maximum difference is smaller than a give tolerance level, $\bar{R}^0$ and $\bar{W}^0$ are taken to be the equilibrium rents and wages, which is the solution to this general equilibrium problem. If the maximum difference is bigger than the given tolerance level, $\bar{R}^0$ and $\bar{W}^0$ should be adiusted untill they become small enough. The tolerances are Ω^r and Ω^w which are small and positive to terminate those adjustments :

(Step 9)Calculate $\bar{R}'$ with given $\bar{R}^0$, $\bar{W}^0$, $\bar{P}^0\bar{X}^0$byusingequation(23).

Then, $\varepsilon^r = \max_i\{\varepsilon_i^r : \varepsilon_i^r = |R_i' - R_i^0| \ \forall\}$

(Step 10)Calculate $\bar{W}'$ with given $\bar{R}^0$, $\bar{W}^0$, $\bar{P}^0$and$\bar{X}^0$byusingequation(24).

Then, Let $\varepsilon^w = \max_i\{\varepsilon_i^w : \varepsilon_i^w = |W_i' - W_i^0| \ \forall_i\}$

(Step 11) If $\varepsilon^r \rangle \Omega^r$ or $\varepsilon^w \rangle \Omega^w$ then

Set$R_i^0 = \sigma^r R^{i'} + (1-\sigma^r)R_i^0$where $0 \langle \sigma^r \langle 1$(we use $\sigma^r = 1$)

and $W_i^0 = \sigma^w W^{i'} + (1-\sigma^w)\ W_i^{i'}$ where $0 < \sigma^w < 1$ (we use $\sigma^w = 1$)

Go to (Step2) in STAGE1.

If $\epsilon^r < \Omega^r$ and $\sigma^w < \Omega^w$ then stop.

V. Conclusion

In this study, a non−monocentric general equilibrium model with transportation congestion was developed, which is computable and theoretically rigorous. This is the first non−monocentric general equilibrium model which considers heterogeneous behavior of households in choosing their residential location, the work place and shopping destinations. The model is based on discrete choice behavior and incorprates the logit model. All prices of commodities including land rent, wage and optimal congestion tolls are decided at general equilibrium, and the location of firms and households, and efficient road allocations are simultaneously determined by this model's non−monocentric structure. These activities are allocated depending on the equilibrium prices of commodities, wages, rents, and optimal congestion tolls which can maxize consumer's utility and maximize profit of the firm, simultaneously. In addition to a transportation O−D table, link flows and transportation congestion levels, shopping trips and working trips are also separately generated at general equilibrium.

REFERENCES

1. Alonso, William, Location and Land Use, Harvard University, Press, Cambridge, Massachusetts 1964
2. Anas, Alex, Residential Location Markets and Urban Transportation : Economic Theory, Econometrics and Policy Analysis with Discrete choice Models, Academic Press, New York, 1982
3. Anas, Alex, Taste Heterogeneity and Urban Spatial Structure : Monocentric Analysis and the Logit Model Reconciled, Working Paper, To appear in the Journal of Regional Science, University of Illinois at Urbana−Champaign, 1987
4. Anas, Alex and Duann, L.S., Dynamic Forecasting of Travel Demand, Residential Location and Land Development, Papers of the Regional Science Association, vol. 56, pp37−58, 1985
5. Anderson, S. and de Plama, Andre and Thisse, J. F., Analysis of Differentiated Markets with Discrete Choice Models, MIT Press, 1990, forthcoming
6. Dixit, A. and Stiglitz, J., Monopolistic Competition and Optimum Product Diversity, American Economic Review, vol. 67, pp297−308, 1977
7. Hartwick, Philip and Hartwick, John, Efficient Resource Allocation in a Multinucleated City with Intermediate Goods, Quarterly Journal of Economics, vol. 88, pp340−352, 1974
8. Herbert, John and Stevens, Benjamin, A Model for the Distribution of Residential Acivity in Urban Areas, Journal of Regional Science, vol. 2, pp 21−36, 1960
9. Kim, Tschangho John, A Model of Zoning for a Metropolis., Environment and Planning A, vol. 10, pp1035

−1045, 1978

10. Kim, Tschangho John, Alternate Transportation Modes in an Urban Land Use Model : A General Equilibrium Approach., Journal of Urban Economics, vol. 6, pp197−216, 1979

11. Kim, Tschangho John, Modeling the Density Variations of Urban Land Uses with Transportation Network Congestion., Journal of Urban Economics, vol. 19, pp264−276, 1986

12. King, Thomas A., Computing General Equilibrium Prices for Spatial Economices, The Review of Economics and Statistics, vol. 59, pp340−350, 1977

13. King, Thomas A., General Equilibrium with Externalities : A Computational Method and Urban Applications, Journal of Urban Economics, vol. 7, pp84−101, 1980

14. MacKinnon, James, Urban General Equilibrium Models and Simplicial Search Algorithms, Journal of Urban Economics, vol. 1, pp161−183, 1974

15. MacKinnon, James, A Tchnique for the Solution of Spatial Equilibrium Models, Journal of Regional Science, vol. 16, pp293−307, 1976

16. Mills, Edwin S., Transportation and Patterns of Urban Development : An Aggregative Model of Resource Allocation in a Metropolitan Area, American Economic Review, vol. 57, pp197−210, 1967

17. Mills, Edwin S., Markets and Efficient Resource Allocation in Urban Areas, Swedish Journal of Economics vol, 74, pp100−113, 1972

18. Mills, Edwin S., Urban Economics, third, Scott, Foresman and Company, Glenview, Illinois, 1984

19. Moore, James E., Linearized, Optimally Configured Urban Systems Models : A Mills Heritage Model with Replaceable Capital, Stanford University, 1986

20. Moses, Leon N., Outputs and Prices in Interindustry Models, Papers of the Regional Science Association, vol. 32, pp7−18, 1974

21. Rho, Jeong Hyun, Implementation and Evaluation of a Nonlinear Three Dimensional Urban Activity Model, University of Illinois at Urbana Champaign, 1988

22. Sheffi, Yosef, Urban Transportation Networks : Equilibrium Analysis with Mathematical Programming Methods, Prentice−Hell, Inc., 1985

23. Sullivan, Arthur, The General Equilibrium Effects of Congestion Externalities, Journal of Urban Economics, vol. 14, pp80−104, 1983

24. Sullivan, Arthur, The General Equilibrium Effects of the Residential Property Tax : Incidence and Excess Burden, Journal of Urban Economics, vol. 18, pp235−250, 1985

25. Sullivan, Arthur, A general Equilibrium Model with Agglomerative Economies and decentrallized Employment, Journal of Urban Economics, vol. 20, pp55−74, 1986