

Stability Properties of Semiconductor Lasers with Optical Feedback from an External Grating

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We report an analysis on the stability properties of external cavity semiconductor lasers exposed to strong feedback from an external grating. The frequency range of stable single mode oscillation is found to depend on the offset between the resonance frequency of the solitary laser and the frequency of maximum reflection from the grating.

I. INTRODUCTION

The semiconductor lasers have been widely employed in various areas such as optical fiber communications^[1], optical fiber sensors^[2], pumping of solid state lasers^[3], high resolution spectroscopy^[4]. However, the typical linewidth of semiconductor lasers is too broad for their direct applications and many methods have been devised to reduce the laser linewidth. Optical feedback method has been commonly used to narrow the laser linewidth, due to its simple construction of experimental system. Typically, external optical feedback can be accomplished using plane mirror^[5], grating^[6], confocal resonator^[7]. The optical feedback from grating and confocal resonator is of particular interest due to its frequency selectivity and tunability. The frequency tunability is quite important for some applications such as optical communication^[1], atomic spectroscopy^[4]. Recently, frequency tunability over the gain bandwidth of semiconductor laser has been demonstrated while the linewidth of semiconductor lasers have been narrowed upto 4 kHz by using strong optical feedback^[5,6]. The tuning characteristics of semiconductor laser with optical feedback from a grating reflector has been studied by Binder et al.^[8] but the dynamical stability properties have not been discussed in detail and this paper may be considered as an extension of

their work.

In this paper, we report the results of dynamical stability analysis for the grating feedback system and demonstrate that the frequency range of stable single mode oscillation depends upon the frequency offset between the resonance frequency of solitary laser and the frequency of maximum reflection.

II. THEORY

The rate equation for the complex electric field $E(t)$, which is normalized such that $|E(t)|^2$ is the photon density, is given by^[9,10]

$$\frac{dE(t)}{dt} = [i(\omega - \omega_M^0) + 0.5\Delta G(1 - i\alpha) \times \frac{1}{\tau_m} \ln\left(\frac{r_{off}}{r_2}\right)]E(t) \quad (1)$$

where ω and ω_M^0 are the optical angular frequencies with and without feedback, respectively. ΔG is the feedback-induced change in the temporal gain G , which is assumed to be a linear function of the carrier density $N(t)$: $G(N) = G_N(N - N_0)$, with G_N the gain slope $\partial G / \partial N$ and N_0 the transparency carrier density. α is the linewidth enhancement factor, τ in is the diode cavity round-trip time and r_2 is the reflection coefficient.

cient of the laser facet facing the external grating. The effective reflection coefficient $r_{eff}(\omega)$ for the combination of the AR-coated facet and the grating is given by^[8,11]

$$r_{eff}(\omega) = \frac{r_2 - r_{\alpha}(\omega)\exp(i\Phi(\omega))}{1 - r_2 r_{\alpha}(\omega)\exp(i\Phi(\omega))} \quad (2)$$

where $r_{\alpha}(\omega)$ is the reflection coefficient of the grating, $\Phi(\omega) = \omega\tau$ is the round-trip phase shift in the external cavity length L_{ex} ($\tau = L_{ex}/c$, c : velocity of light in vacuum). When the product $r_2 r_{\alpha}$ is small compared to 1, Eqn. (2) can be approximated to be

$$r_{eff}(\omega) \cong r_2 - (1 - r_2^2)r_{\alpha}(\omega)\exp(i\Phi(\omega)) - (1 - r_2^2)r_2 r_{\alpha}^2(\omega)\exp(2i\Phi(\omega)) \quad (3)$$

As in [8], we assume $r_{\alpha}(\omega)$ to be a Gaussian function

$$r_{\alpha}(\omega) = r_G \exp\{-\ln 2(\omega - \omega_G)^2 / (\Delta\omega_G)^2\} \quad (4)$$

where ω_G is the frequency of maximum reflection (amplitude r_G) from the grating and $\Delta\omega_G$ the full width at half-maximum (FWHM) of the spectral amplitude response of the grating.

The rate equation for the carrier density $N(t)$ is written as

$$\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - G(N) |E(t)|^2 \quad (5)$$

where J is the carrier injection rate per unit volume and τ_s is the carrier lifetime.

The frequency of the longitudinal modes of the compound-cavity laser are the solutions to the phase condition for the semiconductor cavity with the two facet reflection coefficient r_1 and $r_{eff}(\omega)$ ^[8]

$$\phi_n(\omega, g) + \phi(\omega) = 2m\pi \quad (6)$$

where $\phi(\omega)$ is the phase of the complex effective reflection coefficient $r_{eff}(\omega)$ and m is an integer. The phase shift within the laser cavity $\phi_n(\omega)$ can be expressed in terms of detuning of ω from the M -th longitudinal mode of the semiconductor cavity $\omega_M(g)$ by

$$\phi_n(\omega, g) = (\omega - \omega_M(g))\tau_m \quad (7)$$

where g is the gain. If ω_M^0 is the longitudinal mode

of the solitary laser with gain profile g^0 , $\omega_M(g)$ is given by

$$\omega_M(g) = \omega_M^0 + 0.5\alpha v_g \Delta g \quad (8)$$

where v_g is the group velocity inside the active medium and $\Delta g = g - g^0 = \Delta G/v_g$. From Eqn. (6)-(8), the following phase condition can be obtained:^[8]

$$\omega - \omega_M^0 = \{\alpha L_{in} \Delta g - \phi_c(\omega)\} / \tau_m \quad (9)$$

where L_{in} is the cavity length of diode laser ($L_{in} = v_g \tau_m$). The loss $\alpha(\omega)$ of the compound cavity mode can be found from the unity gain condition over one round trip;

$$\alpha(\omega) = -(1/L_{in}) \ln(r_1 |r_{eff}(\omega)|) \quad (10)$$

where r_1 is the reflection coefficient of the uncoated laser facet. The condition for the oscillation of the solitary laser is $g^0 = \alpha_{sol}$, where α_{sol} is the loss of the solitary laser calculated from (10) by replacing $|r_{eff}(\omega)|$ with r_2 . Eqns. (9) and (10) can also be obtained from the steady state solution of Eqn. (1).

The dynamical stability of the external feedback system can be determined by linearizing the rate equations about the steady-state solutions. Then the system determinant $D(z)$ can be found from the Laplace transformed rate equations to be^[9,10]

$$D(z) = (z + 1/\tau_R)[(z - \xi_c + \eta f_s)^2 + (\xi_s + \eta f_c)^2] + \omega_R^2 [z + \eta f_s - \xi_c + \alpha(\xi_s + \eta f_c)] \quad (11)$$

with

$$f_c = \frac{1}{\tau_m} \sum_{m=1}^2 \cos(m\omega\tau) c_m [1 - \exp(-mz\tau)]$$

$$f_s = \frac{1}{\tau_m} \sum_{m=1}^2 \sin(m\omega\tau) c_m [1 - \exp(-mz\tau)]$$

$$\frac{1}{\tau_R} = \frac{1}{\tau_s} + G_N E_{st}^2, \quad \omega_R^2 = G_N G(N_{st}) E_{st}^2$$

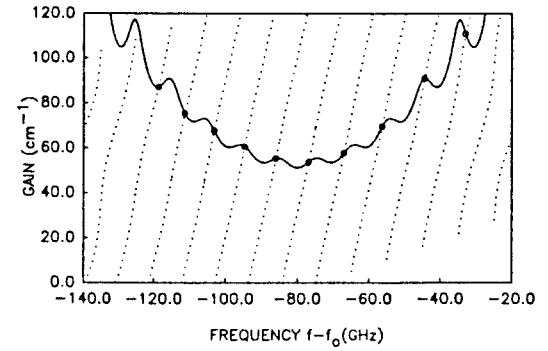
where E_{st} and N_{st} are the respective steady-state values of the electric-field amplitude and the carrier density, ξ and η are the real and imaginary parts of the steady-state value of r_2/r_{eff} , respectively.

III. RESULTS AND DISCUSSIONS

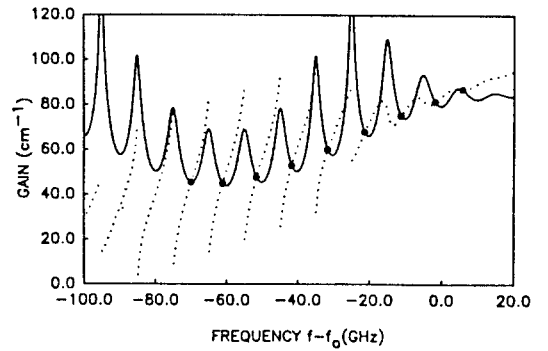
We assume the FWHM of the spectral response of the grating to be $\Delta\omega_g = 2\pi \times 30$ GHz and used the following external feedback parameters: $L_{\alpha} = 3$ cm ($\tau = 0.1$ ns), $r_1^2 = 32\%$. To study the dynamical stability of the system, the secular equation $D(z) = 0$ is solved numerically under various conditions using the following laser parameters: $G_N = 1.1 \times 10^{-12}$ m³sec⁻¹, $N_0 = 1.1 \times 10^{24}$ m⁻³, $\tau_s = 2$ ns, $\tau_p = 2$ ps, $\tau_n = 8$ ns, and $\alpha = 3$. The stability of the system is determined by checking whether the root of secular equation has positive real part (unstable) or all of roots have negative real parts (stable). The imaginary part of the root of secular equation gives the characteristic oscillation frequency.

From the intersections of constant phase curve and loss curve at fixed value of ω_c , we can find compound cavity modes ω to determine feedback phase $\omega\tau$ in f_c and f_s used in Eq. (11). Fig. 1(a)-(c) show the computed loss and constant phase curves at the various reflectivities of laser facet and the various frequency detuning of maximum reflection of the external grating semiconductor laser. The stable compound cavity modes are marked with filled circles while the unstable modes are indicated with open circles. If the reflectivity of laser facet is low, there can be many stable compound cavity modes which can oscillate together. However, the number of stable modes reduces to one or two with increasing facet reflectivity.

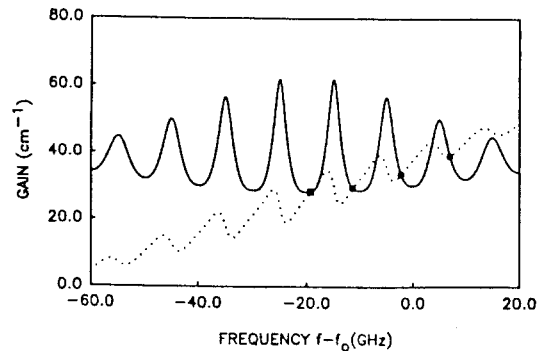
Changing the external cavity length on the order of wavelength results in the shift of the oscillation frequency. The effect of cavity length detuning can be analyzed by changing the feedback phase $\omega\tau$ to $\omega\tau + \delta$ in Eqns. (2) and (3) and by finding the compound cavity modes. Change of the feedback phase results in the shift of loss curve and constant phase curve simultaneously. Fig. 2(a)-(c) show the locus of the compound cavity modes and their chirp reduction factor $F(=d\omega_M^0/d\omega)$ obtained at various detuning. The same parameters as in Fig. 1 are used in Fig. 2. The chirp reduction factor is calculated using the formular in Ref. [8] and relatively low value of it (0-30) has been obtained. The stable modes are shown with closed circles while the unstable modes are shown with open circles. By exa-



(a)

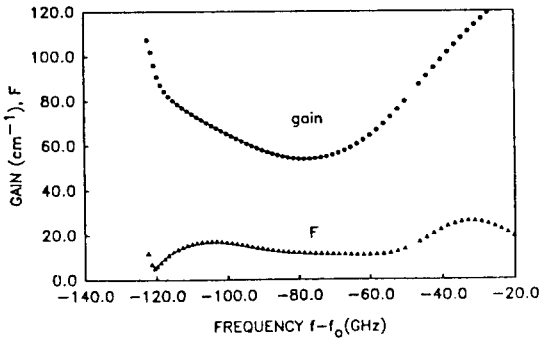


(b)

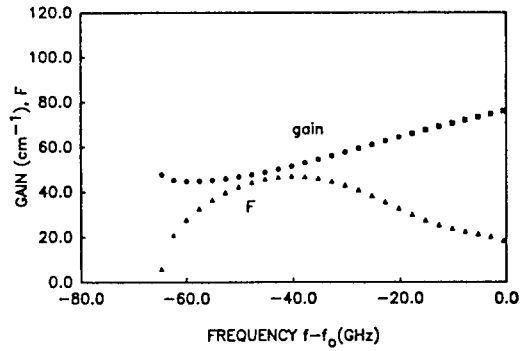


(c)

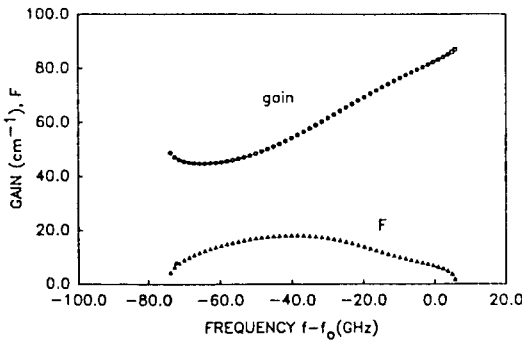
Fig. 1. Computed loss (solid line) and constant phase (dotted lines) curves for an external grating laser with $r_g = 13\%$ and (a) $r_2^2 = 0.05\%$, $\Delta f_G = -80$ GHz, (b) $r_2^2 = 2\%$, $\Delta f_G = -60$ GHz, (c) $r_2^2 = 32\%$, $\Delta f_G = -20$ GHz. The stable and unstable modes are indicated with closed circles and open circles, respectively.



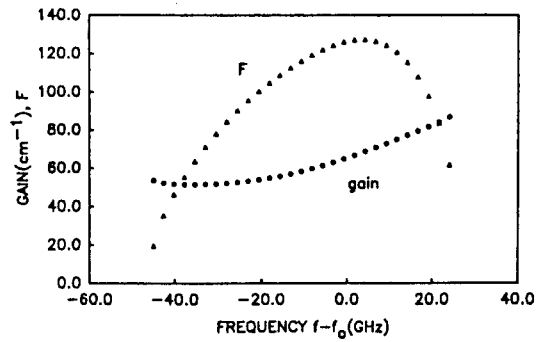
(a)



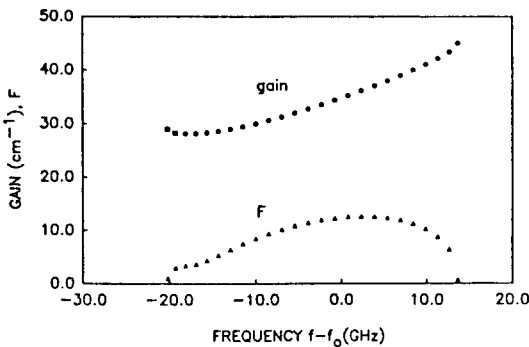
(a)



(b)



(b)



(c)

Fig. 2. The locus of compound cavity modes and chirp reduction factor at the same conditions of Fig. 1, respectively. The stable modes and unstable modes are shown which closed circles and open circles, respectively. The unstable modes due to the undamping of relaxation oscillation are marked with open squares.

Fig. 3. The locus of compound cavity modes and chirp reduction factor with laser facet reflectivity $r_2^2 = 3\%$, $r_g^2 = 10\%$, $\tau = 0.4$ ns and (a) $\Delta f_G = -60$ GHz, (b) $\Delta f_G = -20$ GHz. The same convention as in Fig. 2 is used to indicate the stability of a mode.

mining the imaginary part of the root of the secular equation, we also found that the mode can be unstable due to the existence of nearby stable modes or due to undamping of relaxation oscillations. The unstable modes belonging to the latter case is marked by open squares in Fig. 2(b) and (c). They have a characteristic frequency corresponding to the relaxation oscillation frequency.

Fig. 3(a) and (b) show the locus of compound cavity modes at relatively long external cavity lengths. The mode with lowest threshold gain is stable with proper frequency offset as shown in Fig. 3(a), while if is uns-

table with improper frequency offset as can be seen in Fig. 3(b). It may be noticed that the stable operation range depends on frequency offset as well as on the external feedback parameters. The attainable maximum chirp reduction factor also depends on the frequency offset and the range of that factor is limited by the stability of the mode. The external optical feedback from a grating has some limitation as shown above due to relatively broad frequency response of the grating. Even though the number of modes is greatly reduced with grating feedback compared to that with plane mirror, additional narrow range frequency selective optical element may be necessary to have a single compound cavity mode oscillation.

IV. CONCLUSIONS

We have analyzed the dynamic stability of semiconductor lasers with strong optical feedback from a grating and found that the dynamically stable frequency range of single mode oscillation depends on the frequency offset between the frequency of solitary laser and the frequency of maximum reflection. We have determined the stable operation range at various laser facet reflectivity and frequency detuning. It is observed that the attainable chirp reduction factor depends on the frequency offset. We also observed that the instability of some modes can be caused by the existence of nearby stable modes or undamping of relaxation oscillations. It may be concluded that dynamic stability analysis is necessary to estimate the performance of external cavity semiconductor lasers.

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