Antagonistic Stiffness Characteristics in Robotic Linkage Systems

로보틱 시스템에 존재하는 antagonistic stiffness 특성

Byung-Ju Yi* Sang Kee Song** Whang Cho***

이 병 주* 송 상 기** 조 황***

Abstract

This work investigates the antagonistic stiffness properties inherent in the coordination of general robotic systems such as in maintenance of static equilibrium (e.g., given posture) of nonlinearly constrained mechanisms through redundant actuation. Such antagonistic situations occur in normal operational modes of many robotic systems including coordinations of multiple manipulators, multi-fingered end-effectors, and walking machines, as well as in the human body. Stiffness due to antagonism is shown to be a good means of characterizing properties of these sort of systems. The concept of antagonistic stiffness is distinguished from that of structural stiffness intrinsic in deformable(i.e., non--rigid) body systems because here no deformations of bodies are involved in the stiffness generation but only relative rigid body displacements. This stiffness, therefore, may be interpreted as the system's effective stiffness due to iput redundancy, and can be utilized as a measure of open-loop stability for generally equilibrated robotic mechanisms, as well as for generation of an active(and therefore controllable) nonlinear spring by preloading the system. In this paper, conditions for full, active stiffness generation and open-loop stability of several models with antagonism are investigated from both analytic and purely geometric points of veiw.

요 약

본 논문은 시스템의 운동학적 자유도보다 많은 수의 input을 사용하여 비선형 구속조건을 갖는 메카니즘의 정역학적 평형(예로써, 주어진 자세)을 유지시키는 경우와 같이 일반 로보트 시스템의 협력 작업시 일어나는 antagonistic stiffness를 연구하였다. 이러한 antagonistic 상황은 coordinations of multiple manipulators, multi-fingered end-efector, walking machine, 그리고 인간의 움직임등을 포함하는 많은 로보트 시스템의 작동시에 일어난다. Antagonism으로 야기되는 stiffness 는 이러한 시스템의 특성을 파악하는 좋은 척도가 될 수 있다. Antagonistic stiffness의 개념은 시스템을 구성하는 강체들의 상대 변위의 함수로 얻어지기 때문에 비강체들이 변형하는 특성을 나타내는 structural stiffness와는 구별된다. 따라서 이 개념은 여유입력들에 의해 얻어지는 시스템의 effective stiffness로 해석될 수 있고 일반 로보트 mecha-

* Department of Mechanical Engineering
THE UNIVERSITY OF TEXAS AUSTIN AT AUSTIN Graduate Research Assistant
** Department of Mechanical Engineering
Inha University, Graduate Research Assistant
*** Department of Control and Measurement
Kwangwoon University, Assistant Professor

nism의 개경로(open-loop) 안정도의 척도로 이용될 수 있으며 목적에 따라서 stiffness의 제어가 가능한 비선형 spring을 만드는 데에도 응용이 가능하다. 본 논문에서는 antagonism이 일어나는 몇가지 상황에서의 stiffness 특성과 개경로 안정성 조건등을 해석적, 기하학적 관점에서 다루었다.

I. Introduction

In the coordination of general dynamic systems, we can observe many dynamic systems that sustain statically equilibrated, and internally or externally constrained, states. In these states, the system becomes a single overconstrained mechanism. This results in an antagonistic situation, which generates a local effective system stiffness. The effective stiffness is important in that it is a generic system property in tasks where forces and motions are imparted to an object, and it is primarily a geometric property of the system as a whole.

There are a limited number of works that deal with antagonism. One of the early current investigation was maded by Hogan [1982]. He modeled equilibrated axes consisting of several passive springs and discussed the concept of open-loop disturbance rejection and nonlinear spring design. Tong and Somerset [1985] suggested a simple active control of single axis using two antagonistic actuations. Jacobson illustrated push and pull type antagonistic fingers. Benedict and Tesar [1978] implicitly addressed the antagonistic property generated from preloaded external springs. Cutkosky and Wright [1985] designed an active wrist, which has a parallel mechanism structure composed of four bars and utilizes the idea of antagonistic preloading. Note that this, so called, "antagonistic stiffness", plays an important role when estimating a system's open-loop stability. In addtion, an active nonlinear spring will be created by antagonistically preloading a mechanism with redundant actuation.

Hanafusa and Asada [1981] studied system stiffness and stability of a three-fingered robot hand in two dimensions in which a potential function for describing a stable grasp is derived. In this early analysis, they assume that the fingers are made of linear springs with a single degree of freedom, and friction is ignored. Fearing [1986] proposed a method for stable planar grasping of two dimensional polygons. Jameson and Leifer [1986] also studied the stability of a frictional point model and stability of a soft-finger contact model. Nakamura, et. al. [1988] defined object stability and contact stability for the dynamic coordination of a multi-fingered system. Tehy defined object stability as the ability of a system to return to the nominal position against positional errors, and contact stability as the ability of a system to maintain contact without slip in the presence of external disturbing forces. Most previous stability analyses concerning grasping have concentrated on object or contact stability. However, the stability factor due to finger configuration has not been widely considered. Also, note that the antagonistic stiffness due to manipulator (finger) configuration has been omitted in the derivation of the effective robot task space stiffness of statically equilibrated systems, such as in force control [West and Asada, 1985] and in grasping analysis [Kao and Cutkosky, 1989]. In this paper the antagonisitic(open-loop) stiffness property that is inherent in any statically, dynamically, or internally constrained robotic mechanism is taken into account.

Nguyen [1987] addresses force closure in three-dimensional objects grasped by virtual springs corresponding to force components transmitted at the finger tips. He proposed a least square solution for specifying the stiffness of the virtual springs to obtain a desired grasp stiffness. However, the

finger geometry and the static load relationship between the virtual springs and the finger joints are not considered. Kao and Cutkosky [1989] express the compliance of a grasp as a function of grasp geometry, contact conditions(including general friction conditions) between the fingers and the grasped object, and the mechanical properties of the fingers. They also study the reverse problem of determining servo gains at the joints of a robotic hand required to achieve a desired overall compliance. In this paper, the concept of active nonlinear spring generation by preloading the system will be addressed from a geometric point of view. Design of active RCC devices was proposed as a prospective application of active nonlinear spring generation [Yi, et. al, 1989].

The purpose of this work is the theoretical investigation of antagonism and its application to physical systems. This understanding should allow for the enhancement of operational performance in internally or externally constrained linkage systems. The organization of this paper is as follows. Initially, we illustrate basic kinematic modeling approches for open-chain and closed-chain systems. Next, the generation of an active nonlinear spring and conditions for full stiffness are addressed in terms of the nonlinear nature of the kinematic constraints in closed-chain mechanisms. Next, several simple mechanisms with antagoinism are illustrated, with emphasis on the relationship between the manipulator configuration and the internal loading mode. One bar and four bar mechanisms under antagonistic internal loading are shown to illustrate the basic stability concept from a purely geometric point of view. The concept is based on actively generated local stiffness and is applied further to stability analysis of dual arms and multi-fingered hands. Finally, conclusions and suggestions for future work are given.

II. Kinematic Formulation

2.1 Open-Chain Kinematics

Here, only the general methodology and result format of the higher-order kinematics is given. The problem of position analysis is not addressable (except in an iterative, or differential displacement sense) by the given procedure. Adopting the standard Jacobian $[G_{\phi}^{u}]$ representation for the velocity of a vector of P dependent (output) parameters u in terms of a set of M independent input coordinates ϕ, one has

$$\dot{u} = [G_{\phi}^{u}]\,\dot{\phi}. \tag{2-1}$$

Here

$$[G_{\phi}^{u}] = [\,\frac{\partial u}{\partial \phi_1}\ \frac{\partial u}{\partial \phi_2}\ \cdots\ \frac{\partial u}{\partial \phi_M}\,] = [\,g_1^{u}\ g_2^{u}\ \cdots\ g_M^{u}\,] \tag{2-2}$$

is the Jacobian relating the coordinates u and ϕ, with the n th column g_n^u being of dimension Px1 . Having stated the first-order kinematics in a fairly common form, the second-order kinematics are presented in a less common from. Here, a particular matrix formulation is chosen in which the non-linear, velocity related components are expressed in terms of a three-dimensional coefficient array $[H_{\phi\phi}^{u}]$, (consisting of position dependent second-order partial derivatives) operated on quadratically in a "plane by plane" sense. Generally, the acceleration vector $\ddot{u}$ of a set of P dependent parameters u is represented in terms of the M independent coordinates ϕ as

$$\ddot{u} = [G_{\phi}^{u}]\,\ddot{\phi} + \dot{\phi}^{T}\,[H_{\phi\phi}^{u}]\,\dot{\phi} \tag{2-3}$$

where the second-order influence coefficient array $[H^{u}_{\phi\phi}]$, with the dimension of PxMxM, is defined as

$$([\dot{G}^{u}_{\phi}])\,\dot{\phi} = \dot{\phi}^{T}\,[H^{u}_{\phi\phi}]\,\dot{\phi} = \begin{pmatrix} \dot{\phi}^{T}\,[{}^{1}H^{u}_{\phi\phi}]\,\dot{\phi} \\ \dot{\phi}^{T}\,[{}^{2}H^{u}_{\phi\phi}]\,\dot{\phi} \\ \vdots \\ \dot{\phi}^{T}\,[{}^{P}H^{u}_{\phi\phi}]\,\dot{\phi} \end{pmatrix} \qquad (2\text{-}4)$$

where the ith plane of $[H^{u}_{\phi\phi i}]$ is

$$[{}^{i}H^{u}_{\phi\phi}] = \frac{\partial^{2} u^{i}}{\partial\phi\,\partial\phi} \;:\; \text{MxM matrix.}$$

In most kinematic papers, $[G^{u}_{\phi}]$ has been used instead of $\dot{\phi}^{T}[H^{u}_{\phi\phi}]$, but the second order purely geometric property $[H^{u}_{\phi\phi}]$ is useful when geometric interpretation is needed such as in the determination of existence, and conditions for, degenerate singularity [Burdick, 1988], and in the definition of antagonistic stiffness herein.

The inverse kinematics problem determining the relative joint velocities $\dot{\phi}$ and acceleration $\ddot{\phi}$ is obtained by an isomorphic transformation. Provided the mechanism is not singular and P is equal to M (square Jacobian), the joint speeds are

$$\dot{\phi} = [G^{\phi}_{u}]\,\dot{u} \qquad (2\text{-}5)$$

where

$$[G^{\phi}_{u}] = \frac{\partial\phi}{\partial u} = [G^{u}_{\phi}]^{-1}. \qquad (2\text{-}6)$$

The joint accelerations are

$$\ddot{\phi} = [G^{\phi}_{u}]\,\ddot{u} + \dot{u}^{T}\,[H^{\phi}_{uu}]\,\dot{u} \qquad (2\text{-}7)$$

Where using the generalized scalar (tensor) product (0) (see Appendix[1])

$$[H^{\phi}_{uu}] = \frac{\partial^{2}\phi}{\partial u^{2}} = -\,[G^{u}_{\phi}]^{-T}\,([G^{u}_{\phi}]^{-1} \circ [H^{u}_{\phi\phi}])\,[G^{u}_{\phi}]^{-1}. \qquad (2\text{-}8)$$

The transfer methodology (Eq.(2-6) and Eq.(2-8)) will be used later when dealing with antagonism from external force(Section 3.2).

2.2 Closed-Chain Kinematics

The methodology is illustrated here in terms of R, M-DOF arms manipulating a common object in an N-dimensional space. Assuming the Jacobians $[{}_{r}G^{u}_{\phi}]$ relating the common object motion parameters ($\dot{u}$) to the relative joint parameters (${}_{r}\phi$, r= 1, 2,⋯, R) of the R manipulators are non-singular, the first step is to determine the total system model in terms of the common object (pseudo) coordinate set (u). To accomplish this, obtain the joint space model (${}_{r}S_{\phi}$) of each manipulator, using the forward kinematics (and kinetic equations), as if it were isolated (independent) from the rest of the arms.

$$({}_{r}S_{\phi}) = [{}_{r}G^{u}_{\phi}],\,[{}_{r}H^{u}_{\phi\phi}]. \qquad (2\text{-}9)$$

The transferance of each chain (Eq.(2-6) and Eq.(2-8)) is not employed here since in the general case, where P is not equal to M, the use of the Jacobian inverse (pseudo-inverse) does not yield the generic solution of the system. However, the open chain model(s) (Eq.(2-9)) will be utilized to formulate the forward kinematics in closed-chain

mechanisms (Eq.(2-19)).

Based on the pervious open-chain kinematics, the following discusses the closed-chain kinematics of general parallel mechanisms. Assuming that the end-effector of each arm rigidly grasps the object, the system can be viewed as a multi-loop parallel mechanism. There are numerous distinct operational subcases of this general situation depending, for instance, on whether M=N, M>N, or M<N. Here, a system is assumed to have general closed kinematic chains. The number (W) of independent loops is represented by the following formula

$$W = J - L + 1 \tag{2-10}$$

Where J and L denote mumber of joints and links, respectively. The number(C) of holonomic constraint equations is

$$C = W \times Q \tag{2-11}$$

where Q is 3 and 6 for planar and spatial mechanisms, respectively.

The holonomic constraint equations will be expressed in terms of system Lagrangian coordinates, or sets of independent(ϕ_a) and dependent (ϕ_p) coordinates, as follows.

$$f(\phi) = f(\phi_a, \phi_p) = 0. \tag{2-12}$$

The first order kinematic influence coefficients (KIC), which relate the dependent coordinates to the independent coordinates, can be obtained by total differentiation of Eq.(2-12)

$$[G_a^f]\dot{\phi}_a + [G_p^f]\dot{\phi}_p = 0 \tag{2-13}$$

where

$$[G_a^f] = [\frac{\partial f}{\partial \phi_a}], [G_p^f] = [\frac{\partial f}{\partial \phi_p}].$$

$[G_a^f]$ is a $C \times N_a$ matrix whose i^{th} row and j^{th} column element is $\partial f_i / \partial \phi_{aj}$ and $[G_p^f]$ is a $C \times N_p$ matrix with $\partial f_i / \partial \phi_{pj}$ as its ith row and j^{th} column element. Here, N_a and N_p are the number of independent and dependent coordinates, respectively. Proceeding further by solving Eq.(2-13) for ϕ_p,

$$\dot{\phi}_p = -[G_p^f]^{-1}[G_a^f]\dot{\phi}_a \tag{2-14}$$

where the nonsingularity of matrix $[G_p^f]$ is assumed. Now we define the first order KIC matrix of a closed loop system as

$$[G_a^p] = -[G_p^f]^{-1}[G_a^f] \tag{2-15}$$

where p and a imply ϕ_p and ϕ_a, respectively. Using this definition, Eq.(2-14) can be written as

$$\dot{\phi}_p = [G_a^p]\dot{\phi}_a \tag{2-16}$$

which relates the system's independent joints to the dependent joints. Since the joints ($_r\phi$) of the rth chain are composed of some of the independent and dependent joints, $_r\phi$ can be expressed in terms of the total system's active joints by decomposing Eq. (2-16) as follows

$$_r\dot{\phi} = [^rG_a^\phi]\dot{\phi}_a \tag{2-17}$$

where an augmented matrix $[^rG_a^\phi]$ is formed according to the order of independent and dependent joints in the rth chain. Now, the forward kinematics for the common object space is obtained from any of the open-chain (r^{th} chain) kinematic relations as follows

$$\dot{u} = [{}_rG^u_\phi]\,{}_r\dot{\phi} = [G^u_a]\,\dot{\phi}_a \tag{2-18}$$

where

$$[G^u_a] = [{}_rG^u_\phi]\,[{}^rG^\phi_a] \tag{2-19}$$

Evaluation of the second order influence matrices $[H^p_{aa}]$ is also straightforward. If we differentiate Eq. (2-13) with respect to time again, the following equation is obtained

$$[G^f_a]\,\ddot{\phi}_a + \dot{\phi}_a{}^T[H^f_{aa}]\,\dot{\phi}_a + \dot{\phi}_p{}^T[H^f_{pa}]\,\dot{\phi}_a + \dot{\phi}_a{}^T[H^f_{ap}]\,\dot{\phi}_p + [G^f_p]\,\ddot{\phi}_p + \dot{\phi}_p{}^T[H^f_{pp}]\,\dot{\phi}_p = 0 \tag{2-20}$$

where, the bilinear operators introduced in Eq.(2-20) are defined as follows

$$[H^f_{aa}]_{kij} = [\frac{\partial^2 f}{\partial\phi_a\partial\phi_a}]_{kij} = \frac{\partial^2 f_k}{\partial a_i\partial a_j}$$

$$[H^f_{pa}]_{kij} = [\frac{\partial^2 f}{\partial\phi_p\partial\phi_a}]_{kij} = \frac{\partial^2 f_k}{\partial p_i\partial a_j}$$

$$[H^f_{ap}]_{kij} = [\frac{\partial^2 f}{\partial\phi_a\partial\phi_p}]_{kij} = \frac{\partial^2 f_k}{\partial a_i\partial p_j}$$

$$[H^f_{pp}]_{kij} = [\frac{\partial^2 f}{\partial\phi_p\partial\phi_p}]_{kij} = \frac{\partial^2 f_k}{\partial p_i\partial p_j}$$

Now, the acceleration of the dependent parameters is expressed as

$$\ddot{\phi}_p = [G^p_a]\,\ddot{\phi}_a + \dot{\phi}_a{}^T[H^p_{aa}]\,\dot{\phi}_a \tag{2-21}$$

where

$$[H^p_{aa}] = [\frac{\partial^2\phi_p}{\partial\phi_a\partial\phi_a}]$$

$$= -[G^f_p]^{-1}\circ([H^f_{aa}] + [H^f_{aa}]^* + [H^f_{aa}]^{**}) \tag{2-22}$$

with

$$[H^f_{aa}]^* = [G^p_a]^T[H^f_{pp}]\,[G^p_a] \tag{2-23}$$

$$[H^f_{aa}]^{**} = ([G^p_a]^T[H^f_{pa}] + [H^f_{ap}]\,[G^p_a]). \tag{2-24}$$

Here, the second order of geometric properties $[H^f_{aa}]$ and $[H^f_{pp}]$ are symmetric in holonomic systems. $[H^f_{aa}]^*$ is still symmetric after the plane by plane congruence transformation between$[G^p_a]$ and each plane of $[H^f_{pp}]$. $[H^f_{aa}]^{**}$ is also symmetric since $[G^p_a]^T\,[H^f_{pa}]$ is the transpose of $[H^f_{ap}]\,[G^p_a]$. Therefore, the left hand-side is symmetric.

That is, each plane of $[H^p_{aa}]$ is a symmetric matrix. This is consistent with the fact that $[H^p_{aa}]$ is the generalization of the Hessian matrix defined for a single function of several variables (see Cho, et. al, for detailed derivation of these KIC). By the same augmentation method implied by Eq.(2-17), the second order forward kinematics can be easily obtained.

Now, an alternate expression of Eq.(2-22) is

$$[H_{paa}] = -[G^f_p]^{-1}([H_{faa}] + [H_{faa}]^* + [H_{faa}]^{**}) \tag{2-25}$$

$$= [\,{}_1h^p_{aa}\ {}_2h^p_{aa}\ {}_3h^p_{aa}\ {}_4h^p_{aa}\ {}_5h^p_{aa}\ {}_6h^p_{aa}\,] \tag{2-26}$$

with

$$[H_{faa}] = [\,{}_1h^f_{aa}\ {}_2h^f_{aa}\ {}_3h^f_{aa}\ {}_4h^f_{aa}\ {}_5h^f_{aa}\ {}_6h^f_{aa}\,] \tag{2-27}$$

$$[H_{faa}]^* = [\,({}_1h^f_{aa})^*\ ({}_2h^f_{aa})^*\ ({}_3h^f_{aa})^*\ ({}_4h^f_{aa})^*\ ({}_5h^f_{aa})^*\ ({}_6h^f_{aa})^*\,] \tag{2-28}$$

$$[H_{faa}]^{**} = [\,({}_1h^f_{aa})^{**}\ ({}_2h^f_{aa})^{**}\ ({}_3h^f_{aa})^{**}\ ({}_4h^f_{aa})^{**}\ ({}_5h^f_{aa})^{**}\ ({}_6h^f_{aa})^{**}\,] \qquad (2\text{-}29)$$

where the matrices (Eq.(2-25)) to Eq. (2-29)) are the collections of the upper triangular elements of the three dimensional matrices in Eq.(2-22), respectively. For example, each column of $[H_{paa}]$ is shown graphically (Fig.1) below in terms of $[H^P_{aa}]$.

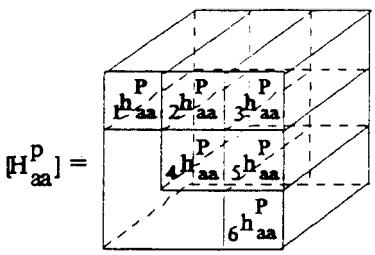


Figure 1

The other matrices are formed in the same manner. It is important to note that lineardependence of the columns of $[H_{paa}]$, $[H_{faa}]^*$, and $[H_{faa}]^{**}$, which implies linear dependence of the planes of the three dimensional matrices, influences the rank of the matrix $[H_{paa}]$. The above open-chain and closed-chain kinematics will be utilized in the following analysis of antagonistic properties.

Ⅲ. The Concept of Antagonism and Its Application

For a system in static equilibrium with another, or with itself, there exists a resistive action between the systems, or within the system. This phenomena is called "antagonism". In the human body, antagonism is defined as opposition in physiological action or active opposition. For example, human arms consist of 29 muscles, showing redundancy in actuation compared to seven joint space and six operational space freedoms. When one needs to hold a heavy object, or respond to external disturbances promptly, antagonism through the redundant actuation of muscles strengthens the system. This action, which is interpreted as a local open--loop control, actually increases system stiffness and simultaneously distributes the required generalized muscle loads.

It will be shown that this phenomenon is inherent in general mechanisms in static equilibrium. In the following, the antagonistic property is characterized and utilized to generate an adjustable nonlinear spring by active preloading, and to measure the system's open-loop stability. System geometry (configuration) is emphasized in the analysis of the system's behavior under antagonism.

3.1 Active Nonlinear Spring Generation using Redundant Actuation of Closed-Chain Parallel Mechanisms

Nguyen[1987] addresses the force closure (stiffness generation) problem in three-dimensional objects grasped by virtual springs corresponding to force components transmitted at the finger tips. He proposed a least square solution for specifying the stiffness of the virtual springs to obtain a desired grasp stiffness. However, he does not illustrate the geometry and the static load relationship between the virtual springs and the finger joints. Here, the active nonlinear spring generation of general closed-chain mechanisms will be addressed with regard to manipulator geometry and redundant actuation, in a purely open-loop fashion, and the conditions for complete stiffness generation will be analyzed from a geometric point of view.

In general constrained parallel mechanisms with redundant actuation, antagonism will be found. Examples of such mechanisms are multiple manipulators, multi-fingered hands, and walking mac-

hines. For general multiple manipulators, with possible motion redundancies in its subchains, the joint space approach [Cho, et. al., 1989] will be employed because of its allowance for the general modeling of kinematically redundant closed chain systems. The effective inertial load(T_a^*)at the independent joints ϕ_a required to drive the system sccording to a specified kinematic trajectory can be distributed to all the joints, including the dependent joints(ϕ_p), according to

$$T_a^* = T_a + [G_a^p]^T T_p$$
$$= [\,I \;:\; [G_a^p]^T\,]\begin{bmatrix} T_a \\ T_p \end{bmatrix} \tag{3-1}$$

yielding,

$$\begin{bmatrix} T_a \\ T_p \end{bmatrix} = [G]^+ T_a^* + (\,I - [G]^+[G]\,)\,\epsilon \tag{3-2}$$

where

$$[G] = [\,I \;:\; [G_a^p]^T\,] \tag{3-3}$$

and T_a, T_p imply the efforts of the independent and dependent joints, respectively. The second term of Eq.(3-2) represents a homogeneous solution which generates internal force characteristics such as stiffness, but no motion. In static equlibrium ($T_a^*=0$), the effective loads generating stiffness are denoted by

$$T_q^K = \begin{bmatrix} T_a^K \\ T_p^K \end{bmatrix} = (\,I - [G]^+[G]\,)\,\epsilon \tag{3-4}$$

where ϵ can in general be selected to yield desired, specified active stiffness.

Antagonistic Stiffness Modeling for Closed-chain Mechanisms

When any independent joint set ϕ_a is in static equilibrium with the dependent joint set ϕ_p, the effective load at the independent joints will be

$$(T_a^K)^* = T_a^K + [G_a^p]^T T_p^K = 0. \tag{3-5}$$

where T_a^K and T_p^K can be decided by selecting ϵ vector in Eq.(3-4) or, as in the following, by direct solution of Eq.(3-5) for T_a^K in terms of T_p^K.

An effective restoring force $\Delta(T_a^K)^*$ is generated against external disturbances and its behavior can be modeled as a spring action as follows.

$$\Delta(T_a^K)^* = \Delta([G_a^p]^T T_p^K) = -[K_{aa}]\,\Delta\phi_a \tag{3-6}$$

Thus, local antagonistic stiffness is defined as

$$[K_{aa}] = -\frac{\partial(T_a^K)^*}{\partial\phi_a} = (-T_p^K)^T \circ [H_{aa}^p] \tag{3-7}$$

where feedback position gains and joint and link compliances are not included in "open loop" active (or antagonistic) stiffness generation. The total system stiffness will of course include these effects. $[K_{aa}]$ can be represented in a vector form as

$$K_a = [H]\,T_p^K \tag{3-8}$$

where vector K_a is the collection of the independent upper triangular elements of $[K_{aa}]$ and $[H]$ is the transpose of $[H_{paa}]$ as follows

$$[H] = [H_{paa}]^T. \tag{3-9}$$

The required actuation effort of the dependent joints for the active stiffness generation will be

$$T_p^K = [H]^+ K_a. \tag{3-10}$$

where $[H]^+$ is a pseudo-inverse solution. The system will be balanced by applying an equilibrating actuation effort T_a^K of the independent joints according to Eq.(3-5).

Here, the desired operational active stiffness $[K_{uu}]$ is related to joint space active stiffness $[K_{aa}]$ as follows

$$[K_{aa}] = [G_a^u]^T [K_{uu}] [G_a^u]. \tag{3-11}$$

where, the forward Jacobian$[G_a^u]$ in mechanisms with closed-chains is obtained by Eq.(2-19).Once a desired operational active stiffness $[K_{uu}]$ is given, the system's inputs will be decided according to Eqs.(3-10) and (3-5).

Now the characteristic of the second order geometric property $[H_{aa}^p]$ will be discussed due to the interdependence between this property and system stiffness. In particular, some kinematic phenomena will be addressed for geometric interpretation. First of all, a closed-chain system has W independent loops according to the formula (Eq. 2-10). For example, there are W=2 independent loops for the linkage shown in Fig.2. It is important to note that out of the C constraint equations, all W angle constraint equations will be linear for planar mechanisms (such as Fig. 2). When differentiating constraint equations twice with respect to time, we have W degeneracies (W zero planes) in the second order geometric properties, $[H_{aa}^f]$, $[H_{pp}^f]$, $[H_{pa}^f]$, and $[H_{ap}^f]$, respectively.In this case, $[H_{paa}^f]$ or $[H]^T$ will have rank (C-W). This implies that $[H]^+$ will be an approximated least square solution. From Eq.(3-10), only (C-W) stiffness elements can be independently generated. Thus, the number of nonlinear constraint equations determines the degree of stiffness generation possible. Note that this relationship does not hold for general spatial mechanisms since the angle constraints are non-linear and configuration dependent.

As an example, a planar shoulder (Fig.2) has six constraint equations with two of them being linear. Thus, only four of the six columns of the 6 by 6 [H] matrix in Eq.(3-8) will be linearly independent. In other words, only four stiffness elements can be independently created through full actuation of all nine joints. However, by adding one more leg (Fig. 3) which results in two more nonlinear constraint equations, full stiffness generation (6 independent elements) will be possible because in this case the system has six independent columns out of the mine columns in the 6 by 9[H] matrix.

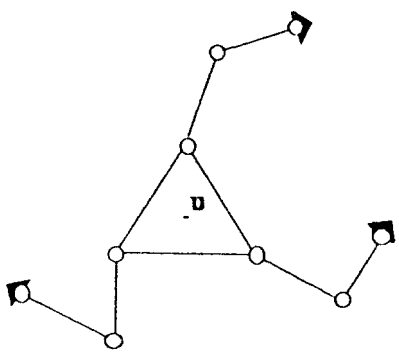


Figure 2

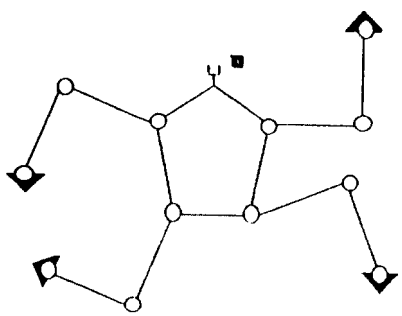


Figure 3

Here, only 9 actuations are required for the full stiffness generation because in this case the resulting 6 by 6 [H] matrix is full rank due to the 6 nonlinear constraint equations. This implies that 6 dependent inputs along with the three independent inputs can be used for full stiffness generation. Conclusively, the minimum required actuation number of a closed-chain system for full stiffness generation is the number of independent inputs plus the number of independent stiffness elements (which is the same as the number of the activated, dependent inputs). Naturally the number of nonlinear constraint equations must not be less than the number of independent stiffness elements.

Again, it is important to note that spatial mechanisms don't have this difficulty (linearity) since all constraint equations are completely nonlinear in nature. As an example, a spherical shoulder module [Cox and Tesar, 1989] (Fig.4), which has just three operational rotational motions, can create full stiffnes (six independent rotational stiffness elements) through six nonlinear angular constraint equations and nine joint actuations.

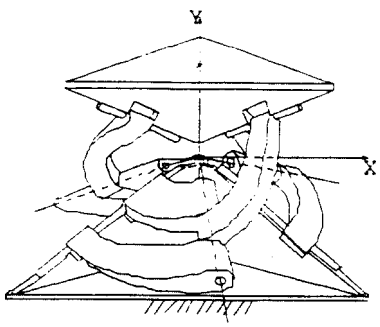


Figure 4

Additionally, there exist geometrically singular configurations which effect the ability to maintain full stiffness generation. Consider when a system happens to be in a locking position (Fig. 5), (i.e., when the three partial instantaneous screws are coincident).

The system loses the ability to resist a moment, and hence, the ability to generate rotational stiffness, in operational space.

Here, our initial investigation is made for the relationship between these singularities and the degeneration of the stiffness map [**H**]. The alternate form of Eq.(3-11) is

$$\mathbf{K_a} = [G]\,\mathbf{K_U}, \tag{3-12}$$

where $\mathbf{K_u}$ is the collection of the independent upper triangular elements of $[K_{uu}]$, and the elements of the 6x6 [G] matrix are quadratic functions of the $[G_K^u]$ elements. Now, we have the following equality from Eq.(3-8) and Eq.(3-12)

$$[G]\,\mathbf{K_u} = [H]\,\mathbf{T_p^K} \tag{3-13}$$

where row dimensions of [G] and [H] are consistent. Assuming that we have at least one additional chain to avoid the inherent degeneracy stemming from the linear nature of the constraint equations, the locking configuration now yields one degeneracy in [G] since $[G_8^u]$ is singular. This results in loss of control of the rotational stiffness of $\mathbf{K_u}$, which in fact directly addresses a degeneracy in [**H**].

In fact, this loss off stiffness control will result if any set of these of nine potential "independent" inputs exhibits this type of locking configuration.

On the other hand, spatial mechanisms have less chance of this "second-order" singularity since all partial instantaneous screws of the system are not in the same plane, but are scattered in 3 dimensional space. Based on the above analytic observations, the following conditions for full stiffness generation will be postulated.

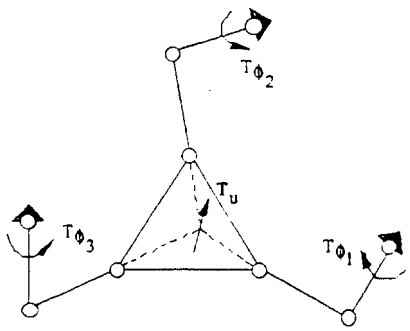


Figure 5

Conditions for full stiffness generation

A closed-chain mechanism is capable of full stiffness generation if and only if it satisfies

i) $V = (C - I) \geq D$ and $J_a \geq V + N$

where

C : Number of Independent Constraint Equations

I : Number of Linear Constraint Equations

V : Number of Independent Nonlinear Constraint Equations

N : Degree of Freedom in Operational Space

D : Number of Independent Stiffness Elements=N(N+1)/2

J_a : Number of Active Joints

ii)Every possible system Jacobian $[G_a^u]$ should be nonsingular

As another example, a stewart platform with 30 independent constraint equations (6 holonomic constraint equations in each of 5 independent loops) and 36 possible active joints (6 joints in each of 6 legs) needs at least 27 actuations to satisfy complete stiffness generation.

Once the previously postulated conditions are satisfied, such a mechanism can be potentially used as an adjustable nonlinear spring, which is an important concept for designing adaptive RCC devices. Effective diagonal stiffness, which is useful for precise position and force control applications, can be generated.

The aforementioned RCC device concept is not without precedent. Cutkosky and Wright [1985] designed an active wrist to control the location of the 'Center of Compliance' and the active stiffness at the same time. It is noted that they also used a parallel type mechanism(four bar)structure to create an active stiffness because of its benificial nonlinearity. However, their device has limitation in the Center of Compliance workspace and in the range of stiffness. Design of active RCC devices was proposed as a prospective application of active control of nonlinear springs [Yi, et.al, 1989]. In that sense, the discussion here will provide a direction for development of general design methodology for adjustable RCC devices in terms of their geometric configuration. Yi, et.al.[1989] employed this methodology for a Feedforward stiffness control concept. It is based on the idea that the redundant actuation of kinematically dependent inputs allows one to preload the system, potentially creating a beneficial restoring force which acts as an effective stiffness to compensate for disturbance, and also to gain a certain force level in an open-loop fashion. Redundant actuation also allows distribution of the load carrying responsibility among a multiplicity of potential actuation sets based on a variety of criteria ([Yi, et.al., 1990], [Walker, et.al., 1988], [Nakamura, 1988], and [Cheng and Orin, 1989]).

3.2 Open-loop Stability Analysis

It is also important to note that the antagonistic prperty has been omitted in the derivation of the effective stiffness of serial robots and the effective stiffness in multifingered hands. In many force control papers, robot stiffness is not explicitly denoted, or only structural stiffness (joint and link) is treated, excluding the antagonistic stiffness that is actually inherent in these systems. This may

be because the typical industrial manipulator is a position control device as such it is made heavy and stiff, and the drive system has a high gear train ratio. In fact, the effect from external disturbances on joint motion in such systems is negligible. This implies that the antagonistic effect is small. However, in Direct Drive systems which have a backdrivable actuator system (constant joint torques) dus to negligible friction and no gear ratio and backlash , the antagonistic effect will be a dominant factor in the system's effective stiffness. Therefore, the open-loop stability of mechanisms with backdrivable actuator systems will be studied in the following.

In static equilibrium, assuming the structural compliances are negligible, the effective operational space stiffness $[K_{UU}]$ of a general robotic system with backdrivable actuators is obtained as follows [Yi, et.al.1990]

$$[K_{UU}] = \sum_{r=1}^{R} [({}_rT_{\phi}^{K})^T \circ [{}^rH_{UU}^{\phi}] + [{}^rG_U^{\phi}]^T [{}_rK_{\phi\phi}] [{}^rG_U^{\phi}]] \qquad (3\text{-}14)$$

where

$$({}_rT_{\phi}^{K})^T = [((T_{\phi}^{K})^A)^T + ((T_{\phi}^{K})^*)^T + ((T_{\phi}^{K})^E)^T] \qquad (3\text{-}15)$$

and

$$[{}_rK_{\phi\phi}] = [{}_rK_{\phi\phi}^{P}]_e + [{}_rK_{\phi\phi}^{F}].$$

The first term in Eq.(3-14) represents the active stiffness created by the antagonistic actuation $((T^K_\phi)^A)^T$, external spring preloads $((T^K_\phi)^*)^T$ [Freeman and Tesar, 1988], and gravity or external load balancing $((T^K_\phi)^E)^T$. And also, the second term in Eq.(3-14) includes joint servo controller action $[{}_rK^F_{\phi\phi}]$, and effective external spring coefficients $[{}_rK^P_{\phi\phi}]_e$. In the following study, feedback $[{}_rK_{\phi\phi}]$ is not taken into account, but the effect of each antagonistic stiffness term on the system's stability will be studied in terms of several simple illustrative examples.

Antagonism from gravity loading

Figure 6 shows a simple example of antagonism. A backdrivable actuator supporting a constant gravitational load at the end of massless bar illustrates an antagonistic situation. The open-loop stability of this system can be easily descibed due to its simple geometry. In Fig. 6, once a disturbance is imposed on the system, the system will produce a restoring force equal to the difference bewteen the constant torque and the moment due to the gravitational force. However, in Fig. 7 the resulting effective load will result in motion away from equilibrium. For more general geometries, we need to measure the open-loop stability analytically since physical interpretation is not immediate.

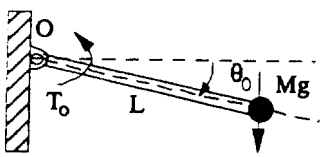


Figure 6

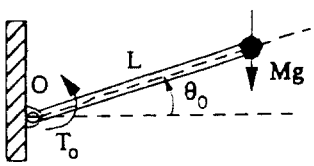


Figure 7

When one bar of Fig. 6 is in static equlibrium, the effective load at point O will be

$$T_0^* = -T_0 + MgL\cos\theta_0 = 0. \quad (3\text{-}16)$$

An effective restoring force is generated against external disturbances, and its behavior can be modeled as a spring action as follows

$$\Delta T_0^* = -MgL\sin\theta_0 \Delta\theta = -K_\theta \Delta\theta \quad (3\text{-}17)$$

where, T_0 is constant

The stiffness property

$$K_\theta = MgL\sin\theta_0 \quad (3\text{-}18)$$

is found as a good measure of antagonism in that it quantifies the resistive nature of the system. It shows a stable system action.

Here, the antagonistic stiffness is equivalent to the second derivative of the system's potential energy with respect to independent variables. Note also that, in static equilibrium, the system's potential energy can be represented as the negative of the system's virtual work.

However, for the configuration given in Fig. 7 , slightly different from Fig. 6, the system's effective stiffness will be obtained as follows(positive θ direction reversed)

$$K_\theta = -MgL\sin\theta_0. \quad (3\text{-}19)$$

Thus, as explicitly shown here, the configuration of Fig. 7 has an unstable dynamic behaviour since the system stiffness is negative definite. Therefore, system geometry is obviously critical with respect to open-loop stability

Antagonism from redundant actuation

As another example, Fig. 8 shows a four bar mechanism with antagonism. A four bar is a simple closed-chain mechanism with one degree of freedom.

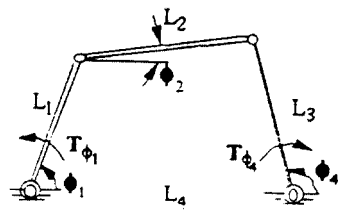


Figure 8

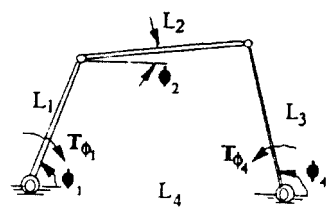


Figure 9

However, if we actuate more inputs than there are kinematic degrees of freedom, the system becomes antagonistic.

Based on the geometry given in Fig. 8, the Jacobian g_1^2, which relates the angular velocity of joint 2 to the angular velocity of joint 1, is obtained as follows

$$g_1^2 = L_1 S_{1\text{-}4} / L_2 S_{4\text{-}2} \quad (3\text{-}20)$$

where $S_{1\text{-}4}$ and $S_{4\text{-}2}$ denote $\sin(\phi_1\text{-}\phi_4)$ $\sin(\phi_4\text{-}\phi_2)$, respectively. And also the Jacobian

$$g_1^4 = L_1 S_{1\text{-}2} / L_3 S_{4\text{-}2} \quad (3\text{-}21)$$

relates the angular velocity of joint 4 to the angular velocity of joint 1. This Jacobian is a first order geometric property. The second order geometric property h_{11}^4, involved in relating the angular

acceleration of joint 4 to the angular acceleration of joint 1, is derived as follows

$$h_{11}^{4} = \frac{L_1 C_{1-2} - L_3 (g_1^4)^2 C_{4-2} + L_2 (g_1^2)^2}{L_3 S_{4-2}}. \qquad (3\text{-}22)$$

In equilibrium, the effective load$(T_{\phi 1})^*$ at joint 1 can be expressed as follows

$$(T_{\phi_1})^* = T_{\phi_1} + (g_1^4)^T T_{\phi_4} = 0, \quad (g_1^4)^T = g_1^4. \qquad (3\text{-}23)$$

Assuming a small displacement, the restoring force will be defined as

$$\Delta(T_{\phi_1})^* = -K_{\phi_1} \Delta\phi_1 = \Delta((g_1^4)^T) T_{\phi_4} \qquad (3\text{-}24)$$

since $T_{\phi 1}$ and $T_{\phi 4}$ are constant (in an open-loop sense). Now, an antagonistic stiffness is defined as follows

$$K_{\phi_4} = -\Delta(T_{\phi_1})^* / \Delta\phi_1 = -h_{11}^4 T_{\phi_4}. \qquad (3\text{-}25)$$

By analogy with the one bar example, $K_{\phi 1}$ is the inherent system's effective stiffness with respect to joint ϕ_1.

Now, for two different antagonistic actuation modes are shown in Fig.8 and Fig.9, a question arises regarding the stability of the two modes. Assume that the four bar is initially in an equilibrium state in both antagonistic modes. If we disturb the system by small amount, then the system will behave like a spring in such a way as to minimize the system potential energy. However, the stability of this springlike action will depend on the positive definiteness of the system stiffness. Therefore, the definiteness of the system stiffness will be studied for the two different antagonistic configurations.

Form Eq.(3-25), if a clockwise actuation is set as a positive effort, we can quantify the value and sign of stiffness $K_{\phi 1}$ based on the sign of h^4_{11}. For the model of Fig.9, where $T_{\phi 4}$ is positive, if h^4_{11} is positive, the system with this mode of antagonistic actuation is unstable with regard to any external disturbance. On the other hand, the model of Fig.8 will be stable. However, it is not easy to see the nature(such as magnitude and sign) of h^4_{11} since it is a highly nonlinear equation. It is concluded that for the same configuration with the different internal loading the system has different open-loop stability.

Antagonism from non-potential external force

Fig.10 represents a force controlled Direct Dirve serial manipulator in contact with a moving or fixed environment.

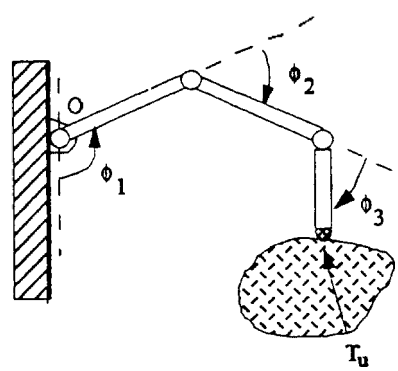


Figure 10

The system's effective load in operational space is

$$T_u^* = T_u + [G_u^\phi]^T T_\phi = 0. \quad (3\text{-}26)$$

Now, an antagonistic stiffness

$$[K_{uu}] = (-T_\phi)^T \circ [H_{uu}^\phi] \quad (3\text{-}27)$$

is defined utilizing the same restoring force analysis as in the one- and four-bar cases.

Here, the same system will be analyzed from both an analytic and a geometric point of view.

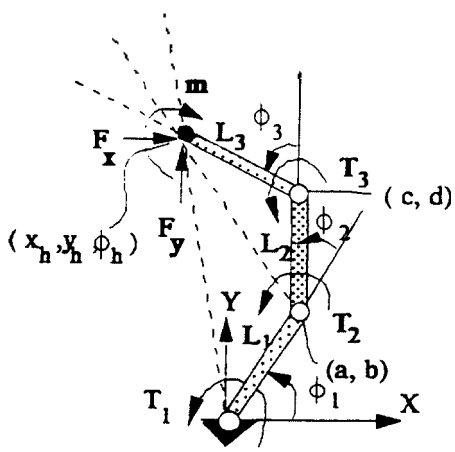


Figure 11

For the open-chain of Fig.11, the end-effector position is generally represented by the following three equations

$$x_h = L_1C_1 + L_2C_{1+2} + L_3C_{1+2+3} \quad (3\text{-}28)$$

$$y_h = L_1S_1 + L_2S_{1+2} + L_3S_{1+2+3} \quad (3\text{-}29)$$

$$\phi_h = \phi_1 + \phi_2 + \phi_3. \quad (3\text{-}30)$$

Where S_{1+2+3}, S_{1+2}, C_{1+2+3} and C_{1+2} denote $\sin(\phi_1+\phi_2+\phi_3)$, $\sin(\phi_1+\phi_2)$, $\cos(\phi_1+\phi_2+\phi_3)$, and $\cos(\phi_1+\phi_2)$, respectively. Restoring force analysis is performed for the first axis (ϕ_1). The effective load for axis ϕ_1 is

$$T_1^* = T_1 - F_x y_h + F_y x_h + m = 0 \quad (3\text{-}31)$$

and a restoring force vector is generated against system disturbances that cause joint displacements $\Delta\phi_1$, $\Delta\phi_2$, and $\Delta\phi_3$ according to

$$\Delta T_1^* = \frac{\partial T_1^*}{\partial \phi_1}\Delta\phi_1 + \frac{\partial T_1^*}{\partial \phi_2}\Delta\phi_2 + \frac{\partial T_1^*}{\partial \phi_3}\Delta\phi_3 \quad (3\text{-}32)$$

where the end-effector force is assumed to be constant. The effective stiffness elements with respect to the first joint are defined as

$$[K_{\phi\phi}]_{1;1} = -\frac{\partial T_1^*}{\partial \phi_1} = F_x x_h + F_y y_h \quad (3\text{-}33)$$

$$[K_{\phi\phi}]_{1;2} = -\frac{\partial T_1^*}{\partial \phi_2} = F_x(x_h - a) + F_y(y_h - b) \quad (3\text{-}34)$$

$$[K_{\phi\phi}]_{1;3} = -\frac{\partial T_1^*}{\partial \phi_3} = F_x(x_h - c) + F_y(y_h - d). \quad (3\text{-}35)$$

where the constant joint torques and end-effector moment m do not contribute to the system's stiffness as linear angle constraint equations do not contribute to stiffness generation.

The effective load for axis ϕ_2 is

$$T_2^* = T_2 - F_x(y_h - b) + F_y(x_h - a) + m = 0 \quad (3\text{-}36)$$

and a restoring force vector is generated as

$$\Delta T_2^* = \frac{\partial T_2^*}{\partial \phi_1}\Delta\phi_1 + \frac{\partial T_2^*}{\partial \phi_2}\Delta\phi_2 + \frac{\partial T_2^*}{\partial \phi_3}\Delta\phi_3 \quad (3\text{-}37)$$

where the effective stiffness elements for the second joint are defined as

$$[K_{\phi\phi}]_{2;1} = -\frac{\partial T_2^*}{\partial \phi_1} = F_x(x_h - a) + F_y(y_h - b) \quad (3\text{-}38)$$

$$[K_{\phi\phi}]_{2;2} = -\frac{\partial T_2^*}{\partial \phi_2} = F_x(x_h - a) + F_y(y_h - b) \quad (3\text{-}39)$$

$$[K_{\phi\phi}]_{2;3} = -\frac{\partial T_2^*}{\partial \phi_3} = F_x(x_h - c) + F_y(y_h - d). \quad (3\text{-}40)$$

Finally, the effective load for axis ϕ_3 is

$$T_3^* = T_3 - F_x(y_h - d) + F_y(x_h - c) + m = 0 \quad (3\text{-}41)$$

and a restoring force vector is generated as

$$\Delta T_3^* = \frac{\partial T_3^*}{\partial \phi_1}\Delta\phi_1 + \frac{\partial T_3^*}{\partial \phi_2}\Delta\phi_2 + \frac{\partial T_3^*}{\partial \phi_3}\Delta\phi_3 \quad (3\text{-}42)$$

where the effective stiffness elements for the third joint are defined as

$$[K_{\phi\phi}]_{3;1} = -\frac{\partial T_3^*}{\partial \phi_1} = F_x(x_h - c) + F_y(y_h - d) \quad (3\text{-}43)$$

$$[K_{\phi\phi}]_{3;2} = -\frac{\partial T_3^*}{\partial \phi_2} = F_x(x_h - c) + F_y(y_h - d) \quad (3\text{-}44)$$

$$[K_{\phi\phi}]_{3;3} = -\frac{\partial T_3^*}{\partial \phi_3} = F_x(x_h - c) + F_y(y_h - d). \quad (3\text{-}45)$$

Combining all the above analytics, the antagonistically generated stiffness matrix $[K_{\phi\phi}]$ (alternate form of Eq.(3-27)) is shaped as follows

$$[K_{\phi\phi}] = \begin{bmatrix} p & s & t \\ s & s & t \\ t & t & t \end{bmatrix} \quad (3\text{-}46)$$

where

$$p = F_x x_h + F_y y_h \quad (3\text{-}47)$$

$$s = F_x(x_h - a) + F_y(y_h - b) \quad (3\text{-}48)$$

$$t = F_x(x_h - c) + F_y(y_h - d). \quad (3\text{-}49)$$

The necessary and sufficient condition for $[K_{\phi\phi}]$ matrix to be positive definite is(see Appendix 2

$$p > s > t > o. \quad (3\text{-}50)$$

Here, three sub-conditions are considered and graphically combined in Fig.12.

$$\text{i) } t > 0 \quad : \quad F_y > -\frac{x_h - c}{y_h - d}F_x \quad (3\text{-}51)$$

$$\text{ii) } s > t \quad : \quad F_y > -\frac{c - a}{d - b}F_x \quad (3\text{-}52)$$

$$\text{iii) } p > s \quad : \quad F_y > -\frac{a}{b}F_x. \quad (3\text{-}53)$$

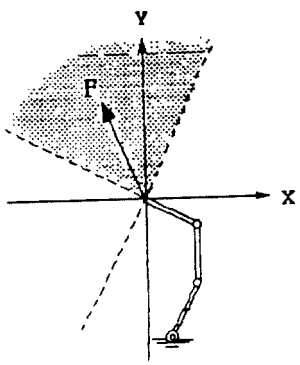


Figure 12

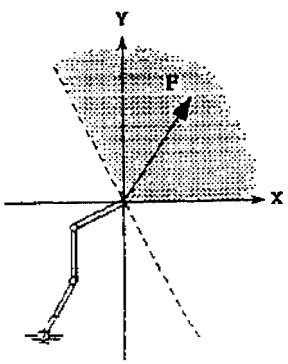


Figure 13

For the given configuration(Fig.12), the system is stable if the resultant force vetor F applied to the end-effector is within the the shaded area(common area of the 3conditions). Note that dashed lines on the boundaries of shaded areas are not included in the stable regions. Fig.13 shows another configuration with its stable region, which is considerably larger than that of the first example.

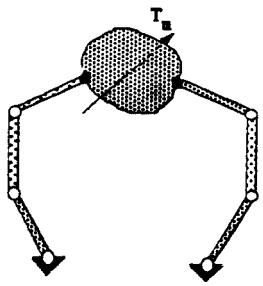


Figure 14

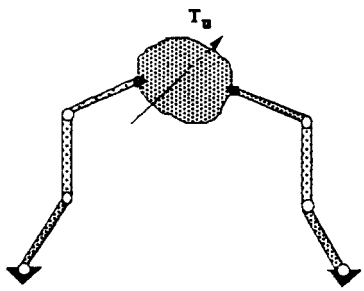


Figure 15

Antagonism in Dual Arms and Finger Grasping

Fig.14 and Fig.15 are intended to illustrate both dual arm and grasping operations. Yi, et.al.[1990] have treated the stability of these systems quantitatively in terms of antagonistic stiffness as follows

$$[K_{uu}] = \sum_{r=1}^{2} [(-{}_rT^K_\phi)^T \circ [{}^rH^\phi_{uu}]]. \qquad (3\text{-}54)$$

where ${}_rT^K_\phi$ and $[{}^rH^\phi_{uu}]$ represent the required joint input for an internal loading and the inverse Hessian for the rth chain(arm, finger), repectively.

Here, as the illustrative treatment of this situation, grasping under internal squeezing will be considered. In Fig.14 and Fig.15, each sub-chain can be considered as an open-chain serial arm(Fig. 12 and Fig.13) encountering point contact.

In grasping operations, stability of each chain is a sufficient condition for object stability(assuming the nominal grasping forces quarantee contact stability), since each chain will transmit a stabiliz ing restoring force to the object. Assuming no moment applied to end-effector, as seen in the previous planar arm example, the squeezing force in finger grasping always yields an unstable factor to system's stability and the stable region varies according to the system's configuration. Therefore, it is imperative to realize that open-loop stability has a strong interdependence with system's internal loading mode and the configuration. In the previous literature concerning grasping stability, this so called "geometric stability" has not been considered. It is interpreted that current finger systems [Cutkosky and Kao, 1989] were designed with nonbackdrivable, tendon and gear driven actuation systems in which case the antagonistic effect caused by the "rigid" shaft displacement is negligible due to the system friction and the high gear ratio between the actuator shaft and the drive link. However, when employing backdrivable actuators, promising for finger design and advanced robotic drive systems, the inherent antagonistic stiffness porperty plays an important role in the system's stabilty as observed in this paper.

Ⅳ. Conclusion

Kinematically constrained, redundantly actuated situations occur in normal operating modes of robotic systems. In these modes, the system becomes

antagonistic. The resulting inherent effective stiffness was found to be a good measure of the antagonism. This knowledge can be used to enhance the operational performance in naturally constrained linkage systems. The conditions for full active stiffness generation were investigated and are expected to be useful in the design of, redundantly actuated, RCC devices capable of adjustable task-space stiffness generation. One bar and four bar mechanisms were given as simple examples to show the stability of two different modes of antagonism, and the basic idea was extended to general closed-chain mechanisms, such as multi-fingered hands and dual arms. Open-loop stability due to the manipulator configuration is considered as another significant factor in system stability, especially in backdrivable systems. Finally, it is shown that geometric insight is of significance in understanding the dynamic nature of a mechanism with antagonism.

Appendix 1

Generalized Scalar Dot Product(o)

[A] o [B] = [C]

where

[A] = P x Q
[B] = Q x M x N
[C] = P x M x N

$$c_{ikl} = \sum_j a_{ij} b_{jkl}$$

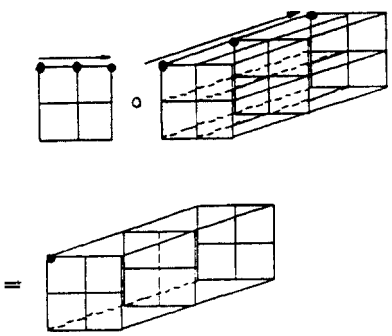
i : plane of C, row of A

k, l : row, column of C and B

j : column of A, plane of B.

This operation was originally defined by Freeman and Tesar[1988]. It plays a primary role for the systematic development of isomorphic transfer techniques. What this operation does is uniformly scale of each matrix in [B] by a scalar(each component of row a_{ij}, j=1,…N) and then sums them all as the following graphic example

Example : c_{111}



Appendix 2

The condition for a matrix to be positive definite is that the determinants of all principle minors should be positive. For a given 3 by 3 matrix,

$$\begin{bmatrix} p & s & t \\ s & s & t \\ t & t & t \end{bmatrix}$$

the following three conditions for the determinant should be satisfied. For the first minor,

$$\det(p) = p > 0$$

and for the second minor,

$$\det\begin{bmatrix} p & s \\ s & s \end{bmatrix} = (p-s)s > 0$$

here, s cannot be negative since(p-s)s will always be negative. Thus, s should be positive and also less than p. For the third minor,

$$\det\begin{bmatrix} p & s & t \\ s & s & t \\ t & t & t \end{bmatrix} = (p-s)t\,(s-t) > 0$$

here, t cannot be negative since(s-t)s will always be negative. Thus, t should be positive and also less than s. Now, by combining the above results the necessary and sufficient conditions for the matix to be positive definite is as follows

$$p > s > t > 0.$$

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▲Buung ju Yi

Education : B.S : Mechanical Engineering, Hanyang University

M.S. : Mechanical Engineering, The University of Texas at Austin

Ph.D : Mechanical Engineering, The University of Texas at Austin

Fields of Interest : Dynanics, Robotics, Bionechanios

▲Sang Kee Song (Vol. 10, No. 5)

▲Whang Cho

Assistant Professor

Dept. of Control and Measurement, Kwangwoon University

Education : B.S. : Mechanical Engineering, Inha University

M.S. : Mechanical Engineering, the University of Texas at Austin

Ph.D. : Mechanical Engineering, the University of Texas at Austin

Fields of Interest : Dynamics, Robotics, Bicmechanics