⦿ Technical Paper

Stochastic Prediction of Rolling of Ships in Irregular Waves

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불규칙 해상의 선박 횡요의 확률론적 예측

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Key Words : White Noise(백색 잡음), Non white Noise(비 백색 잡음), Nonlinear roll Equation(비선형 횡요 방정식), Equivalent Linearization(등가선형화), Average Method(평균화법)

초 록

불규칙 해상에서 선박의 큰 횡요각의 예측이 중요한 과제로 대두 되고 있다. 본 논문에서는 통계적 해석에 의한 이의 예측 방법을 제시한다. 즉 주어진 비 선형 횡요운동 방정식으로부터 배의 횡요각과 각속도의 결합 확률 밀도 함수를 구하는 방법을 도입하고 각종 계수들의 값의 변화에 따른 예측 결과를 다른 논문에서 제시한 시뮬레이션 결과와 비교하였다.

1. Introduction

The present study investigates a method of predicting the threshold crossing time by solving a nonlinear rolling equation of motion of a ship in irregular waves. Since the nonlinear nature of the rolling motion of a ship in waves is very complicated, a complete analytic solution of the problem has not been proposed so far.

In the past, those approaches to solve this problem can be illustrated as an equivalent linear equiation,[1] a perturbation method,[2] and functional representation method[3),4)]. But these yielded only limited information on the roll response statistics such as a mean square of the roll angle.

Another approach which is able to predict the form of the response distribution for nonlinear systems response in Fokker-Planck-Kolmogorov(FPK) mothod. Dunne[5] has developed a new approximate method for dealing with a nonlinear systems which are disturbed by excitation that can not be adequately classed as wideband, and modeled as white noise. He combined the method

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of equivalent linearization and the FPK technique to obtain some useful results. The method has been applied for estimating threshold crossing rates. The theoretical results were tested with intensive simulation results. That is, their method is based on the assumption that the crossing properties of the response can be approximately replaced by the excitation with a white noise process of suitable intensity. Then they reinstated the nonlinear restoring function from the equivalent linearized equation of motion.

The present study reinstates the full nonlinear damping and nonlinear restoring function with the equivalent white-noise intensity so that the non-linearity in the damping can be adequately modeled. This white-noise exited nonlinear equation of motion is solved by average method to obtain the needed joint probability density function.

2. Roll Response Model in Random Beam Seas

If the influence of all other degrees of freedom can be neglected, the equation of motion of a ship rolling in random beam waves can be written in the following form

$$\ddot{\theta}+D(\dot{\theta})+R(\theta)=n(t) \quad \cdots\cdots (1)$$

wher θ is roll angle, $D(\dot{\theta})$ represents damping function, and $R(\theta)$ represents a restoring function, and $n(t)$ is a Gaussian random process with zero mean and spectrum $S(\omega)$. The time scale is chosen so that the undamped natural roll frequency is unity. The damping and restoring function can be represented as

$$D(\dot{\theta})=C_1\dot{\theta}+C_{C3}\dot{\theta}^3 \quad \cdots\cdots (2)$$

$$R(\theta)=\theta+K_3\theta^3 \quad \cdots\cdots (3)$$

Solving equation (1) means the determination of a limited amount of information about the solution process. The threshold crossing probability can be considered as useful statistics. So the thresholds $\theta=\pm a$ considdered in this study are high enough to be regarded as dangerous. The number of upcrossins per unit time of θ at thresthold $a>o$ can be written as[6)]

$$\lambda(a)=\int_0^{\infty}\dot{\theta}\ f(a,\ \dot{\theta})\ d\dot{\theta} \quad \cdots\cdots (4)$$

where $f(\theta,\ \dot{\theta})$ is the joint probability density function, and the mean time $\mu(a)$ between upcrossings is the reciprocal of $\lambda(a)$

$$\mu(a)=\frac{1}{\lambda(a)} \quad \cdots\cdots (5)$$

3. Exact Solution

When a dynamic system is subjected to white noise excitation, the exact solution can be obtained thruogh the FPK equation. A few exact analytical solutions of FPK equation exist for random vibration problems. The most general solution is due to Caughey.[7)] Consider the following equation of motion for a single degree of freedom system.

$$\ddot{\theta}+\dot{\theta}H(E)+g(\theta)=Z(t) \quad \cdots\cdots (6)$$

where

$$E=\frac{1}{2}\dot{\theta}^2+V(\theta) \quad \cdots\cdots (7)$$

Where E represents the total energy of the system, $V(\theta)$ is the potential energy of the system, $Z(t)$ is a white noise which has a constant spectral density J. Caughey has shown an exact solution for the probability density function $f(\theta,\ \dot{\theta})$ is as follows

$$f(\theta,\ \dot{\theta})=C\ \exp\left[-\frac{2}{J}\int_0^E H(\xi)d\xi\right] \quad \cdots\cdots (8)$$

When $D(\dot{\theta})=C_1\dot{\theta}$, the stationary joint probability density function can be obtained using equation (8) as

$$f(\theta, \dot{\theta}) = A \exp\{-\frac{2C_1}{J}\int_0^{\theta} R(u)du + \frac{1}{2}\dot{\theta}^2\} \quad \cdots\cdots (9)$$

where A is a normalizing constant. The mean upcrossing time can be shown as

$$\mu(a) = \frac{\int_{-\infty}^{\infty} \exp\{-\frac{2C_1}{J}\int_0^{v} R(u)du\}dv}{\frac{1}{2}\sqrt{\frac{J}{\pi C_1}}\exp\{-\frac{2C_1}{J}\int_0^{a} R(u)du\}} \quad \cdots (10)$$

4. Nonlinear Damping, Non-White Excitation

The present method is based on the assumption that the crossing properties of the response of an excitation can be approximated by replacing the excitation with a white noise process of suitable intensity J.

4. 1 Equivalent Linearization

First the system is linearized in a conventional way. Consider an alternative form of equation (1)

$$\ddot{\theta} + g(\theta, \dot{\theta}) = n(t) \quad \cdots\cdots (11)$$

The equivalent linearization technique applied to equation (1) replaces the system by an equivalent linear system

$$\ddot{\theta} + C_{eq}\dot{\theta} + K_{eq}\theta = n(t) \quad \cdots\cdots (12)$$

The equivalent linear system in (12) are determined by minimizing the mean square of the linearization error

$$\varepsilon = C_{eq}\dot{\theta} + K_{eq}\theta - g(\theta, \dot{\theta}) \quad \cdots\cdots (13)$$

The optimum linear terms can be expressed as

$$C_{eq} = \frac{E\{\theta^2\}E\{\dot{\theta}g(\theta, \dot{\theta})\} - E\{\theta\dot{\theta}\}E\{\theta g(\theta, \dot{\theta})\}}{E\{\theta^2\}E\{\dot{\theta}^2\} - E^2\{\theta\dot{\theta}\}} \quad \cdots\cdots (14)$$

$$K_{eq} = \frac{2E\{\theta^2\}E\{\theta g(\theta, \dot{\theta})\} - 2E\{\theta\dot{\theta}\}E\{\dot{\theta}g(\theta, \dot{\theta})\}}{2E\{\theta^2\}E\{\dot{\theta}^2\} - 2E^2\{\theta\dot{\theta}\}} \quad \cdots\cdots (15)$$

Where E{ } represents the expectation process. The expectation in equation (14) & (15) are not available. The present study employed the iterative scheme which initiated with the initial guess. For the linearized system, the mean upcrossing times $\mu_{S(\omega)}(a)$ and $\mu_J(a)$ can be readily obtained. That is

$$\mu_{S(\omega)}(a) = 2\pi\frac{\sigma_{S1}}{\sigma_{S2}} \exp\{\frac{a^2}{2\sigma_{S1}}\} \quad \cdots\cdots (16)$$

$$\mu_J(a) = 2\pi\frac{\sigma_{J1}}{\sigma_{J2}} \exp\{\frac{a^2}{2\sigma_{J1}}\} \quad \cdots\cdots (17)$$

where

$$\sigma_{S1}^2 = \int_{-\infty}^{\infty} |H(\omega)|^2 S(\omega)\, d\omega$$

$$\sigma_{S2}^2 = \int_{-\infty}^{\infty} \omega^2 |H(\omega)|^2 S(\omega) d\omega$$

$$\sigma_{J1}^2 = J\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega$$

$$\sigma_{J2}^2 = J\int_{-\infty}^{\infty} \omega^2 |H(\omega)|^2 d\omega$$

and where $H(\omega)$ is the frequency response function which can be represented as

$$H(\omega) = \frac{1}{[(K_{eq} - \omega^2) + iC_{eq}\,\omega]} \quad \cdots\cdots (18)$$

The value of J is chosen to minimize the square error

$$\int_0^{a_{max}} [\mu_{S(\omega)}(a) - \mu_J(a)]^2\, da \quad \cdots\cdots (19)$$

Where a_{max} is a suitable high threshold. The newly determined J is the best fit for the mean crossing time functions over the range of threshold of interest. If one reinstate the nonlinear damping and the nonlinear restoring, the equation (1) can be rewritten as

$$\ddot{\theta} + D(\dot{\theta}) + R(\theta) = \sqrt{J}\; Z(t) \quad \cdots\cdots (20)$$

Where Z(t) represents the white noise excitation. If we can get the probability density function of equation (20), then the expected values in equation (14) and (15) can be obtained. Thus the whole process can be an iteration scheme. The whole process is shown in Fig. 1.

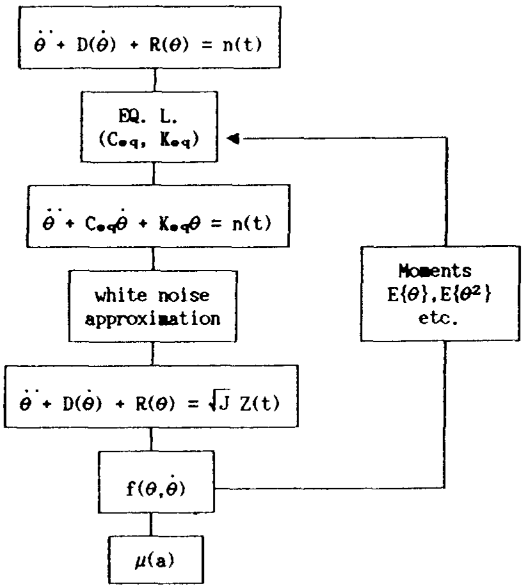

Fig. 1 Structure of the prediction method

4. 2 Average Method

The average method for the random vibration studies described by Roberts and Spanos[8] i adopted to calculate the approximate value of f(θ, θ̇) for the equation (20).

The basic concept of the method for randomly excited oscillators is as follows when the energy dissipated per cycle is due to light damping, the total energy can be treated as approximately constant over one cycle of oscillation. The period of free oscillation is found to be given by

$$T(E)=4\int_0^{\theta_c} \frac{d\theta}{\sqrt{2(E-V)}} \quad \cdots\cdots (21)$$

where θ_c is such that

$$V(\theta_c)=E \quad \cdots\cdots (22)$$

The error integral

$$I=\int_0^{T(E)} \varepsilon^2 \, dt \quad \cdots\cdots (23)$$

can be minimized with respect to H(E). This yields the following expression for H(E) for use in equation (20)

$$H(E)=\frac{\int_0^{\theta_c} D([(E-V(\theta))]^{1/2})\, d\theta}{\int_0^{\theta_c} [2(E-V(\theta))]^{1/2}\, d\theta} \quad \cdots\cdots (24)$$

A combination of equation (8) and (24) now gives an approximate expression by f(θ, θ̇).

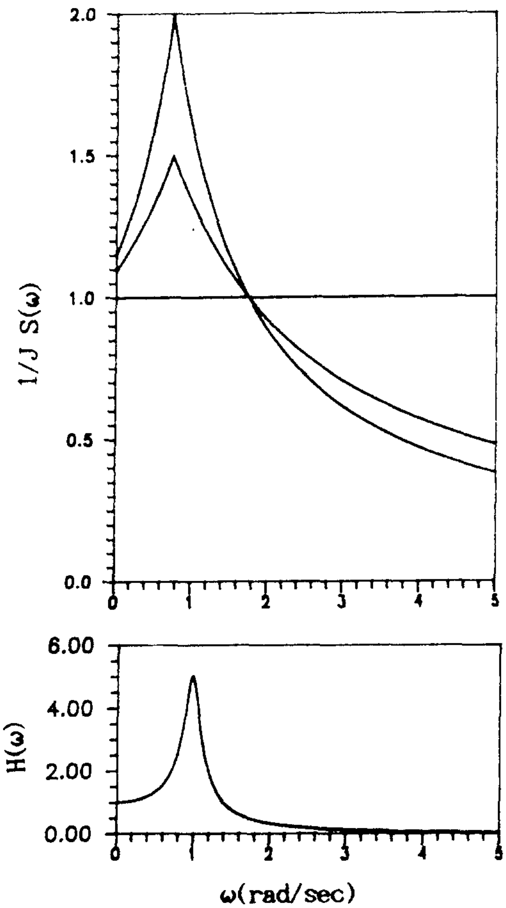

Fig. 2 Test spectrum & frequency response function
(ω_o=0.75, ω_c=5, C_{eq}=0.2, K_{eq}=1)

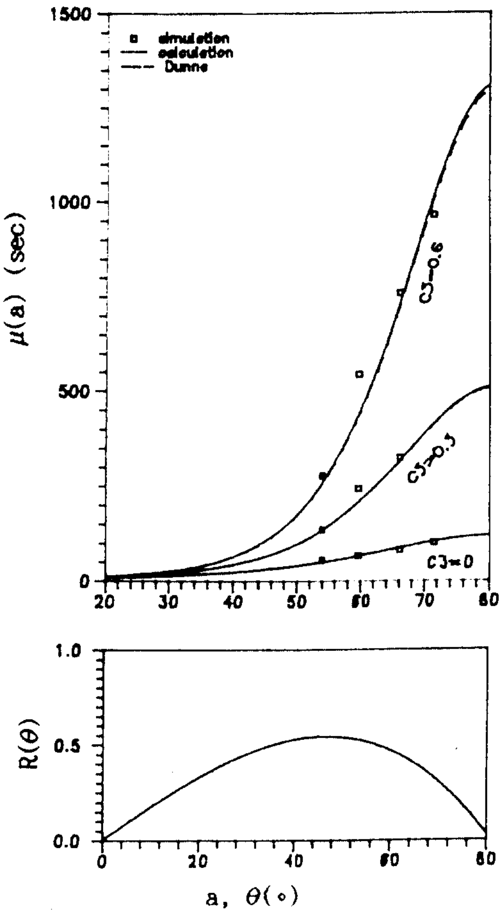

Fig. 3 Mean upcrossing time vs. threshold ($J=0.07$, $C_1=0.2$, $K_3=-0.5$)

5. Numerical Results

To apply the proposed scheme to non-white excitation, the following test spectrum was adopted following Dunne[5] for the mutual comparison purpose

$$S(\omega)=\begin{cases}\dfrac{PJ}{1-(P-1)\,|\,\omega-\omega_0\,|} & \omega<\omega_c\\ 0 & \omega>\omega_c\end{cases} \qquad \cdots\cdots (25)$$

Where J, ω_0, and P are scale, location and shape parameters respectively, with $P=1$ represents the white-noise process. Fig. 2 shows the shape of the test spectrum, and the frequency response spectrum.

Fig. 3 shows mean crossing time against threshold for white noise intensity $J=0.07$, nonlinear coefficient $K_3=-0.5$, linear damping ration $C_1=0.2$ and three values of nonlinear damping coefficient C_3. The square represents the simulation points which are given by Dunne for comparison purposes. The dotted line represents the results obtained by Dunne. Dunne obtained by reinstating the nonlinear restoring function, but with linear damping. The solid line represents the proposed scheme. The higher the damping, the larger the mean crossing time as expected. The restoring function is plotted against the roll angle to show the non linear region of restoring. The proposed scheme shows better results over the Dunne's scheme. Fig. 4 is similar except that $J=0$.

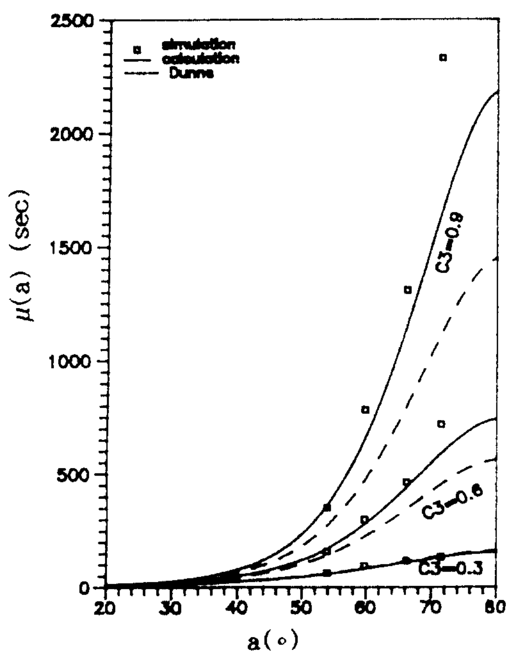

Fig. 4 Mean upcrossing time vs. threshold ($J=0.05$, $C_1=0.02$, $K_3=-0.5$)

05 and $C_1=0.02$. Even though the nonlinear damping coefficient is much larger than the linear coefficient, the proposed scheme agress well with the simulation results.

The linear damping is considered next. The results in Fig. 5 shows that the proposed scheme and the Dunne's scheme give very similar results with each other as expected.

Fig. 6 and Fig. 7 are similar to Figs. 3 and 4, except that $C_3=0.6$ and the parameter P varies. The proposed scheme shows better results than that proposed by Dunne when the nonlinear damping dominates.

6. Conclusions

The following conclusions are drawn from this study.

1) The crossing properties of the response can be approximately replaced by the excitation with a white noise process of suitable intensity.

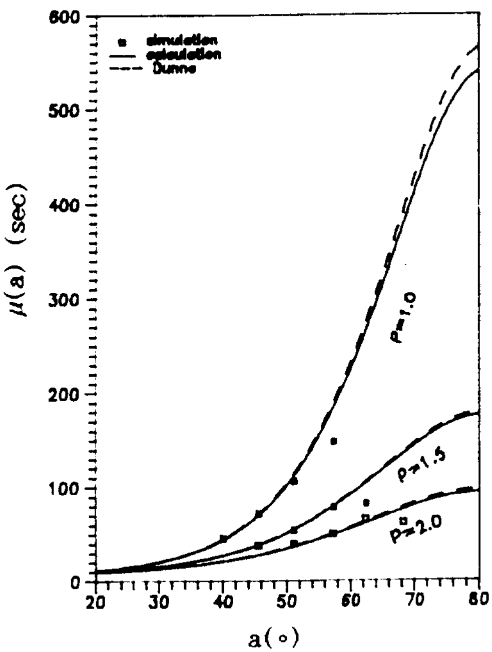

Fig. 5 Mean upcrossing time vs. threshold ($J=0.09$, $C_1=0.4$, $C_3=0$, $K_3=-0.5$)

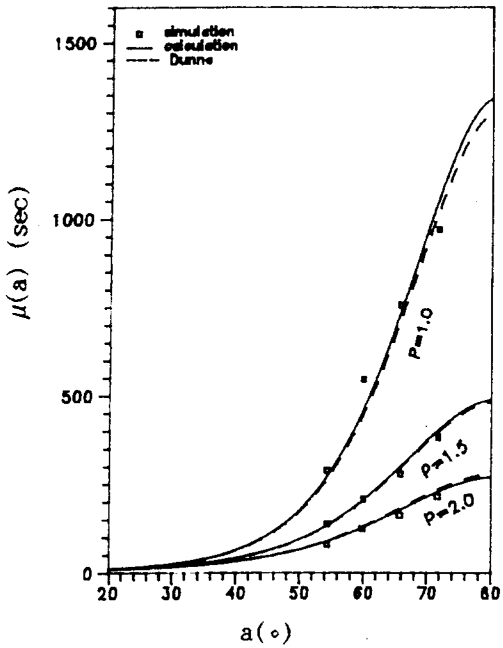

Fig. 6 Mean upcrossing time vs. threshold ($J=0.07$, $C_1=0.2$, $C_3=0.6$, $K_3=-0.5$)

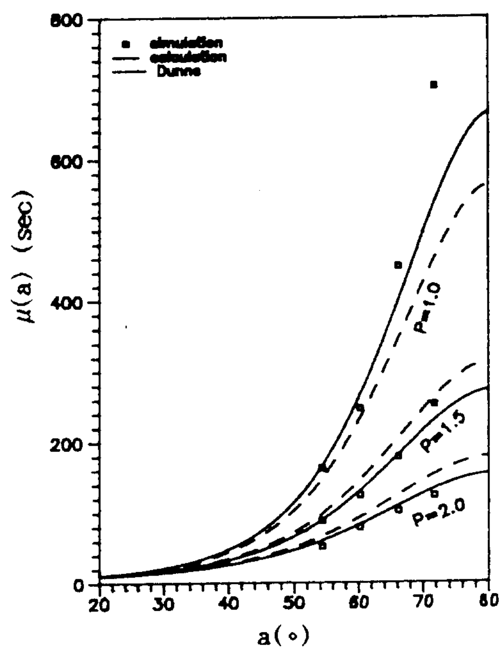

Fig. 7 Mean upcrossing time vs. threshold ($J=0.05$, $C_1=0.02$, $C_3=0.6$, $K_3=-0.5$)

2) The proposed scheme which reinstates the full nonlinear damping and nonlinear restoring with the equivalent white - noise intensity gives a relatively good agreement with the simulation results.

3) The proposed scheme can be readily extended to a real sea spectrum with the introduction of appropriate frequency response function.

References

1) Kaplan, P., "Lecture Notes on Nonlinear Theory of Ship Roll Motion in a Random Seaway", Proceedings, 11th International Towing Tank Conference, Tokyo, Japan, pp. 393 -396, 1966
2) Flower, J. O., "A Perturbational Approach to Non - Linear Rolling in a Stochastic Sea", International Shipbuilding Progress, Vol. 23, pp. 209-212, 1976
3) Hasselman K., "On Nonlinear Ship Motions in Irregular Seas", Journal of Ship Research, Vol. 10, No. 1, pp. 64-68, 1966
4) Vassilopoulos, L., "The Application of Statistical Theory of Nonlinear Systems to Ship Motion Performance in Random Seas", International Shipbuilding Progress, Vol. 14, No. 150, pp. 54-65, 1967
5) Dunne, J. F., "Ship Roll Response and Capsize Prediction in Random Beam Seas", Ph. d. thesis, Univ. of Bristol, UK, 1982
6) Soong, T. T., "Random Differential Equations in Science and Engineering", Academic Press, 1973
7) Caughey, T. K., "On the Response of a Class of Nonlinear Oscillators to Stochastic Excitation", Proc, Coll. Int. du Ceutre Nat. de la Rechercher Scient. No. 148, Morseille, France, 1964
8) Roberts, J. B. and Spanos, P. D., "Random Vibration and Statistical Linearization", Publisher John Wiley and Sons, 1990