◉ Technical Paper

A Study on Fatigue Life Distribution of SM45C under Constant Rotating Bending Stress

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(*Received Oct.* 21, 1991)

SM45C의 회전굽힘 응력하의 피로수명분포에 관한 연구

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Key Words : Fatigue Failure(피로파손), Damage Tolerance Design(손상허용설계), Probability Theory(확률이론), Cumulative Damage(누적손상), Random(랜덤)

초 록

피로파괴연구의 급격한 발전에 따라 최근의 기계나 구조물들은 많은 분야에서 손상허용설계원리에 근거하여 설계되고 있다. 이러한 상황 하에서 피로파손의 정확한 특성을 밝히는 것은 신뢰성을 고려한 기계나 구조물의 설계에 있어 가장 중요한 요인이 된다. 피로파손은 많은 랜덤요소를 내포하고 있으므로 실험결과 분석 및 수명예측을 분석하기 위해서는 통계학적 해석이 요구되고 있다.

본 연구의 목적은 회전굽힘피로시험을 수행하고 피로수명을 분석하는데 정규분포, 대수분포, 지수분포 및 Weibull 분포를 이용하여 실험결과와 비교하고 파손확률을 찾는데 있다.

Nomenclature

CDF	Cumulative distribution function
f(t)	Failure law in terms of density function
N_f	Number of cycles to failure
P	Failure probability of the i-th data
S_a	Stress amplitude
β	Shape parameter or Weibull distribution slope
λ	Scale parameter in Weibull distribution
λ_1	Scale parameter in exponential distribution
μ	Mean fatigue life
μ_y	Log mean of population
σ^2	Variance of fatigue life
σ_y^2	Log variance of population

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1. Introduction

The fatigue phenomenon is complex and influenced by many factors. Until now, there have been many problems not understood well about fatigue.

Experimental results obtained by either constant or variable amplitude fatigue tests show wide scattering of fatigue life, which do not give just single correlation, even if the conditions of specimen preparation and testing conditions are strictly specified.

In recent years, for the reliable design of a component, failure probability became an important quantity. To evaluate this quantity, the true distribution of cumulative damage in the component and its critical value must be known. Unfortunately, since it is impossible to know the true distribution of this quantity, the use of distribution functions estimated from experimental data is necessary. Failure probability has been frequently evaluated using the parametric approach.

A few papers have discussed fatigue failure from statistical standpoints.[1-3] In the papers referred above, it was common to employ either Weibull or log-normal distribution to represent distribution of fatigue life, but the effects of distribution functions related to loading conditions and specimen shapes were not sufficiently clarified.

In this study, constant rotating bending fatigue tests were first carried out on SM45C round bar specimens having a small drilled center hole of 2*mm* dia. Furthermore, in order to examine the fatigue life distribution, the normal, log-normal, exponential, and Weibull distribution functions were used for the distribution model of fatigue life. The experimental results and the calculated distribution of fatigue life were compared.

2. Experimental Procedures and Results

The material used in the present study was SM45C of carbon steel, and its mean chemical compositions and mechanical properties are listed in Tables 1 and 2. The configuration of the test specimen is shown in Fig. 1.

Table 1 Mean chemical compositions of material (Wt. %)

Material	C	Si	Mn	P	S	Cu	Ni
SM45C	0.45	0.243	0.753	0.0173	0.0667	0.0133	0.01

Table 2 Mean mechanical properties of material (*kgf/mm*²)

Material	σ_y	σ_u	ε(%)	σ_y : Yield strength σ_u : Ultimate strength ε : Percent of elongation
SM45C	34.1	63.3	22.0	

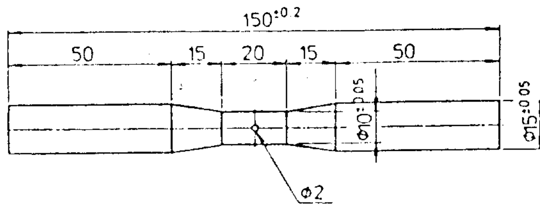


Fig. 1 Configuration of fatigue test specimen Center holed specimen in *mm*

In order to confine crack to initiation place and to observe the crack easily the specimen is provided with a center hole.

Since the yield phenomenon is sensitive to surface roughness, the specimen surface was carefully polished by emery papers and buff-finished by using powdered Fe_2O_3.

The stress concentration factor of the specimen was 2.02, which was used by Peterson[4] and Nishida[5] independently.

Fatigue tests were performed by means of the Ono-type rotating bending fatigue testing machine at room temperature. The mechanism of the Ono-type rotating bending fatigue testing machine is shown in Fig. 2. 70 specimens were prepared for test, although the experimental sample size was small from a statistical point of view. The maximum bending moment was 10 *kgf-m* and the rotating speed could be controlled from minimum 100 *cpm* to maximum 3600 *cpm*. In this study, fatigue tests were carried out at 3000 cpm and the specimen at $N_f = 10^7$ was regarded as a run-out one. For each of these run-out specimens, the surface around the drilled hole was carefully checked by the optical microscope but no crack was found for any specimen.

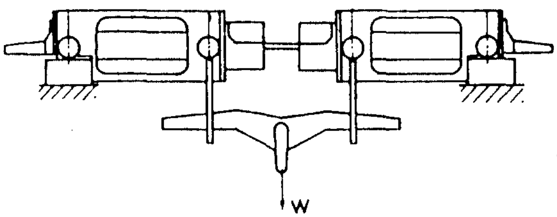


Fig. 2 Mechanism of the Ono-type rotating bending fatigue testing machine

The material micro-structure, initial nonuniformity of specimens in terms of dimension, if occurred, in the specimen setting on the fatigue test machine were neglectied in this study.

More details of the experimental information may be found in the previous articles,[6-8] and only a brief description is given here. Constant amplitude fatigue test results are shown in Table 3. In Table 3 $N_f = 10^7$ represents "run-out", where fatigue limit was determined by staircase method, and the obtained fatigue limit was 12.37 kgf/mm^2. The mean and standard deviation of test results are given in Table 4. In Fig. 3, a scatter diagram of the data is shown. The fatigue life of individual specimen and the mean fatigue life of specimens at the constant stress levels are also plotted by the marks of ○ and ● respectively. By using the least square method, relationship between stress and number of cycles to failure was indicated as a regression line equation at the right hand upper corner in Fig. 3. It is also shown that the degree of scattering of the experimental results in the higher stress amplitude is relatively smaller than that in the lower stress amplitude.

The results are used in the present analysis to assess the fatigue life.

3. Statistical Approach

The following four distributions are used alternatively :

Table 3 Fatigue life at each stress level at 3000 *cpm*

Stress	N_f	Stress	N_f	Stress	N_f	Stress	N_f	Stress	N_f
40	1.80E+4	30	4.38E+4	22.5	1.50E+5	17	7.03E+6	14	4.72E+6
	1.13E+4		5.28E+4		1.74E+5				
	2.05E+4		6.74E+4		2.62E+5	16.5	8.14E+6	13.5	5.98E+6
	2.57E+4		8.33E+4		3.19E+5		8.75E+6		
	4.44E+4		1.04E+5		7.41E+5	13			7.92E+6
			1.61E+5		7.72E+5	16	8.84E+6		8.10E+6
							9.37E+6		9.56E+6
35	0.08E+4								
	2.10E+4	27.5	6.06E+4	20	2.37E+5		1.00E+7		
	3.71E+4		6.47E+4		2.96E+5			12.5	8.67E+6
	5.24E+4		1.57E+5		1.35E+6	15.5	8.41E+6		1.00E+7
	7.43E+4		1.71E+5		4.02E+6		9.36E+6		1.00E+7
32.5	3.35E+4		1.96E+5		4.38E+6		1.00E+7	12	6.98E+6
	3.52E+4	25	1.02E+5	17.5	6.07E+5	15	1.60E+6		1.00E+7
	4.36E+4		2.40E+4		6.25E+5		1.95E+6		1.00E+7
	4.75E+4		3.14E+5		9.55E+5		2.37E+6		
	9.55E+4		4.07E+5		5.38E+6		1.00E+7	11.5	1.00E+7

Note : Stress in kgf/mm^2 and N_f in cycles

Table 4 Calculated means standard deviations of the fatigue life N_f

Stress (kgf/mm^2)	Mean	Standard deviation
40	2.25E+4	1.23E+4
35	4.11E+4	2.03E+4
32.5	5.11E+4	2.28E+4
30	8.54E+4	3.91E+4
27.5	1.30E+5	5.63E+4
25	2.66E+5	1.12E+5
22.5	4.03E+5	2.56E+5
20	2.06E+6	1.80E+6
17.5	7.29E+5	1.60E+5
15	1.96E+6	3.16E+5

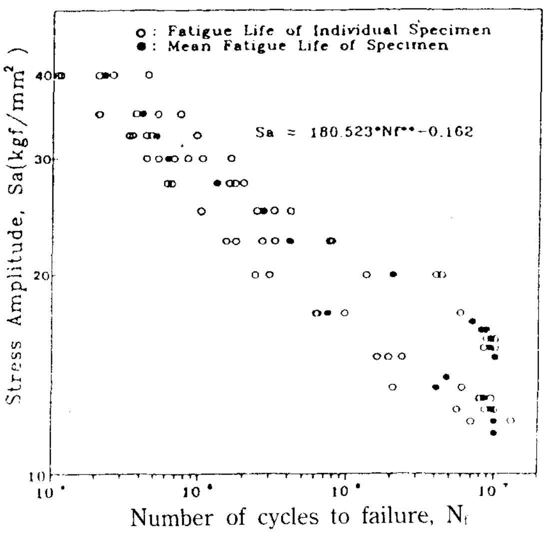


Fig. 3 Scatter diagram of the data N_f

Normal distribution :

$$F(t)=\int_0^t \frac{1}{\sigma\sqrt{2\Pi}} e^{-(t'-\mu)^2/2\sigma^2} dt' \cdots \quad (1)$$

where μ : mean fatigue life

σ^2 variance of fatigue life

t' dummy variable

Log-normal distribution :

$$F(t)=\int_1^t \frac{1}{\sigma\sqrt{2\Pi}} \frac{1}{t'} e^{-(\ln t'-\mu^y)^2/2\sigma^2} dt \quad \cdots\cdots\cdots\cdots\cdots (2)$$

where μ_y : log mean of population

σ_y^2: log variance of population

Exponential distribution :

$$F(t)=1-e^{-(\lambda_1 t)} \quad \cdots\cdots\cdots\cdots\cdots (3)$$

where λ_1 : scale parameter

Weibull distribution :

$$F(t)=1-e^{-(\lambda t^\beta)} \quad \cdots\cdots\cdots\cdots\cdots (4)$$

where λ : scale parameter

β : shape parameter or Weibull slope

Experimental data were plotted on a probability paper to see, among the four statistical distributions, which distribution provided a better fit to them, however, the process was omitted here for the sake of simplicity. It was seen that normal and log-normal distribution provided a better fit to the constant amplitude fatigue life data than the exponential and Weibull distributions did.

The median-rank estimation was used[9] All the life data at each stress level were first rearranged in the order of life, and then failure probability of the ith data, P, was calculated by (q-0.3)/(n+0.4), where n is the total number of specimens assigned to a stress level, and q is the failure order. The results are shown in Table 5.

It is easy to estimate the values of the parameters μ, μ_y, σ^2, and σ_y^2 statistically, however, estimation of the parameters μ, μ_l, and β is not an easy task, so they must be determined both graphically and by means of the least sqaure method.[10] The obtained values of the parameters μ, μ_l, and β are given in Table 6.

The calculated results using the four cumulative distribution functions have been shown in Figs. 4 to 11 at each stress level. In these Figs. 4 to 11, thick solid curve represents the CDF in normal distribution calculated by Eq.(1) ; thick dashed curve represents the CDF in log-normal distribution calculated by Eq.(2) ; centercurve represents the CDF in exponential distribution

Table 5 Rearranged fatigue life with median-rank estimates of the percent population failed corresponding to failure order in sample

p = 1 3000*cpm*							
Stress (*kgf/mm*^{2})							
40		35	32.5	30		27.5	
P	N_f	N_f	N_f	P	N_f	P	N_f
12.94	1.08E+4	2.08E+4	3.35E+4	10.91	4.38E+4	12.94	6.06E+4
31.47	1.13E+4	2.10E+4	3.52E+4	26.55	5.28E+4	31.47	6.47E+4
50.00	2.05E+4	3.71E+4	4.36E+4	42.18	6.74E+4	50.00	1.57E+5
68.53	2.57E+4	5.24E+4	4.75E+4	57.82	8.33E+4	68.53	1.71E+5
87.06	4.44E+4	7.43E+4	9.55E+4	73.45	1.04E+5	87.06	1.96E+5
				89.09	1.61E+5		

25		22.5		20		17.5		15
15.91	1.02E+5	10.91	1.50E+5	12.94	2.37E+5	20.63	6.07E+5	1.60E+6
38.64	2.40E+5	26.55	1.74E+5	31.47	2.96E+5	50.00	6.25E+5	1.92E+6
61.36	3.14E+5	42.18	2.62E+5	50.00	1.35E+6	79.37	9.55E+5	2.37E+6
84.09	4.07E+5	57.82	3.19E+5	68.53	4.02E+6			
		73.45	7.41E+5	87.06	4.38E+6			
		89.09	7.72E+5					

Table 6 Parameters of exponential and Weibull distributions

Stress	Exponential	Weibull	
σ(*kgf/mm*^{2})	λ_1	λ	β
40	3.00E−5	1.26E−3	0.625
35	1.64E−5	2.34E−3	0.532
32.5	1.28E−5	2.18E−3	0.527
30	7.81E−6	1.08E−4	0.767
27.5	5.13E−6	8.47E−4	0.566
25	2.55E−6	3.46E−4	0.606
22.5	1.72E−6	1.54E−4	0.649
20	4.31E−7	3.41E−4	0.529
17.5	9.11E−7	2.77E−4	0.577
15	3.43E−7	1.53E−13	2.066

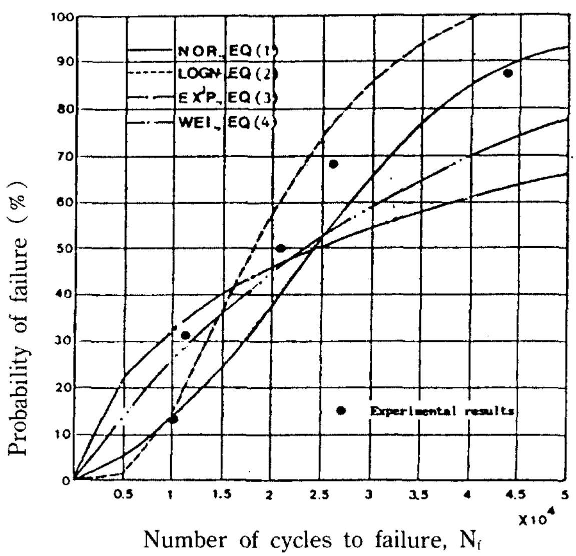


Fig. 4 Fatigue life distribution, $\sigma=40$(*kgf/mm*^{2})

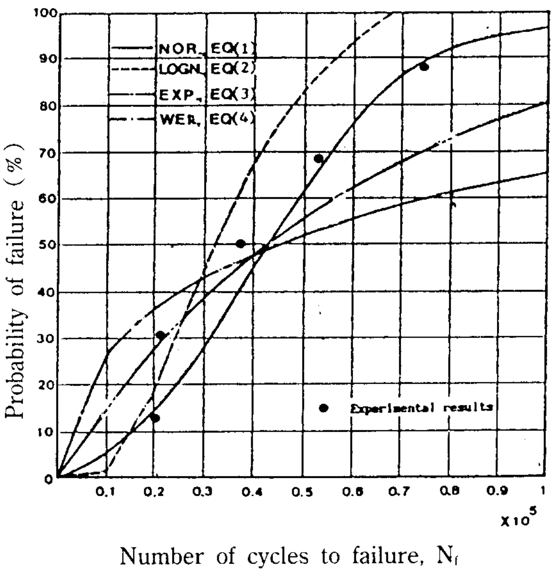


Fig. 5 Fatigue life distribution, $\sigma=35(kgf/mm^2)$

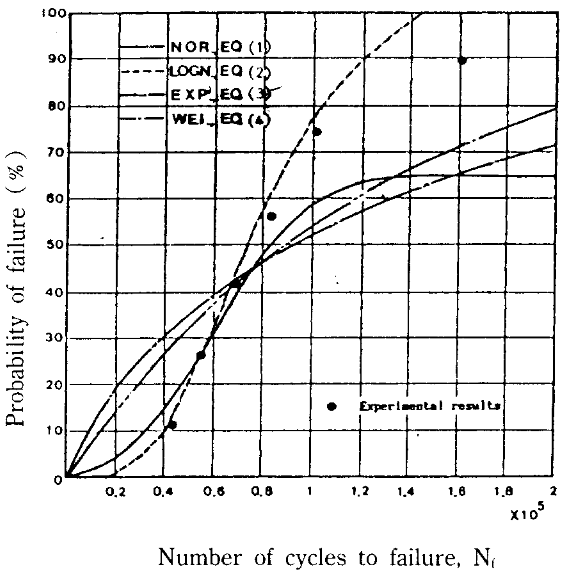


Fig. 7 Fatigue life distribution, $\sigma=30(kgf/mm^2)$

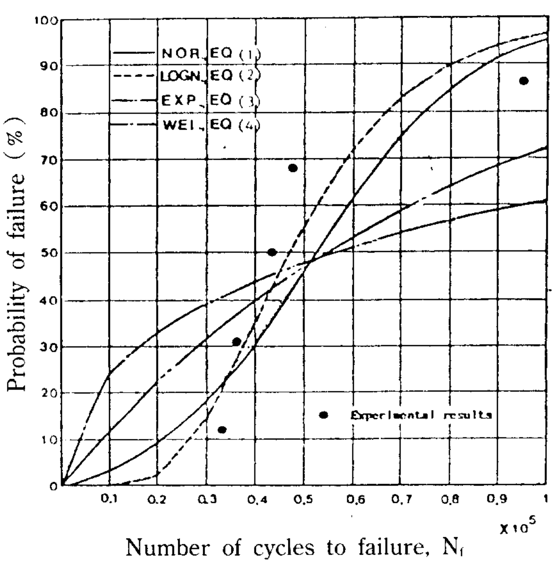


Fig. 6 Fatigue life distribution, $\sigma=32.5(kgf/mm^2)$

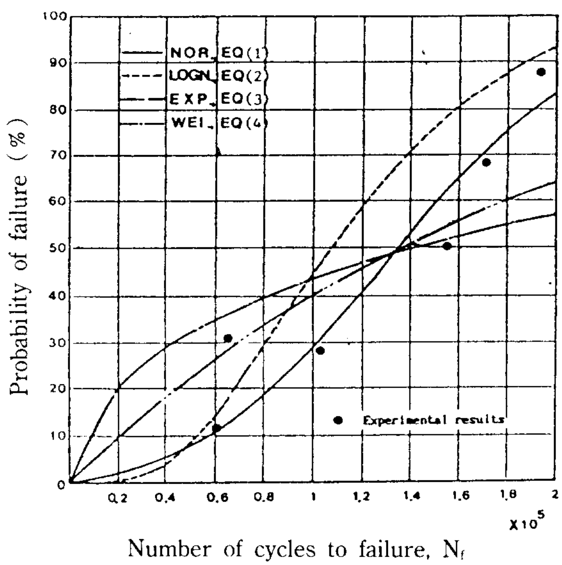


Fig. 8 Fatigue life distribution, $\sigma=27.5(kgf/mm^2)$

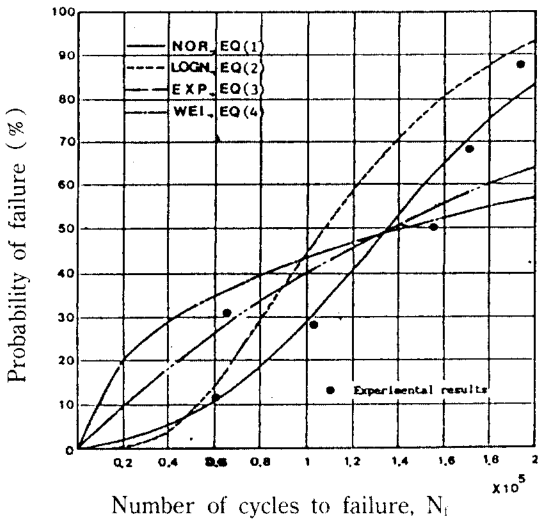


Fig. 8 Fatigue life distribution, $\sigma=20(kgf/mm^2)$

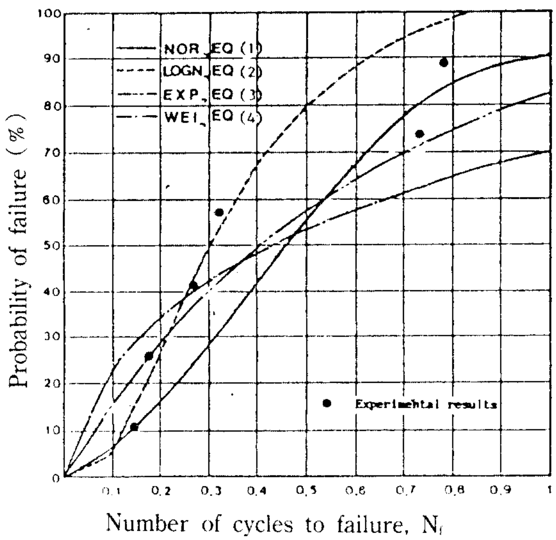


Fig. 10 Fatigue life distribution, $\sigma=22.5(kgf/mm^2)$

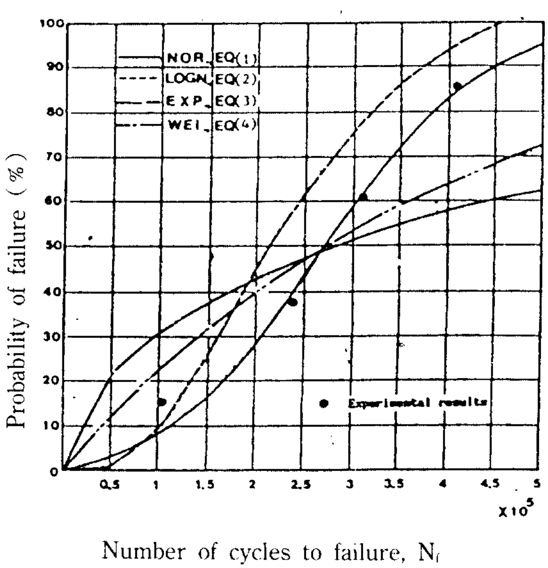
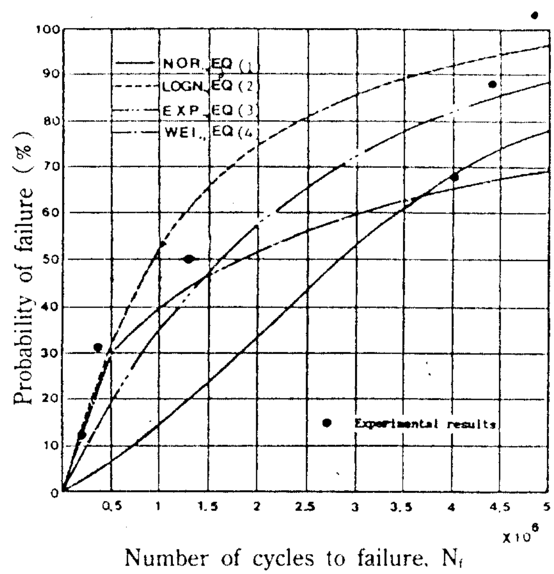


Fig. 9 Fatigue life distribution, $\sigma=25(kgf/mm^2)$



Fatigue life distribution, $\sigma=25(kgf/mm^2)$

calculated by Eq.(3) ; and dashdot curve represents the CDF in Weibull distribution calculated by Eq.(4), for the final fatigue life N_f.

These curves were compared with the data experimentally obtained in Tabel 3 and results of this comparison at the 50 percent failure probability are listed in Table 7. The result of this comparison, when applied to SM45C round bar specimens having a small drilled center hole of 2*mm* dia. Which have been subjected to fatigue cycles with constant rotating bending stress, shows good agreement with both log-normal CDF at the st-

Table 7 Comparison of experimental results with theoretical analysis at the 50 percent of failure probability

Stress level (kgf/mm^2)	Exptl. (N_f)	Normal Theor. (N_f)	Log-normal Theor. (N_f)	Exponential Theor. (N_f)	Weibull Theor. (N_f)
40	2.05E+4	2.25E+4	1.96E+4	2.31E+4	2.42E+4
35	3.71E+4	4.11E+4	3.54E+4	4.23E+4	4.43E+4
32.5	4.36E+4	5.11E+4	4.71E+4	5.40E+4	5.61E+4
27.5	1.57E+5	1.30E+5	1.16E+5	1.35E+5	1.40E+5
20	1.35E+6	2.06E+6	1.11E+6	1.61E+6	1.78E+6

ress level of 40, 32.5, 30, 22.5 and 20 kgf/mm^2 and normal CDF at the stress level of 35, 27.5, 25, 17. 5, and 15 kgf/mm^2 from the experimental results.

·A log-normal distribution, which provide the smallest value of N_f at the 50 percent failure probability at each stress level from the experimental results, may have safer predictive capabilities than any other distribution function.

Calculated results on the final fatigue life are in good agreement with the experimental results plotted by circular dots, although not passing through the experimental results. In these cases, calculated CDF coincides approximately well with that of experimental results, except for a few data points in extremely short and long life regions.

This disagreement in this study would be due to the smallness of the number of specimens, and the inhomogeneity of micro-composition, defects, micro-organization, geometric shape, and size.

Consequently, it was found that theoretical analysis developed in this paper was successfully applied to SM45C round bar specimens which have a center hole.

4. Conclusions

1) The cumulative distribution functions of the specimens with a center hole are derived from the constant rotating bending test results to examine fatigue life at each stress level.

2) The log-normal CDF gave the closes fit to the scatter in experimental results on constant amplitude fatigue life, at the stress level of 40, 32.5, 30, 22.5, and 20 kgf/mm^2, and the normal CDF gave the closest fit at the stress level of 35, 27.5, 25, 17.5, and 15 kgf/mm^2.

3) Taking safety design into account, log-normal CDF provides the smallest value of N_f at the 50 percent failure probability at all stress levels of the experimental results, so it may have safer predictive capabilities than any other distribution function.

More data would have reduced both experimental and statistical uncertainties.

Acknowledgment

The authors would like to thank Dr. Joon Bae Kim for useful advice and Mr. Gale Matheny for helpful encouragement.

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