論 文

Performance Analysis of Common Spreading Code CDMA Packet Radio Systems with Multiple Capture Capability

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多重캡처 특성의 單一擴散코드 CDMA 패킷 라디오 시스템들의 性能 分析

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ABSTRACT In this paper we present a multiple capture model for common spreading code CDMA packet radio systems with star topology. Basic equations for the collision-free, header detection, and multiple capture probabilities are derived at the central receiver. Link performances, including the average number of packet captures, allowable number of simultaneous transmission, and system throughput, are theoretically evaluated for a hybrid system, combining envelope header detection and differential data detection. Using the Block Oriented Systems Simulator(BOSS), simulations were carried out for the central receivers with envelope or differential header detection. It is shown that for a threshold approximation to the probability of data packet success, the multiple capture model significantly improves system throughput.

要 約 本 論文은 單一擴散코드 CDMA 方式의 中央 集中 패킷 라디오 시스템들을 위한 多重캡처 모델을 提示하였다. 中央 受信機에서 collision-free, 헤더 檢出, 多重캡처 確率들을 計算하기 위한 基本式들을 誘導하였다. 包絡線(envelope) 헤더 檢出方式과 差動(differential) 데이타 檢出方式을 結合한 hybrid 시스템에서 平均 패킷캡처의 數, 同時傳送可能한 패킷의 數, 시스템 throughput 등 link 레벨의 性能들이 理論的으로 評價되었다. 包絡線 또는 差動 헤더 檢出方式의 中央 受信機에 대해 블럭별 回路의 BOSS 시뮬레이션을 遂行하였다. 데이타패킷 成功確率에 threshold 모델을 適用하여 多重캡처 모델이 시스템의 throughput을 顯著히 增加시킬 수 있음을 確認하였다.

I. INTRODUCTION

This paper is concerned with the multiple capture in centralized packet radio systems in which all radios use spread-spectrum signals(1) with a common code for spectral spreading. It is well known that spread-spectrum signals have an especially strong capture property due to the processing gain after signal detection. Compared to the power capture that causes the strongest of two or more overlapping packets to be received correctly, here "capture" refers to the delay capture in which the first arrived packet is usually received correctly if the time offset between the first two arrivals is greater than

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the duration needed for packet synchronization.

There have been many efforts to improve system performance by using the capture effect existing in a spread-spectrum radio receiver. However, most previous works[2][3] focused on a single capture model which allows the first arrived packet to be captured and successfully received while the later packets are rejected as noise. Here we present a multiple capture model [4][5] in which the multiple capture is assumed to occur whenever some number of packets are collision-free, concurrently received signals are treated like noise in evaluating collision-free packet performance, and their common headers preceding a data packet are correctly detected.

The multiple capture model incorporates identification of the header of a packet to decide whether the packet is correctly captured or not. For analysis of multiple capture, we must combine the effects of interfering signals on header detection at the physical level with transmission processes at the link or network level which affect the distribution of packet arrival times at the receiver. Based on the proposed multiple capture model, the performance of packet radio systems with common spreading code and code division multiple access (CDMA) scheme is then analyzed.

II. MULTIPLE CAPTURE MODEL

1. System Description

A multiple capture model is developed for a star network in which a finite number K of mobile radios communicate with a single central receiver by the use of spread-spectrum signals. We are here interested with an asynchronous data transmission at the bit time level between the central receiver and its surrounding mobile radios while the packet transmissions are operated in a slotted random-access mode.

All radio terminals introduce some amount of random delays in their transmissions in order to randomize packet arrival times at the receiver, but for synchronization at the packet level, the random delays are required to be small compared to the packet duration. Each radio performs range measurements to eliminate the effect of propagation delays on the distribution of packet arrival times, by adjusting its transmission time as a function of the difference between the maximum and its propagation delay. The radio also utilizes power control to normalize received signal strengths, since in the centralized packet radio system, the central receiver can measure the signal strengths of received packets and send this information back to the transmitting radios.

For multiple successful transmissions, we require that all radio terminals use the common code for spectral spreading. At the central receiver, time-shifted signals using the same spreading code then appear like components of a multipath channel output with one radio transmission. Thus it is possible to receive a number of transmissions at the same time by properly resolving the overlapped signals with random time of arrivals.

2. Transmitted Signal Formats

A packet consists of a common header and data packet, in which the data packet contains an address and real data, and the address includes a source and destination address. The header sequence may be chosen from two different types of frame sync words, which are the Barker sequences with the maximum sidelobe correlation one and prefixes of PN sequences, *i.e.*, pseudonoise or maximal-length binary sequences (m-sequences).

We consider spreading codes which are the auto-optimal phase m sequences with least sidelobe energy (AO / LSE)[6]. These sequences are optimal with respect to the peak correlation parameters and the mean square correlation parameters. Each bit of the header and data packet is encoded by multiplying the spreading code of period N, resulting in a number of binary chips, each of length T_c, where N is also equal to the number of chips per data bit T_b.

As the modulation format, we employ CDMA direct sequence binary phase shifting keying (DS / BPSK) for envelope detection of received packets, while for differential detection, employ direct sequence differential phase shift keying (DS / DPSK).

3. Spread-Spectrum Multiple Capture Receiver

Generally, the radio channel can be equivalently modeled as a complex baseband in Figure 1. At the spread spectrum multiple capture receiver, we introduce a high rate sampling scheme with a code matched filter in which the sampling rate is equal to a chip rate or even a higher rate. Using the sampling technique, we do not acquire time synchronization at the bit time level, instead the header of every received packet is processed in order to properly identify the beginning of the packet as well as the presence of the packet.

For multiple capture, we consider two different types of header detection schemes, that is, envelope and differential detections, which require the observation of a received signal over a number of bit intervals and comparison with an appropriate threshold. These schemes will provide some gains over the case of conventional detection schemes making a bit by bit decision[2].

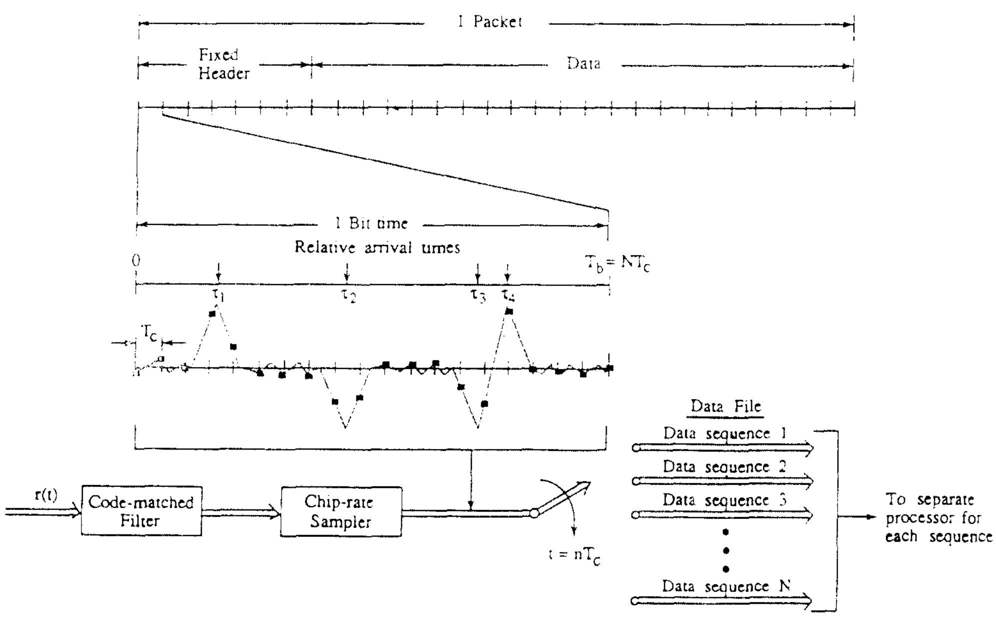


Fig.1. Spread spectrum multiple capture receiver.

Ⅲ. MULTIPLE CAPTURE PROBABILITY

A minimum time offset between arrival times of incoming packets needed to ensure the multiple capture with high probability depends on the chosen sampling rate and the time resolution of the code-matched filter output. We first define *collision-free region* as the set of times at which the code-matched filter output can be sampled in which any one transmission is effectively sampled in the mainlobe. If the minimum time offset Δ is chosen to be $T_c(1+\frac{1}{2\lambda})$ for some λ (the nubmer of samples per chip time T_c), the sampling time which gives rise to the largest correlation value of a desired signal will always lie within the collision-free region.

For analytical purposes, we assume that a packet is *collision-free* if the time offsets of the packet from its adjacent packets are larger than the minimum time offset $\Delta = T_c(1+\frac{1}{2\lambda})$ for a given λ. In this case, there always exists a collision-free sampling time that gives rise to the largest correlation value of a collision-free signal. Then *collision* is assumed to occur when the collision-free region of a packet does not exist. From this, it follows that a packet is *partially collision-free* if the collision-free region of the packet exists, but at least one of the time offsets is smaller than the minimum time offset Δ. Based upon the above assumptions, each received packet may be classified as collision-free, collided, or partially collision-free.

An exact analysis of the packet collisions in a continuous way is not tractable, and we approximate the real distribution of packet arrival times as follows: The packet arrival times $\{\tau_i\}_{i=1}^{m}$ (modulo T_c) are mutually independent and uniformly distributed among the set of discrete times, *i.e.*, $\{t_0,\cdots,t_{L-1}\}$, where the t_l's $(l=0,1,\cdots,L-1)$ are equally spaced over $[0,T_c]$, with spacing $\frac{\Delta}{2}$, given the minimum time offset Δ. By observing Figure 2, we find that if some arrivals are arranged among the L discrete times, no two consecutive, they can be classified as collision-free. On the other hand, if two or more arrivals

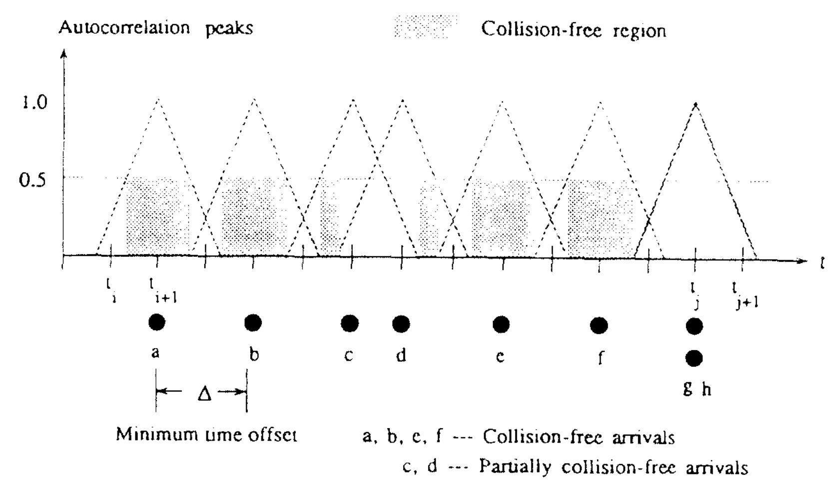


Fig.2. Classification of packets with discrete random arrival times when a chip-rate sampling is employed.

hit the same discrete time, they must be collided. But the other consecutive arrivals cannot be easily classified as collision free, which depends on the sampling rate λ. For simplicity, we define consecutive arrivals to be partially collision free.

Dependence among collision-free packet events can be handled by properly mapping these events into distinct elements of a set U as follows. Let an index set $A=\{a_1,a_2,\cdots,a_m\}$ denote the m labeled arrivals that entered the central receiver in the slot, and an index set $T=\{t_0,t_1,\cdots,t_{L-1}\}$ denote the L discrete time among which the m labeled arrivals are arranged, where L is given by $\frac{2T}{\Delta}$ with $\Delta=T/(1+\frac{1}{2\lambda})$ for a given λ. Define a set $U=\{x|A\rightarrow T$ where order counts, but repetition is allowed$\}$ where $|U|=L^m$. The set U is equivalent to the collection of all possible ways in which the m labeled arrivals may be arranged among the L discrete times with repetition.

Define for $\alpha\in A$,

$P_\alpha=\{x\in U|x$ has the property that a packet α is collision-free$\}$.

The Principle of Inclusion and Exclusion[8] states that for $f=1,2,\cdots,m$,

$$|\{x\in U\,|\,f(x)=f\}|$$

$$=\sum_{k\geq 0}(-1)^k\binom{f+k}{f}\sum_{\substack{J\subset A\\|J|=f+k}}\left|\bigcap_{\alpha\in J}P_\alpha\right| \qquad (1)$$

where $f(x)$ denotes the number of properties satisfied by an element $x\in U$, and J is any possible subset of the index set A with size $|J|=f+k$.

For derivation of the number of occurrences of the event that exactly f of the m labeled arrivals are collision free, we need to find the number of the events $\bigcap_{\alpha\in J}P_\alpha$ for $|J|=1,2,\cdots,m$. Since there are $\binom{m}{f+k}$ possible ways of choosing $(f+k)$ from total m properties and each way is equally likely to occur, conditioned on $|J|=q$, we find that

$$\sum_{\substack{J\subset A\\|J|=q}}\left|\bigcap_{\alpha\in J}P_\alpha\right|=\binom{m}{q}\cdot\left|\bigcap_{\alpha=a_1}^{a_q}P_\alpha\right|. \qquad (2)$$

We then have the following proposition.

Proposition 1 : For the subset $J=\{a_1,\cdots,a_q\}$, the number of occurrences of the event that at least q of the m labeled arrivals are collision free is given by (L even)

$$\left|\bigcap_{\alpha=a_1}^{a_q}P_\alpha\right|=\begin{cases}\sum_{r=1}^{q}(q-1)!\,L\binom{q}{r}\binom{L-2q-1}{r-1}(L-2q-r)^{m-q} & \text{if } 0<q<L/2,\\ L^m & \text{if } q=0,\\ 2(L/2)! & \text{if } q=m=L/2,\\ 0 & \text{otherwise}.\end{cases} \qquad (3)$$

Proof : Appendix A.

We now define $N(f)$ to be the number of ways of arranging the m labeled arrivals among the L discrete times so that exactly f packets are collision free. From (1), (2), and (3), we derive that for $1\leq f\leq m$,

$$N(f)\;=\;\sum_{k\geq 0}(-1)^k\binom{f+k}{f}\binom{m}{f+k}$$

$$\sum_{r=1}^{f+k}(f+k-1)!\,L\binom{f+k}{r}\binom{L-2(f+k)-1}{r-1}[L-2(f+k)-r]^{m-f-k} \qquad (4)$$

Let a random variable F and M denote the number of collision-free packets and the number of simultaneously transmitted packets in a slot, respectively. The probability $P_{F|M}(f|m)$ which some number f of m simultaneously transmitted packets are collision-free is given by

$$P_{F|M}(f|m) \triangleq \Pr\{F=f|M=m\} \qquad (5)$$

$$= \frac{N(f)}{L^m} \qquad \text{for } 0 \le f \le m \qquad (6)$$

where

$$N(0) = L^m - \textstyle\sum_{f=1}^{m} N(f).$$

We may also have some successful captures resulting from the partially collision-free transmissions that are arranged among the L discrete times. In some situations it may be important to evaluate the capture probability for partially collision-free packets, *i.e.*, the probability that the code-matched filter output will be sampled at a time when a partially collision-free packet will be detected. The following proposition may be useful in this evaluation.

Proposition 2 : We define $N_c(f)$ to be the number of ways of arranging the m labeled arrivals among the L discrete times so that $(m-f)$ packets are collided, *i.e.*, two or more arrivals at exactly the same discrete time, given that f packets are collision-free. For $1 \le f \le \min\{(m-2), (L/2-1)\}$, we then have

$$N_c(f) = \binom{m}{f}\sum_{r=1}^{f}(f-1)!\,L\binom{f}{r}\binom{L-2f-1}{r-1}{}_{(L-2f-r)}H_{(m-f)} \qquad (7)$$

$${}_nH_k = \sum_{h=1}^{\lfloor k/2 \rfloor}\binom{n}{h}\sum_{\substack{e_i \ge 2\ \forall i \\ e_1+\cdots+e_h=k}}\binom{k}{e_1,\ldots,e_h} \qquad (8)$$

where the symbol $\binom{k}{e_1,\ldots,e_h}$ denotes the multinomial coefficient, and

$$N_c(f) = \begin{cases} {}_LH_m & \text{if } f=0, \\ 0 & \text{if } f > \min\{(m-2), (L/2-1)\}. \end{cases}$$

Proof : Appendix B.

We define $N_p(f)$ to be the number of ways of having at least one of the $(m-f)$ packets partially collision-free and the remaining packets collided, given that f packets are collision free. From (4) and (8), we find that

$$N_p(f) = N(f) - N_c(f)\,. \qquad (9)$$

Let a random variable F_c and F_p denote the number of collision-free packets in a slot, provided that F_c is defined for the subevent of the remaining $(M-F_c)$ packets being collided, and F_p defined for the subevent of at least one of the $(M-F_p)$ packets being partially collision-free and the remaining packets collided. We then obtain the first-order approximation to $P_{F|M}(f|m)$ that accounts for the one-step transition due to the partially collision free transmissions

$$\bar{P}_{F|M}(f|m) \triangleq \Pr\{F_c = f|M = m\}$$
$$+\alpha_{f-1}\Pr\{E_p|F_p = f-1, M = m\}$$
$$\Pr\{F_p = f-1|M = m\}$$
$$+(1-\alpha_f\Pr\{E_p|F_p = f, M = m\})$$
$$\Pr\{F_p = f|M = m\} \qquad (10)$$

$$\cong \frac{N_c(f) + \eta N_p(f-1) + (1-\eta)N_p(f)}{L^m}$$
$$\text{for } 0 \le f \le m \qquad (11)$$

where $N_p(-1)=0$, E_p denotes an event of partially collision free transmissions being sampled at a collision free time, α_f indicates an average loss of such packets to be expected in the header detection process when compared to collision free packets, and $\eta \simeq \alpha_f \Pr\{E_p|F_p=f, M=m\}$ $(0 \le f \le m)$ is an average factor depending on the sampling rate λ that can be determined by locating the collision free region. For example, when $\eta=0$, we have $\bar{P}_{F|M}(f|m) = P_{F|M}(f|m)$, while if we increase η, then more weight is being placed on partially collision free packets being collision free.

Let $\Delta\tau_i$ denote the distance of the arrival time τ_i from the nearest sampling instant that yields the largest correlation value of the i th desired signal. If we define $\epsilon_i = \frac{\Delta\tau_i}{T}$, we find that ϵ_i is uniformly distributed over $[0, \frac{1}{2\lambda}]$. For an envelope detection with the multiple bit observation, assuming the unknown signal phases are constant over the header duration, and if the multiple access noise can be modeled as Gaussian noise, the probability of correct header detection, conditioned on (ϵ_i, m), can be shown to be

$$P_{h_i}(\epsilon_i, m)$$
$$= Q\left(\frac{\sqrt{2N_h}[1-\epsilon_i[1-\bar{\rho}_c(1)-\bar{\gamma}_h(1)\bar{\rho}_c(N-1)]]}{\sqrt{(m-1)\bar{\sigma}^2_{h_i}}}, \frac{\sqrt{2N_h}\kappa_h}{\sqrt{(m-1)\bar{\sigma}^2_{h_i}}}\right) \qquad (12)$$

where $Q(\alpha,\beta)$ is the Marcum Q function defined by

$$Q(\alpha,\beta) = \int_\beta^\infty v \exp\left[\frac{v^2+\alpha^2}{-2}\right] \mathrm{I}(\alpha v)\, dv,$$

N_h is the header length, $\rho_c(j)$ is the normalized partial autocorrelation function of the spreading code and $\gamma_h(j)$ is the normalized aperiodic autocorrelation function of the header sequence both at time offset j, κ_h is the threshold level for header detection, and $\sigma_{h_i}^2$ is the normalized second order moment of the multiple access noise over the header duration. By the symmetry, we can drop the subscript i in $P_{h_i}(\epsilon_i, m)$, and the probability of correct header detection, conditioned on m, is given by

$$P_h(m) = \mathbf{E}_\epsilon\{P_h(\epsilon, m)\}. \qquad (13)$$

By taking into account the effects of data, the multiple access noise is assumed to be independent from sample to sample when the randomization time is sufficiently large. Let a random variable C denote the number of packet captures per slot resulting from collision free packets. Given f of m simultaneously transmitted packets are collision-free, the conditional distribution of the number of packet captures is defined by

$$P_{C|F,M}(c|f,m) = \Pr\{C = c|F = f, M = m\}. \tag{14}$$

From the above independence assumption, we finally derive the multiple capture probability

$$P_{C|M}(c|m) = \sum_{f=c}^{m} P_{C|F,M}(c|f,m)P_{F|M}(f,m) \tag{15}$$

$$= \sum_{f=c}^{m} \binom{f}{c} P_h(m)^c[1 - P_h(m)]^{f-c} P_{F|M}(f|m). \tag{16}$$

We can also replace $P_{F|M}(f|m)$ by $\overline{P}_{F|M}(f|m)$ in (16) to obtain $P_{C|M}(c|m)$ that accounts for the average captures resulting from partially collision-free packets.

Ⅳ. LINK PERFORMANCES

At the link-level, the key design parameters in common spreading code systems are the average number of packet captures, allowable number of simultaneous transmissions, and system throughput. These link performance measures are theoretically evaluated for a hybrid system which combines envelope header detection and differential data detection. In the common code system, a desired signal is highly correlated with the other interfering signals, so the effect of captures on the packet success at the link-level will become significant as the number of simultaneous transmissions increases. In order to emphasize the capture effect on link performance, let us first assume perfect reception of a data packet after the correct header detection. There are K transmitting radios in the network, and each transmits its packet with probability δ. Under the slotted network, we assume the heavy traffic condition, so there is a packet available for transmission at the beginning of every slot. The transmission processes for different radios are assumed to be independent and identical, so that channel traffic is modeled as a binomial random variable with parameters K and δ. We ignore the effect of acknowledgements, assuming that a perfect and instantaneous acknowledgement channel is available. We note that the central receiver with multiple capture capability is equivalent to a multi-receiver which has as many channels as simultaneously captured packets in a given slot. We define $\overline{C}$ as the expected number of packet captures per slot at the central receiver. Here the first-order approximation $\overline{P}_{F|M(f|m)}$ is applied to compte the average number of packet captures. We then have the following expression.

$$\overline{C} = \mathbf{E}\{C\} = \mathbf{E}_M\{\mathbf{E}\{C|M\}\} \tag{17}$$

$$= \sum_{m=1}^{K}\sum_{c=1}^{m} cP_{C|M}(c|m)f_M(m) \quad \textbf{packets / slot} \tag{18}$$

where $f_M(m)$ is the probability of m packets being transmitted with $f_{M(m)} = \binom{k}{m} \delta^m(1-\delta)^{k-m}$ for $m \leq k$.

We proceed to evaluate the allowable number of simultaneous transmissions as an important parameter in evaluating the multiple capture capability of a common spreading code system. Here we determine the number of transmissions

K_T subject to a constraint on both the probability of packet capture and the probability of data bit error for given system parameters. By the symmetry, the probability of packet capture, conditioned on the m active radios in the slot, is given by

$$\bar{P}_{cs}(m) = \frac{1}{m}\mathbf{E}\{C|M=m\} = \frac{1}{m}\sum_{c=1}^{m} cP_{C|M}(c|m) . \qquad (19)$$

As the number of simultaneously transmitted packets increases, the Gaussian assumption on the multiple-access noise over a bit time is expected to be a good one. So we can apply the Gaussian assumption to obtain the probability of data bit error of the i th captured packet, conditioned on (ϵ_i, m). For differential data detection, it can be shown that

$$P_{b_i}(\epsilon_i, m) \cong \frac{1}{2}\left(1 - \frac{2\tilde{\sigma}_i^2}{\sigma_{b_i}^2}\right) \exp\left[\frac{[1-\epsilon_i[1-\bar{\rho}_c(1)]]^2}{-(m-1)\sigma_{b_i}^2}\right] \qquad (20)$$

where $\sigma_{b_i}^2$ is the second-order moment of the multiple-access noise over a bit time and $\tilde{\sigma}_i^2$ is the covariance of the in-phase desired signal components at two adjacent data bits. Dropping the subscript i because of the symmetry, the probability of data bit error, conditioned on m simultaneously transmitted packets, is given by

$$P_b(m) = \mathbf{E}_\epsilon\{P_b(\epsilon, m)\} \qquad (21)$$

where ϵ is uniformly distributed over $[0, \frac{1}{2\lambda}]$ for a given λ.

Throughput expression for the multiple capture model is derived by making a threshold approximation to the probability of data packet success, in which a data packet after being captured is assumed to be successfully received if $P_b(m)$ is maintained below a specified bit-error probability P_b^* during the data packet duration, and otherwise destroyed. We then approximate the probability of packet success as follows : If some number m of simultaneous transmissions does not exceed the cutoff K_T at which $P_b(K_T) \simeq P_b^*$, this probability is given by $\bar{P}_{cs}(m)$ itself, and zero otherwise. Given the $(m-1)$ interfering packets in the common channel, the probability of packet success takes the form,

$$P_{su}(m) = \begin{cases} \bar{P}_{cs}(m) & \text{if } m \le K_T , \\ 0 & \text{if } m > K_T . \end{cases} \qquad (22)$$

Computation of system throughput is straightforward, assuming channel traffic is a binomial random variable with parameters K and δ. Let a random variable S denote the number of packet successes per slot resulting from captured packets. We define the steady-state system throughput $\bar{S}$ as the expected number of packet successes per slot at the central receiver, that is,

$$\bar{S} = \sum_{m=1}^{K} mP_{su}(m)f_M(m) , \qquad (23)$$

and upon substitution from (19) and (22) in (23) we derive

$$\bar{S} = \sum_{m=1}^{\min(K,K_T)} \sum_{c=1}^{m} c P_{C|M}(c|m) f_M(m) \quad \textbf{packets / slot}. \qquad (24)$$

V. RESULTS

Using the Block Oriented Systems Simulator (BOSS), simulations were carried out for the central receivers with envelope or differential header detection, where the common header consists of 16-bit frame sync word with the maximum sidelobe correlation 3, generated by the 5-stage linear feedback shift register that corresponds to the polynomial $h(x)=x^5+x^2+1$ with initial sequence $\alpha_0=(0,1,1,1,0)$, the common spreading code is given by the AO / LSE m-sequences of period 63, 127, the sampling rate λ is chosen to be 1, and the header detection threshold k_h is set to 0.5.

We provide theoretical results for $P_{C|M}(c|m)$ as the first-order approximation with $\eta=1$ as well as simulation results when we employ envelope or differential header detection for multiple capture. Table 1 shows the theoretical and simulation results on $P_{C|M}(c|m)$ when $m=5$, $N=63$, and $m=10$, $N=127$. For this limited case, we find that the theoretical evaluation of $P_{C|M}(c|m)$ is within 5 percent of all of the simulation results. We also observe that in the simulations envelope detection has a higher packet

Table 1. Multiple capture probability $P_{C|M}(c|m)$ for $m=5$, $N=63$, and $m=10$, $N=12$:
* denotes simulation results and for the simulations, 16,000bits were tested.

(a) Envelope header detection

p.d.f. ($m=5$)	$c=1$	$c=2$	$c=3$	$c=4$	$c=5$
$P_{C\|M}(c\|m)$	0.003	0.021	0.117	0.222	0.637
$P^{*}_{C\|M}(c\|m)$	0.005	0.019	0.078	0.252	0.646

p.d.f. ($m=10$)	$c=3$	$c=4$	$c=5$	$c=6$	$c=7$	$c=8$	$c=9$	$c=10$
$P_{C\|M}(c\|m)$	0.001	0.003	0.014	0.035	0.112	0.180	0.284	0.371
$P^{*}_{C\|M}(c\|m)$	0.001	0.002	0.005	0.027	0.069	0.172	0.321	0.403

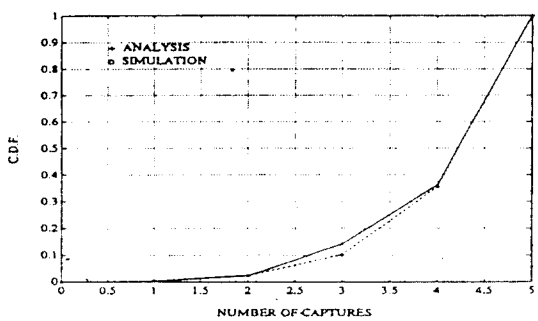
(b) Differential header detection

p.d.f. ($m=5$)	$c=1$	$c=2$	$c=3$	$c=4$	$c=5$
$P^{*}_{C\|M}(c\|m)$	0.003	0.022	0.084	0.279	0.612

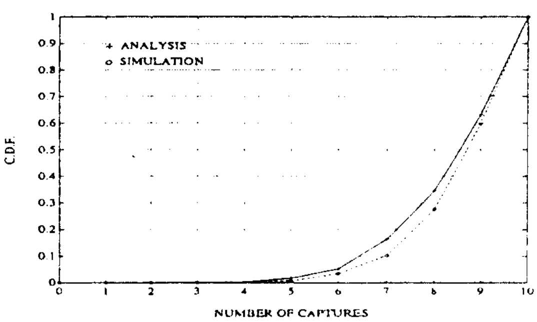
p.d.f. ($m=10$)	$c=3$	$c=4$	$c=5$	$c=6$	$c=7$	$c=8$	$c=9$	$c=10$
$P^{*}_{C\|M}(c\|m)$	0.001	0.003	0.006	0.023	0.092	0.203	0.304	0.368

capture rate than differential detection.

To see how closely the theoretical results resemble the simulation results, we plot the cumulative distribution function $\Pr(C \le c \mid M=m)$ in Figure 3 for the same parameters. For a moderate range of m, we see that theoretical results from the first-order approximation closely approach the simulation results. But when m is relatively large, theoretical results give slightly poor performance, since successful captures from partially collision-free packets increase in proportion to m.



(a) $m=5, N=63$



(b) $m=10, N=127$

Fig.3. Cumulative distribution function $\Pr(C \le c \mid M=m)$ for envelope header detection.

Figure 4 shows the average number of packet captures $\bar{C}$ versus the transmission probability δ for envelope header detection where the appropriate threshold levels are chosen depending on K. As K increases, the channel reaches a point where $\bar{P}_d(m)$ becomes small near at the value of $m=K$. At this point, as δ increases, $\bar{C}$ begins to become saturated because of lower packet capture rate for relatively larger $m(\le K)$. Here we see that the use of envelope header detection results in a significant improvement in performance compared to a single capture receiver ($C \le 1$).

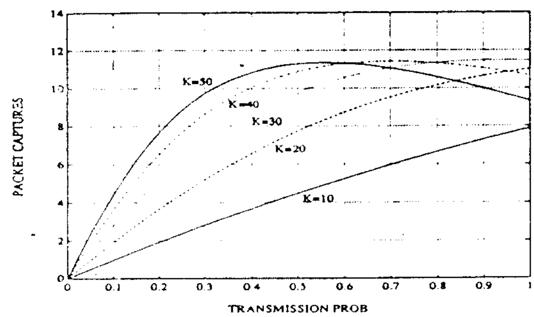


Fig.4. Average number of packet captures $\bar{C}$ for envelope header detection when $N=63$.

In Table 2, we present the allowable number of simultaneous transmissions $\bar{K}_t$ for the hybrid system supportable at the following specified probabilities of packet capture $\bar{P}_d^*=0.736, 0.530$ in which the data bit error probabilities $P_b(\bar{K}_t)$ are maintained near at the specified bit error probabilities $P_b^*=10^{-2}, 10^{-3}$, respectively.

In Figure 5, we plot the system throughput $\bar{S}$ as a function of δ for the hybrid system with the cutoff $K_t=21$ at which $P_b(K_t)=10^{-3}$. By the threshold approximation to the probability of data packet success, at the larger K exceeding K_t, the average number of packet successes drops off immediately after reaching the maximum throughput. So the performance is actually limited by the multiple access noise resulting from interfering data packets.

Table 2. Allowable number of simultaneous transmissions k_T for the hybrid system.

no chips	$P^{*}_{ca} = 0.736$		$P^{*}_{ca} = 0.530$	
N	$\bar{K}_T$	$P_b(\bar{K}_T)$	$\bar{K}_T$	$P_b(\bar{K}_T)$
31	7	2.8×10^{-5}	12	1.6×10^{-3}
63	12	1.9×10^{-5}	21	1.2×10^{-3}
127	22	0.4×10^{-5}	40	0.6×10^{-3}

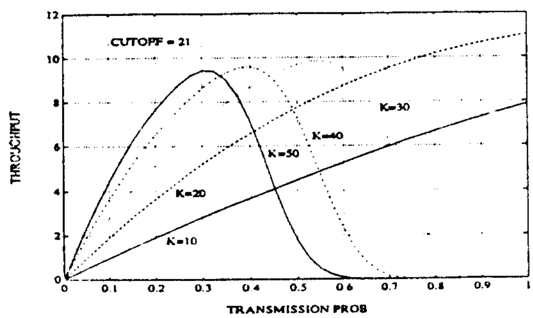


Fig.5. Average number of packet successes $\bar{S}$ for the hybrid system when $N=63$.

Ⅵ. CONCLUSION

In this paper we have analyzed common spreading code CDMA systems under the proposed multiple capture model. We presented theoretical results that indicate various characteristics of collision-free packet performance including the multiple capture and packet success of the hybrid system. Simulation results were also provided to validate theoretical results for the multiple capture probability. First, we found that the envelope detection with the multiple bit observation is more suitable for the common header reception compared to the differential header detection. Secondly, we have shown that we can improve significantly system performance by using the multiple capture property existing in the hybrid spread spectrum radio receiver. Finally, at the link-level, we evaluated two key design parameters in common spreading code systems, namely, the average number of packet captures and the allowable number of simultaneous transmissions that are supportable at a specified probability of packet capture and data bit error rate.

Appendix A

Proof of Proposition 1 :

First, the q arrival times are arranged in a circle, and one free discrete time is inserted between any two arrival times. We then choose r from the q arrival times in $\binom{q}{r}$ ways and place one more free discrete time in front of the r chosen arrival times to make r partitions, in which each partition looks like "$xaxa \cdots xax$" for which 'a' denotes one of the q arrival times and 'x' denotes one free discrete time. In order to arrange total L discrete times in the circle, the remaining $(L-2q-r)$ free discrete times are placed in front of the r partitions with repetition. The number of ways of doing this can be seen as the number of ways of choosing a redundant $(L-2q-r)$ partitions from the r partitions. By the redundant combination, this number is $\binom{L-2q-1}{r-1}$. For each arrangement associated with r partitions, we fix the first arrival time, say a_1, to avoid any possible circular symmetry, and rotate the arrangement one-by-one to the right so that this yields L different types of the arrangement. Finally, we permute the remaining $(q-1)$ arrival times to label the arrangement. Let $f_r(L,q)$ denote the number of ways of arranging q arrival times in order, no two consecutive, among total L discrete times in the circle to have r partitions ($r=$

$1,2,\cdots,q$). By the rule of product, we find that

$$f_r(L,q) = (q-1)!\,L\binom{q}{r}\binom{L-2q-1}{r-1}.$$

Next, for the corresponding subevent associated with r partitions, there are $(L-2q-r)$ discrete times available for arrangement of the remaining $(m-q)$ arrival times. Therfore, the number of ways of arranging the $(m-q)$ arrival times among the $(L-2q-r)$ discrete times, where order counts, but repetition is allowed, is $(L-2q-r)^{m-q}$ for a given r. So the number of occurrences of the event $\bigcap_{\alpha=\alpha_1}^{\alpha_q} P_\alpha$ becomes

$$\left|\bigcap_{\alpha=\alpha_1}^{\alpha_q} P_\alpha\right| = \sum_{r=1}^{q} f_r(L,q)(L-2q-r)^{m-q}.$$

This completes the proof of Proposition 1.

Appendix B

Proof of Proposition 2 :

First, arrange the arrival times of f packets among the L discrete times in a circle so that these packets are collision free. This event can be further classified into distinct subevents according to the number of partitions in them. Note that the number of occurrences of these subevents is given by $f_r(L,f)$ for $r=1,2,\cdots,f$. For a subevent with r partitions, there are $(L-2f-r)$ discrete times available for arrangement of the remaining $(m-f)$ arrival times.

Next, to insure that the remaining $(m-f)$ packets are collided, the arrival times of these packets are arranged among some number h ($1 \le h \le \lfloor \frac{(m-f)}{2} \rfloor$) of the $(L-2f-r)$ discrete times so that each discrete time contains at least two of them. From the multinomial theorem, we find that the number of ways of arranging them is equal to $\binom{m-f}{e_1,\cdots,e_h}$ subject to $e_i \ge 2\ \forall_i$ and $e_1+\cdots+e_h=m-f$. Since there are $\binom{L-2f-r}{h}$ possible ways of choosing the h discrete times, the results follow from the rule of product.

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