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Effect of Departures from Independence for a System

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ABSTRACT

For a series or parallel system, though the component lifetimes have the absolutely continuous bivariate exponential distributions(ACBVE) by Block and Basu(1974), the common assumption that the component lifetimes are independent is used. The purpose of this paper, in this case, is to investigate the magnitude of the error caused by erroneous assumption, using the measure proposed by Klein and Moeschberger(1986). Estimation of the measure is conducted by maximum likelihood estimator(MLE) and those estimators are compared with corresponding jackknifed MLE through the Monte Carlo study.

1. Introduction

Consider a system consisting of two components linked in a series or parallel. For this system, a common assumption is that the component lifetimes are independent. But in practice the situations that component lifetimes are dependent can be found very often. For instance, one thinks that his two eyes are operating independently. However, if one eye of the two was broken, then he could be apt to lose the insight of the other eye, too, because the other carries out the heavier duties than before.

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Some authors (Lagakos 1979, Easterling 1980 and so on)pointed out the need to determine quantitatively how off one might be if an analysis is based on an incorrect assumption of independence. Klein and Moeschberger (1984, 1986) studied the consequences of departures from independence in exponential series systems. They also studied the consequences of assuming independence when the component lifetimes follow the bivariate exponential distribution (BVE) of Marshall and Olkin (1967) and the BVE of Freund (1961). Successively, they investigated in 1987, the error of assuming independence that the component lifetimes follow the BVE models of Gumbel (1960), Downton (1970), and Oakes (1982). They concluded that the independence assumption consistently led to overestimate the system reliability and system mean life.

The purpose of this paper is to investigate the magnitude of the error caused by erroneously assumption that the component lifetmes have the independent exponential distribution, though, in fact, the component lifetimes follow the ACBVE by Block and Basu(1974), which has the bivariate loss of memory property(LMP). Section 2 introduces the ACBVE model of Block and Basu and obtain the system reliability and system mean life under the ACBVE model. Section 3 obtains the magnitude of the error caused by incorrectly assuming independence in terms of the measure which has been proposed by (incorrectly assyming independence in terms of kthe measure which has been proposed by)Klein and Moeschberger (1986). In Section, 4, the maximum likelihood estimator(MLE) for the measure is proposed and compared with the corresponding jackknifed MLE through the Monte Carlo study.

2. System Reliability and the System Mean Life under ACBVE

In this section, we introduce the ACBVE model and obtain some properties of the ACBVE model.

2.1 ACBVE Model

Let X and Y denote the component lifetimes of a system. Assume that X and Y have the ACBVE of Block and Basu (1974) with parameters λ_1, $\lambda_2>0$, $\lambda_{12}\geq 0$ whose density is given by

$$f(x, y)=\begin{cases}\lambda_1\lambda(\lambda_2+\lambda_{12})(\lambda_1+\lambda_2)^{-1}\exp[-\lambda_1 x-(\lambda_2+\lambda_{12})y] & \text{if } x<y \\ \lambda_2\lambda(\lambda_1+\lambda_{12})(\lambda_1+\lambda_2)^{-1}\exp[-(\lambda_1+\lambda_{12})x-\lambda_2 y] & \text{if } x>y,\end{cases} \tag{2.1}$$

where $\lambda=\lambda_1+\lambda_2+\lambda_{12}$.

This model has the so−called bivariate loss of memory property and does not have exponential marginals. The reliability function $\overline{F}(x, y) = Pr(X>x, Y>y)$ is given by

$$\overline{F}(x, y) = \frac{\lambda}{\lambda_1+\lambda_2} \exp[-\lambda_1 x-\lambda_2 y-\lambda_{12}\max(x, y)] - \frac{\lambda_{12}}{\lambda_1+\lambda_2} \exp[-\lambda \max(x, y)],\ x, y>0, \tag{2.2}$$

and marginals $\overline{F}_1(x)$ and $\overline{F}_2(y)$ are given by

$$\overline{F}_1(x) = \frac{\lambda}{\lambda_1+\lambda_2} \exp[-(\lambda_1+\lambda_{12})x] - \frac{\lambda_{12}}{\lambda_1+\lambda_2}\exp(-\lambda x),\ x>0, \tag{2.3}$$

$$\overline{F}_2(y) = \frac{\lambda}{\lambda_1+\lambda_2} \exp[-(\lambda_1+\lambda_{12})y] - \frac{\lambda_{12}}{\lambda_1+\lambda_2}\exp(-\lambda y),\ y>0, \tag{2.4}$$

where $\lambda = \lambda_1+\lambda_2+\lambda_{12}$.

Also the correlation coefficient between X and Y, denoted by ρ, is given by

$$\begin{aligned}\rho = &\lambda_{12}[(\lambda_1^2+\lambda_2^2)\lambda+\lambda_1\lambda_2\lambda_{12}] \\ &\times[(\lambda_1+\lambda_2)^2(\lambda_1+\lambda_{12})^2+\lambda_2(\lambda_2+2\lambda_1)\lambda^2]^{-1/2} \\ &\times[(\lambda_1+\lambda_2)^2(\lambda_1+\lambda_{12})^2+\lambda_1(\lambda_1+2\lambda_2)\lambda^2]^{-1/2}.\end{aligned} \tag{2.5}$$

Note that $0 \le \rho \le 1$.

2.2 System Reliability and System Mean Life

For a two component system, the system reliability is the probability that the system survives until time t, that is, $Pr[\min(X, Y)>t]$ in case of series and $Pr[\max(X, Y)>t]$ in case of parallel.

[1] Series System

First, assume that the component lifetimes X and Y have an independent exponential distribution with parameters λ_1 and λ_2, respectively. Then the system reliability, $\bar{F}_{I,s}(t)$, of this series system is

$$\begin{aligned}\bar{F}_{I,s}(t) &= Pr[\min(X, Y) >]t \mid \textit{independence}] \\ &= Pr[X>t]\cdot Pr[Y>t] \\ &= \exp[-(\lambda_1+\lambda_2)t] \qquad (2.6)\end{aligned}$$

and the system mean life, $\mu_{I,S}$, is

$$\mu_{I,S} = \frac{1}{\lambda_1+\lambda_2} \qquad (2.7)$$

Let $t_{I,S}$ be the point at which the system relicbility is p under the assumption of independence. Then,

$$t_{I,S} = \frac{-\ln p}{\lambda_1+\lambda_2},$$

where

$$p = \bar{F}_{I,S}(t_{I,S}). \qquad (2.8)$$

Now assume that X and Y have the ACBVE of Block and Basu(1974). Then the system reliability of series system, $\bar{F}_{D,S}(t)$, is

$$\begin{aligned}\bar{F}_{D,S}(t) &= Pr[\min(X, Y) > t \mid ACBVE] \\ &= Pr[X>t, Y>t] \mid ACBVE\lambda] \\ &= \bar{F}(t, t) \\ &= \frac{\lambda}{\lambda_1+\lambda_2}\exp[-(\lambda_1+\lambda_2+\lambda_{12})t] - \frac{\lambda_{12}}{\lambda_1+\lambda_2}\exp(-\lambda t) \\ &= \exp(-\lambda t) \qquad (2.9)\end{aligned}$$

and the system mean life, $\mu_{D,S}$, is

$$\mu_{D,S} = \frac{1}{\mu} = \frac{1}{\lambda_1+\lambda_2+\lambda_{12}} \qquad (2.10)$$

[2] Parallel System

First, assume that the component lifetimes X and Y are independently exponentially distributed. Then the system reliability of the parallel system is

$$\begin{aligned}\bar{F}_{I,P}(t) &= Pr[\max(X, Y) > t \mid \textit{independence}] \\ &= 1 - Pr[\max(X, Y) \le t \mid \textit{independence}] \\ &= 1 - Pr[X < t] \cdot Pr[Y < t] \\ &= \exp(-\lambda_1 t) + \exp(-\lambda_2 t) - \exp[-(\lambda_1 + \lambda_2)t] \end{aligned} \tag{2.11}$$

and the system mean life is

$$\begin{aligned}\mu_{I,P} &= \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2} \\ &= \frac{(\lambda_1 + \lambda_2)^2 - \lambda_1\lambda_2)}{\lambda_1\lambda_2(\lambda_1 + \lambda_2)} \end{aligned} \tag{2.12}$$

Let $t_{I,P}$ be the point at which the system reliability is p under the assumption of independence. Then one can get the following relationship.

$$\exp(-\lambda_1 t_{I,P}) + \exp(-\lambda_2 t_{I,P}) - \exp[-(\lambda_1 + \lambda_2) t_{I,P}] = p, \tag{2.13}$$

where $p = \bar{F}_{I,P}(t)$.

Now assume that X and Y have the ACBVE of Block Basu(1974). Then the system reliability of the parallel system is

$$\begin{aligned}\bar{F}_{D,P}(t) &= Pr[\max(X,Y) > t \mid ACBVE] \\ &= 1 - Pr[\max(X, Y) \le t \mid ACBVE] \\ &= Pr(X > t) + Pr(Y > t) - Pr(X > t, Y > t) \\ &= \bar{F}_1(t) + \bar{F}_2(t) - \bar{F}(t, t) \\ &= \frac{\lambda}{\lambda_1 + \lambda_2} \{\exp[-(\lambda_1 + \lambda_{12})t] + \exp[-(\lambda_2 + \lambda_{12})t]\} \\ &\quad - \frac{\lambda + \lambda_{12}}{\lambda_1 + \lambda_2} \exp(-\lambda t), \end{aligned} \tag{2.14}$$

and the system mean life is

$$\mu_{D,P} = \frac{1}{\lambda_1+\lambda_2}\left[\frac{\lambda}{\lambda_1+\lambda_{12}} + \frac{\lambda}{\lambda_2+\lambda_{12}} - \frac{\lambda+\lambda_{12}}{\lambda}\right]. \qquad (2.15)$$

3. The Effect of Departures from Independence

In this section, we investigate the effect of incorrectly assuming independence for predicting the system reliability and the system mean life.

3.1 Measures of Effect

Klein and Moeschberger (1986) introduced the convenient measures of the effect of incorrectly assuming independence, which are

$$\Delta(t) = \frac{\bar{F}_D(t) - \bar{F}_I(t)}{\bar{F}_I(t)}$$

and

$$\delta = \frac{\mu_D - \mu_I}{\mu_I} \qquad (3.1)$$

for predicting the system reliability and the system mean life, respectively. Here, $\bar{F}_D(t)$ and μ_D are computed on the dependent model.

We use this measure not only when the system is series but also when the system is parallel.

First, under the series system, consider the effect of departures from independence. Then for the system reliability, from (2. 6) and (2. 9)

$$\Delta_S(t) = \frac{\bar{F}_{D,s}(t) - \bar{F}_{I,S}(t)}{\bar{F}_{I,S}(t)} = \exp(-\lambda_{12}t) - 1. \qquad (3.2)$$

For the system mean life, from (2. 7) and (2. 10)

$$\delta_S = \frac{\mu_{D,S} - \mu_{I,S}}{\mu_{IS}} = -\frac{\lambda_{12}}{\lambda}. \qquad (3.3)$$

Now, under the parallel system, consider the effect of departures from independence. Then for the system reliabilability, form (2. 11) and (2. 14)

$$\Delta_P(t) = \frac{\bar{F}_{D,P}(t) - \bar{F}_{I,P}(t)}{\bar{F}_{I,P}(t)}$$

$$= \frac{\exp(-\lambda_{12}t)}{\lambda_1+\lambda_2}\{\lambda - \lambda_{12}[\exp(\lambda_1 t) + \exp(\lambda_2 t) - 1]^{-1}\} - 1. \quad (3.4)$$

For the system mean life, from (2, 12) and (2. 15)

$$\delta_P = \frac{\mu_{D,P} - \mu_{I,P}}{\mu_{IP}}$$

$$= [\frac{\lambda}{\lambda_1+\lambda_{12}} + \frac{\lambda}{\lambda_2+\lambda_{12}} - \frac{\lambda+\lambda_{12}}{\lambda}] \times [\frac{(\lambda_1+\lambda_2)^2 - \lambda_1\lambda_2}{\lambda_1\lambda_2}]^{-1} - 1. \quad (3.5)$$

3.2 Interpretations of the Effect of Departures from Independence

First, under the parallel system, consider the effects of departures from independence.

For the fixed values of $p=\bar{F}_{I,P}(t)$ given $\lambda_1=0.30$ and $\lambda_2=0.30$, 0.55, 0.80, 1.05, the values of t are obtained from (2.13), and are given in Table 3.1. Figures 3.1 and 3.2 are plots of $\Delta_P(t)$ and δ_P for $p=0.9$ and for $\lambda_1=0.30$ and $\lambda_2=0.30$, 0.55, 0.8, 1.05 as a function of correlation. From Figures 3.1 and 3.2, one can observe the following facts:

(1) $\Delta_P(t)$ and δ_P become consistently less than zero as the correlation coefficient increases. This means that $\bar{F}_{I,P}(t) > \bar{F}_{D,P}(t)$ and $\mu_{I,P} > \mu_{D,P}$. That is, the independence assumption for the system whose component lifetimes, in fact, follow the ACBVE model, consistently leads to over-estimate the system reliability and the system mean life.

(2) $\Delta_P(t)$ and δ_P have the decreasing tendency according as the correlation increases. This means that the more strong independence assumption is, the more it may cause the error. Similarly, as shown in Tables 3.3 and 3.4, the case of the series system is the same as the case of parallel system.

(3) For series system, if the difference between λ_1 and λ_2 is large, the errors of departures from independence, $\Delta_S(t)$ and δ_S, are also large. But the case of parallel system is not.

4. Estimation of $\Delta(t)$ and δ

4.1 MLE and Jackknifed MLE

Block and Basu (1974) obtained the maximum likelihood estimators (MLE) for parameters λ_1, λ_2 and λ_{12} of the ACBVE. For the special case of $\lambda_1=\lambda_2=\alpha$ and $\lambda_{12}=\beta$, Klein and Basu (1985) showed that the MLE's α and β of α and β, respectively, are

$$\hat{\alpha}=n\left(\frac{1}{u_2-u_1}-\frac{1}{2u_1-u_2}\right)$$

and

$$\hat{\beta}=n\left(\frac{2}{2u_1-u_2}-\frac{1}{u_2-u_1}\right), \tag{4.1}$$

where $u_1=\Sigma_{i=1}^{n}\max(x_i, y_i)$ and $u_2=\Sigma_{i=1}^{n}(x_i+y_i)$.

Now, to obtain the MLE's of $\Delta(t)$ and δ, we also assume that the component lifetimes X and Y have the same marginals, that is, $\lambda_1=\lambda_2=\alpha$ and $\lambda_{12}=\beta$. Then, substitsting (4.1) in (3.2), (3, 3), (3,4) and (3.5), one can obtain the MLE's of $\Delta(t)$ and δ by the invariance property of the MLE.

For the series system, the $\hat{\Delta}_S(t)$ and $\hat{\delta}_S$ of $\Delta_S(t)$ and δ_S are

$$\hat{\Delta}_S=\exp(-\hat{\beta}t)-1 \text{ and } \hat{\delta}_S=-\frac{\hat{\beta}}{2\hat{\alpha}+\hat{\beta}}, \tag{4.2}$$

respectively.

For the parallel system, the MLE's $\hat{\Delta}_P(t)$ and $\hat{\delta}_P$ of $\Delta_P(t)$ and δ_P are

$$\hat{\Delta}_P(t)=\frac{\exp(-\hat{\beta}t)}{2\hat{\alpha}}\{(2\hat{\alpha}+\hat{\beta})-\hat{\beta}[2\exp(\hat{\alpha}t)-1]^{-1}\}-1$$

and

$$\hat{\delta}_P=\frac{2\hat{\alpha}(3\hat{\alpha}+2\hat{\beta})}{3(\hat{\alpha}+\hat{\beta})(2\hat{\alpha}+\hat{\beta})}-1, \tag{4.3}$$

respectively.

Now, we consider this jackknifed version of the MLE. This jackknifed MLE is constructed as follows:

For the series system, let $\hat{\Delta}^{(j)}_{n-1,S}(t)$ be the MLE of $\Delta_S(t)$ based on the subsample of size of n −1 obtained by deleting the j−th observation from the original samples. Then the jackknifed MLE of $\Delta_S(t)$, $\hat{\Delta}_{jack,S}(t)$, is

$$\hat{\Delta}_{jack,S}(t) = n\hat{\Delta}_S(t) - (n-1)\bar{\hat{\Delta}}^{(j)}_{n-1,S}(t)$$

$$= n\hat{\Delta}_S(t) - \frac{n-1}{n} \sum_{j=1}^{n} \hat{\Delta}^{(j)}_{n-1,S}(t). \tag{4.4}$$

For the jackknifed version of MLE of δ_S, $\hat{\delta}_{jack,S}$, the similar procedure can be applied.

For the parallel system, let $\hat{\Delta}^{(j)}_{n-1,P}(t)$ be the MLE of $\Delta_P(t)$ based on the subsample of size of n−1 obtained by deleting the j−th observation from the original samples. Then the jackknifed MLE of $\Delta_P(t)$, $\hat{\Delta}_{jack,P}(t)$, is

$$\hat{\Delta}_{jack,P}(t) = n\hat{\Delta}_P(t) - (n-1)\bar{\hat{\Delta}}^{(j)}_{n-1,P}(t)$$

$$= n\hat{\Delta}_P(t) - \frac{n-1}{n} \sum_{j=1}^{n} \hat{\Delta}^{(j)}_{n-1,P}(t). \tag{4.5}$$

For the jackknifed version of MLE of δ_S, $\hat{\delta}_{jack,S}$, the similar procedure can be applied.

4.2 Monte Carlo Simulation Study

We compare the performances of MLE and jackknifed MLE for $\Delta_S(t)$, δ_S, $\Delta_P(t)$ and δ_P in terms of bias and mean square error(MSE) through Monte Carlo study. The random samples were generated by the method of Friday and Patil(1977). 1000 replications was done. The design of simulations is summarized as the following Table 4.1 and Table 4.2 shows their results:

From Table 4.2 one can observe the followings:

(1) In terms of bias and MSE, the jackknifed MLE performs better than the corresponding MLE.

(2) As correlation increases, the MSE's of the jackknifed MLE and the MLE tend to increase slightly.

(3) As sample size increases, the MSE has a decreasing tendency for the jackknifed MLE and the MLE.

Table 3.1 Values of t when $\overline{F}_I(t)=0.9$,
given $\lambda_1=0.30$, $\lambda_2=0.30$, 0.55, 0.80, 1.05.

Series system		
λ_1	λ_2	$\lambda_{I,S}$
0.30	0.30	0.076
0.30	0.55	0.054
0.30	0.80	0.042
0.30	1.05	0.034

Parallel system		
λ_1	λ_2	$\lambda_{I,P}$
0.30	0.30	1.267
0.30	0.55	0.945
0.30	0.80	0.796
0.30	1.05	0.706

Table 4.1 Design of Simulations

α $=\lambda_1=\lambda_2$	β $=\lambda_{12}$	Sample Size n	True	t
	0.20	30	LDS $=\Delta_S(t)$	$\overline{F}_{I,S}(t)=0.9$
1.50	0.40	40	SDS $=\delta_S$	
	0.60	50	LDP $=\Delta_P(t)$	$\overline{F}_{I,P}(t)=0.9$
	0.80	100	SDP $=\delta_P$	
	1.00			

PARALLEL SYSTEM

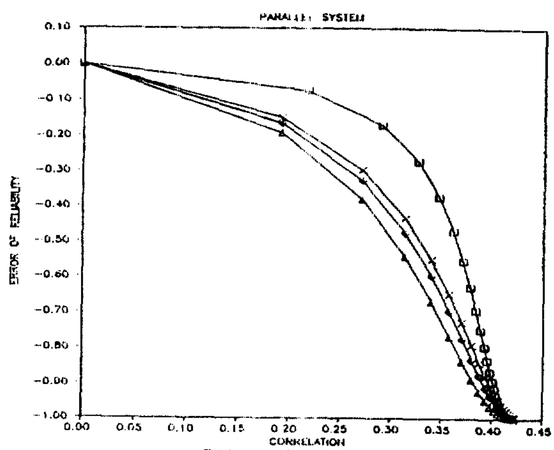

Figure 3.1 Plot of $\Delta_P(t)$

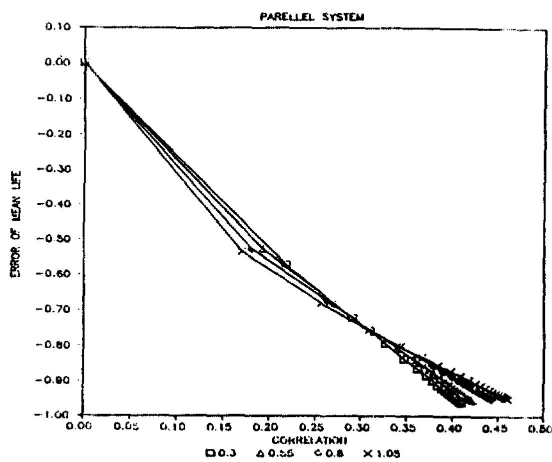

Figure 3.2 Plot of δ_P

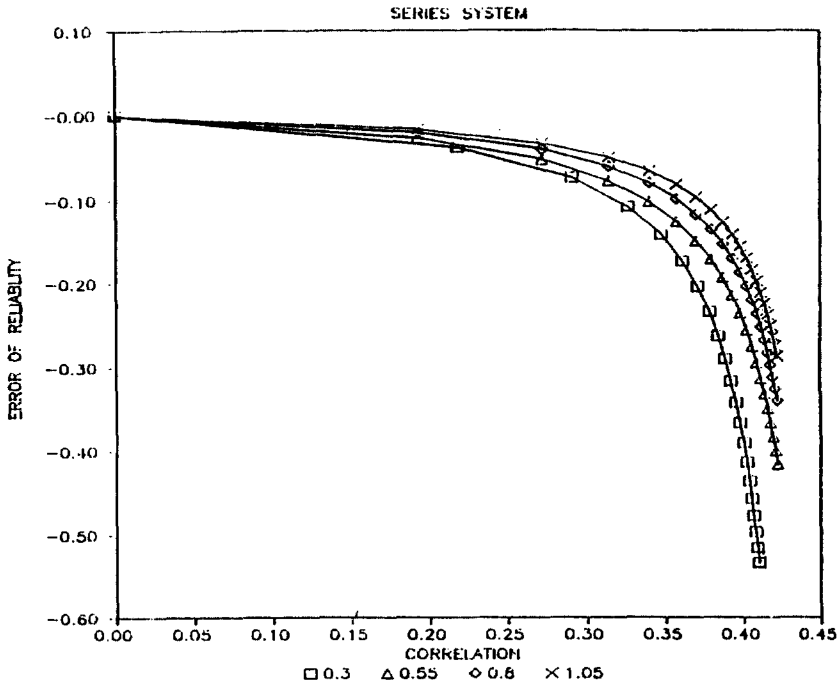

Figure 3.3 Plot of $\Delta_S(t)$

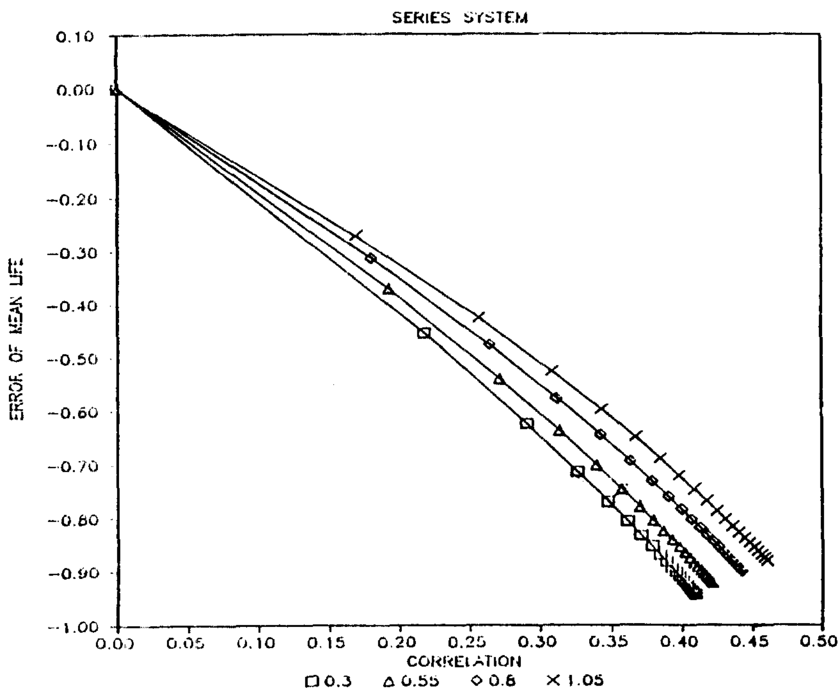

Figure 3.4 Plot of δ_S

Table 4.2 Bias and MSE of Estimators of $\Delta(t)$ and δ.

$n=30$

BETA	CORR	TYPE	TRUE	BIAS MLE	BIAS JNMLE	MSE MLE	MSE JNMLE
200	.0312	LDS	−.0070	−.0002	−.0006	.0010	.0009
		SDS	−.0625	−.0396	−.0009	.0825	.0750
		LDP	−.0190	.0148	−.0051	.0092	.0075
		SDP	−.0993	.0230	−.0094	.1820	.1682
.400	.0586	LDS	−.0139	−.0002	−.0003	.0012	.0012
		SDS	−.1176	−.0422	−.0031	.0998	.0922
		LDP	−.0385	.0175	−.0028	.0104	.0088
		SDP	−.1796	.0276	−.0011	.1919	.1785
.600	.0828	LDS	−.0209	−.0004	−.0003	.0013	.0012
		SDS	−.1667	−.0418	−.0001	.1979	.0895
		LDP	−.0585	.0177	−.0042	.0108	.0092
		SDP	−.2460	.0191	−.0060	.1613	.1509
.800	.1042	LDS	−0.277	.0005	,0006	.0018	.0017
		SDS	−.2105	−.0448	−.0012	.1350	.1241
		LDP	−.0789	.0266	−.0003	.0140	.0119
		SDP	−.3021	.0311	−.0065	.1879	.1744
1.000	.1233	LDS	−.0345	.0013	.0013	.0019	.0018
		SDS	−.2500	−.0275	.0166	.1156	.1080
		LDP	−.0995	.0272	−.0042	.0161	.0162
		SDP	−.3500	.0424	.0182	.1694	.1564

Table 4.2 (continued)

n=40

BETA	CORR	TYPE	TRUE	BIAS MLE	BIAS JNMLE	MSE MLE	MSE JNMLE
.200	.0312	LDS	−.0070	.0012	.0009	.0007	.0007
		SDS	−.0625	−.0185	.0092	.0595	.0562
		LDP	−.0190	.0141	.0010	.0060	.0051
		SDP	−.0993	.0361	.0022	.1408	.1324

.400	.0586	LDS	−.0139	−.0008	−.0009	.0009	.0008
		SDS	−.1176	−.1176	−.0065	.0701	.0653
		LDP	−.0385	.0105	−.0040	.0066	.0059
		SDP	−.1796	.0118	−.0100	.1334	.1265
.600	0.828	LDS	−.0209	.0016	.0015	.0010	.0009
		SDS	−.1667	−.0167	.0140	.1746	.0707
		LDP	−.0585	.0181	.0020	.0074	.0066
		SDP	−.2460	.0370	.0165	.1276	.1204
.800	.1042	LDS	−0.277	−.0009	−.0007	.0011	.0010
		SDS	−.2105	−.0353	−.0031	.0772	.0724
		LDP	−.0789	.0121	−.0049	.0077	.0069
		SDP	−.3021	.0097	−.0083	.1146	.1088
1.000	.1233	LDS	−.0345	−.0004	−.0002	.0013	.0012
		SDS	−.2500	−.0338	−.0012	.0850	.0796
		LDP	−.0995	.0160	−.0026	.0093	.0085
		SDP	−.3500	.0138	−.0030	.1163	.1105

Table 4.2(continued)

n=50

				BIAS		MSE	
BETA	CORR	TYPE	TRUE	MLE	JNMLE	MLE	JNMLE
.200	.0312	LDS	−.0070	.0001	−.0001	.0006	.0005
		SDS	−.0625	−.0212	.0013	.0469	.0446
		LDP	−.0190	.0088	−.0015	.0046	.0041
		SDP	−.0993	.0181	−.0011	.1089	.1039
.400	.0586	LDS	−.0139	.0007	.0006	.0006	.0006
		SDS	−.1176	−.1165	.0067	.0500	.0479
		LDP	−.0385	.0114	.0003	.0051	.0045
		SDP	−.1796	.0232	.0057	.1004	.0959
.600	0.828	LDS	−.0209	−.0016	−.0007	.0008	.0008
		SDS	−.1667	−.0293	−.0053	.0620	.0586
		LDP	−.0585	.0094	−.0025	.0056	.0052
		SDP	−.2460	.0091	−.0063	.1001	.0962
.800	.1042	LDS	−0.277	.0000	.0001	.0009	.0009
		SDS	−.2105	−.0256	−.0005	.0656	.0624
		LDP	−.0789	.0129	−.0001	.0070	.0064
		SDP	−.3021	.0153	.0010	.1015	.0976

		LDS	−.0345	−.0016	−.0015	.0010	.0010
1.000	.1233	SDS	−.2500	−.0355	−.0092	.0667	.0629
		LDP	−.0995	.0186	−.0059	.0067	.0062
		SDP	−.3500	.0004	−.0131	.0904	.0869

Table 4.2(continued)

n=100

				BIAS		MSE	
BETA	CORR	TYPE	TRUE	MLE	JNMLE	MLE	JNMLE
		LDS	−.0070	.0003	.0002	.0003	.0003
.200	.0312	SDS	−.0625	−.0097	.0012	.0258	.0252
		LDP	−.0190	.0052	.0006	.0022	.0021
		SDP	−.0993	.0131	.0040	.0583	.0570
		LDS	−.0139	.0007	.0006	.0003	.0003
.400	.0586	SDS	−.1176	−.0067	.0046	.0272	.0266
		LDP	−.0385	.0067	.0017	.0024	.0023
		SDP	−.1796	.0160	.0075	.0535	.0522
		LDS	−.0209	−.0010	−.0010	.0004	.0004
.600	0.828	SDS	−.1667	−.0214	−.0094	.0315	.0313
		LDP	−.0585	.0031	−.0022	.0026	.0025
		SDP	−.2460	−.0033	−.0109	.0512	.0503
		LDS	−0.277	.0005	.0004	.0005	.0004
.800	.1042	SDS	−.2105	−.0170	−.0047	.0322	.0313
		LDP	−.0789	.0049	−.0010	.0030	.0028
		SDP	−.3021	.0023	.0048	.0490	.0480
		LDS	−.0345	−.0007	−.0006	.0005	.0005
1.000	.1233	SDS	−.2500	−.0179	−.0051	.0323	.0314
		LDP	−.0995	.0044	−.0020	.0030	.0029
		SDP	−.3500	−.0001	−.0066	.0444	.0435

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