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Cost Analysis of Manufacturer's View Point Under Stepdown Warranty Policy

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ABSTRACT

This article is concerned with cost analysis in warranty policy. The warranty cost can be different according to warranty rate and warranty renewal policy. In this paper the stepdown warranty policy is analyzed. Assuming the nonrepairable item, manufacturer's cost is calculated in stepdown warranty policy and free replacement, pro-rata, hybrid policy. Numerical examples are given over Weibull time-to-failure distribution.

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Ⅰ. Introduction

With the advance of manufacturing technologies, feature and performance of the products are rapidly increasing and customers tend to choose products which have the highest reliability among the ones having identical features and performances.

For this reason, warranty policy has been introduced by the manufacturers for active Quality Assurance. Manufactures take the full responsibility for products failures occurred during specified period after sales under warranty policy.

In this section, cost analysis procedures studied so far with irrepairable products under this warranty policy are reviewed.

First, Menke(1969) calculated replacement cost of irrepairable product which has exponential failure distribution function. He also handled price determination problem considering warranty cost when prodata policy is used.

Blischke and Scheuer(1981) calculated manufacturer's profit under free replacement policy for irrepairable products which should be replaced with new ones whenever failure occurs and over some hazard functions like exponential using renewal process.

Thomas(1983) calculated manufacturer's cost of irrepairable products under the free replacement and the hybrid policy, where customers receive renewal warranties whenever failure occurs during warranty periodqs. Some hazard functions like weibull are considered and sensitivity analysis was performed for hazard function parameters on manufacturer's cost aspect.

Kim(1988) suggested stepdown warranty policy model. He assumed repairable and irrepairable products and calculated costs of both manufacturer and customer for each product under each warranty renwal policy. He derived manufacturer's warranty period under three special cases of stepdown warranty policy like free replacement, prorata and hybrid policy.

This paper explains stepdown warranty policy and discusses its utility. Manufacturer's warranty cost is calculated by proposing warranty period renewal policy under three warranty policies.

Ⅱ. Contents and Scope of the Research

Study in warranty policy depends on warranty rate and warranty renewal policy during the warranty period. Manufacturers should determine a liable rate for the products during the warranty period.

Three warranty policies are commonly used according to warranty rate. In free replacement policy, manufacturers take the full responsibility for all costs incurred during the warranty period. Prorata policy increases customer's liability as the product usage time goes on. Hybrid policy adopts free replacement policy within a certain period of time and then uses prorata policy.

In this paper, stepdown warranty policy which generalize these three policies is introduced.

Cost to manufacturer varies as the warranty contract renewed, the cost is calculated under such warranty policy as failed product is replaced with new one and warranty period renewed whenever a product replaced during warranty period.

The costs are also calculated, compared and analyzed under those three different warranty polices.

Ⅲ. Stepdown Warranty Policy

1. Model Description

Under stepdown warranty policy, warranty period W is divided into K steps like figure 3.1 and manufacturers compensate customers for cost C_i when a product fails during the the time period $[W_{i-1}, W_i]$ where, $W_0=0$, $W_K=W$ and $C_1=C$ $C_1>C_2>C_3>C_4>\cdots\cdots\cdots>C_K$.

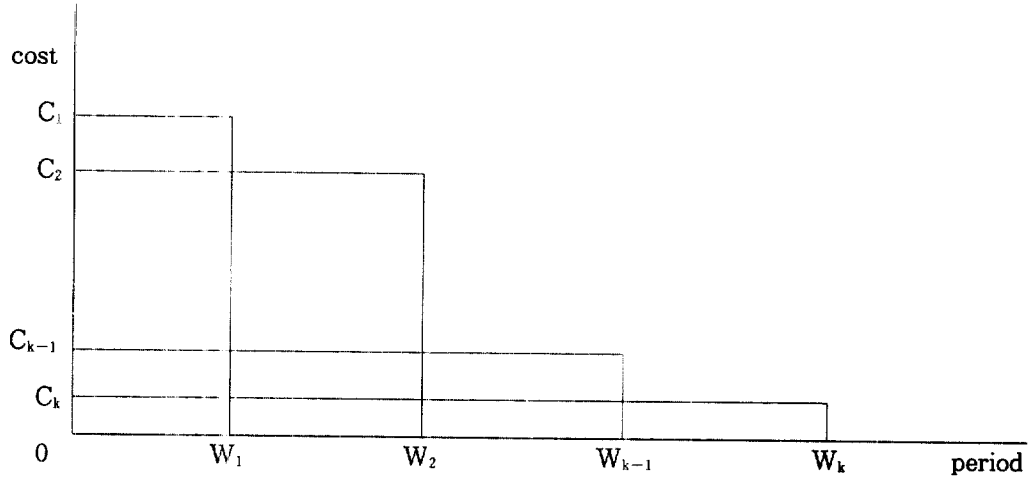

Fig. 3.1 Stepdown Warranty Policy

Figure 3.2 illustrates free replacement policy model, where manufacturers take the full responsibility for the cost incurred during warranty interval W, stepdown warranty policy becomes this model when warranty period step K is 1.

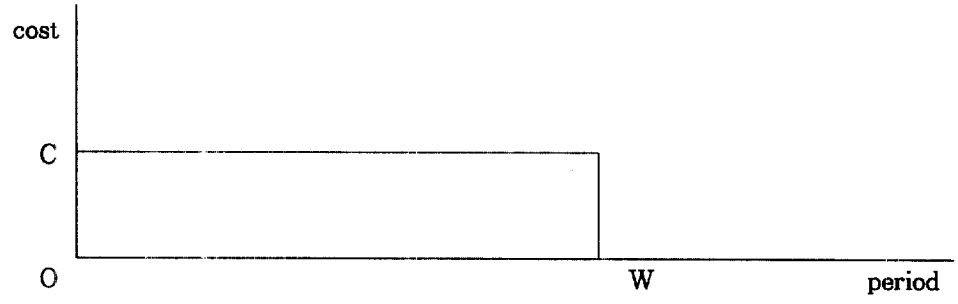

Fig. 3.2 Free Replacement Policy

Prorata Policy is depicted in figure 3.3 In this policy, manufacturer's cost declined in the constant ratio by the warranty period W. Stepdown warranty policy becomes this one by increasing K, and as the result making all $[W_i - W_{i-1}]$ to zero while fixing W.

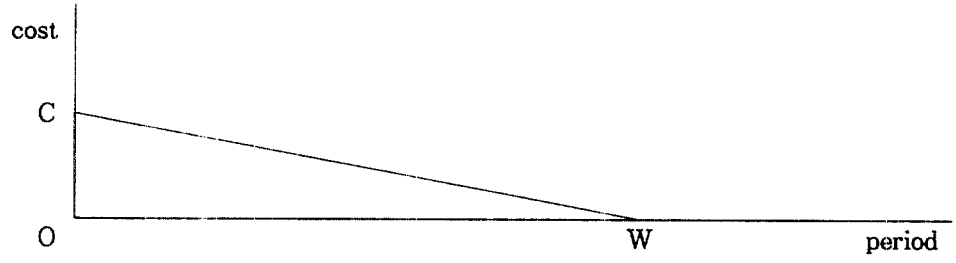

Fig. 3.3 Prorata Policy

Fiugure 3.4 illustrates hybrid policy, which implements free replacement policy with a certain period of time and then uses prorata policy. We can get this model from stepdown warranty policy by approaching $[W_i - W_{i-1}]$, where $i \geq 2$ to 0 while fixing warranty period W_1.

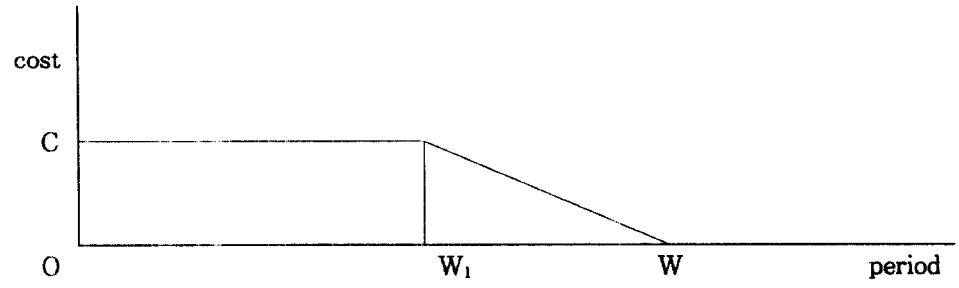

Fig. 3.4 Hybrid Policy

It is impossible to place any comparative advantage over those three policies. If manufacturers have identical monetary plan, prorata policy has longer warranty period than that of free replaccmont policy. If they have identical warranty period, free replacement warranty policy is more useful for increasing customer's interest even though their costs increase.

Because exact product usage time should be known like prorata policy, hybrid policy also has some difficulties to implement in the real life.

Therefore, this paper introduces stepdown warranty policy that generalize existing warranty policies. In this paper we analyze manufacturer warranty cost.

2. Assumption and Notation

Assumption

* Product is irrepairable.
* Manufacturers replace products with identical ones immediately when failures occur.
* Each warranty period $[W_i - W_{i-1}]$ has equal intervals when warranty period is K.
* Warranty cost decreases by same amounts at each warranty period.
* Manufacturer's cost is the expected cost during warranty period when a product sold.

Notation

$x_1\ x_2\ x_3$: Time interval of product failure
$F(x)$: Product failure distribution function
$Q(x)$: Cost to manufacturer when a product fails at time x
N : Number of warranty period renewals occured during warranty period

C_i : Cost manufacturer pays ccustomer at each warranty period $[W_i - W_{i-1}]$ when failure occurs

$\triangle C$: $C_i - C_{i-1}$

K : warranty interval steps

λ : Scale parameter of F(x)

β : Shape parameter of F(x)

A^M : Expected cost to manufacturer by warranty period w

A^M_F : Expected cost to manufacturer by warranty period w under free replacement policy

A^M_P : Expected cost to manufacturer by wrranty period w under prorata policy

A^M_H : Expected cost to manufacturer by warranty period w under hybrid policy

3. Cost Analysis

The eastablishment of warranty policy in product items importantly affects in management fund planning needed to warranty policy of manufacturer and decisively affects in purchasing products.

So warranty policies are appraised with important decision making analysis.

Therefore in order to make appropriated warranty policy eastablishiment, it is needed to consider both warranty rate and warranty renewal poliy.

In this section, the following warranty renewal policy is suggested to calculate manufacturer's cost incurring after a product sale.

Warranty policy

As one item sold, the failed product is replaced with a new one and warranty period renewed whenever one item is replaced during warranty period.

Manufacturer's costs incurred during warranty period are calculated.

Let xi be a random variable which is represented failure time inteval in i^{th} new item.

Let Q(xi) be manufacture's cost, as i^{th} new item is failed in xi time period.

By the stepdown warrardy poliy's definition.

$$Q(xi) = \begin{cases} C_i & W_{i-1} < xi \leq W_{i-1} \\ 0, & xi > W_k \end{cases} \qquad 1 \leq i \leq k$$

If N is the number of renewed warranty period by warranty period w,

$$\{N=n\} = \{X_1 \leq W, X_2 \leq W, X_3 \leq W, \cdots\cdots, Xn \leq W, X_{n+1} > W\}$$
$$= \{F(W)\}^n \{1-F(W)\} \qquad n=0, 1, 2, \cdots\cdots$$

Therorem 1

Under stepdown warranty policy the expeted cost to manufacturer by warranty period w is

$$A^M = \frac{1}{1-F(W)} \sum_{i=1}^{k} C_i \{F(W_i)-F(W_{i-1})\} \cdots\cdots\cdots\cdots\cdots (1)$$

proof

$$A^M = E\{ \sum_{i=1}^{k} Q(x_i) \}$$

$$= \sum_{n=0}^{\infty} \sum_{i=1}^{n} E\{ Q(x_i) \mid N = n \} \cdot Pr\{ N = n \}$$

$$= \sum_{n=0}^{\infty} nPr\{ N = n \} \cdot E\{ Q(x_1) \mid x_1 \leq W \}$$

$$= \sum_{i=1}^{k} \frac{F(W)}{1-F(W)} \int_{W_{i-1}}^{W_i} C_i \, dF(x) / F(W)$$

$$= \frac{1}{1-F(W)} \sum_{i=1}^{k} C_i \{F(W_i)-F(W_{i-1})\}$$

Theorem 2

i) Under free replacement policy manufacturer's cost by warranty period w is

$$A^{M_P} = \frac{C}{1-F(W)} F(W)$$

ii) Under prorata policy manufacturer's cost by warranty period w is

$$A^{M_P} = \frac{C}{1-F(W)} \int_0^W \frac{1}{W} F(x) dx$$

iii) Under hybrid policy manufacturer's cost by warranty period w is

$$A^{M}{}_{H} = \frac{C}{1-F(W)} \int_{w1}^{w} \frac{F(x)}{(W-W_1)} dx$$

proof

i) Manufacturer's cost of free replacement cost is calculated by putting K = 1 in (1) equation.

$$A^{M}{}_{F} = \frac{C}{1-F(W)} F(W)$$

ii) When warranty peiod w is fixed, $\lim_{k\to\infty} \max | W_i - W_{i-1} |$ is zero. As

$$\lim_{k\to\infty} W_i = x \text{ therefore } \lim_{k\to\infty} C_i = (C/W) * (W-x)$$

$$A^{M}{}_{P} = \lim_{k\to\infty} A^{M}$$

$$= \frac{1}{1-F(x)} \lim_{k\to\infty} \sum_{i=1}^{k} C_i \{F(W_i) - F(W_{i-1})\}$$

$$= \frac{1}{1-F(w)} \frac{C}{W} \int_0^w (W-x)\, dF(x)$$

$$= \frac{1}{1-F(w)} \frac{C}{W} \int_0^w F(x)\, dx$$

iii) Warranty period w and w_i is fixed and as $i \geq 2$

in case of $\lim_{k\to\infty} \max | W_i - W_{i-1} | = 0$ when $\lim_{k\to\infty} W_i = x$

therefore $\lim_{k\to\infty} C_i = \frac{(W-x)}{W-W_i} C$

so manufacturer's cost of hybrid policy is

$$A^{M}{}_{H} = \lim_{k\to\infty} A^{M}$$

$$= \frac{i}{1-F(x)} [CF(W_1) + \lim_{k\to\infty} \sum_{i=2}^{K} \{F(W_i) - F(W_{i-1})\}]$$

$$= \frac{i}{1-F(w)} [CF(W) + \frac{C}{W-W_1} \int_{w1}^{w} (W-x)\, dF(x)\}$$

$$= \frac{i}{1-F(w)} \frac{C}{(W-W_1)} \int_{w1}^{w} F(x)\, dx$$

4. Case Study

The following example is analyed through stepdown warranty policy and thee warranty policies.

Example

As one item sold, manufacturer compensate customer for $C_1 = 50{,}000$ \$ which is warranty cost of item failing during warranty period w_1. The difference of cost in each step, $\triangle C$, is 10,000 \$.

Manufacturer's costs are calculated in weibull time−to−failure distribution having parameter lambda, beta. The warranty period w is 1.2 years.

In table 3.1 manufacturer's cost is decreasing under stepdown warranty policy as warranty period step is increased. But when parameter beta values are increased in each warranty peiod step, manufacturer's cost is increasing remarkably.

Table 3.1 Stepdown Warranty Policy

STEP	BETA	LAMBDA	MANUFACTURER'S COST
2	1	1	107785.
2	2	1	141588.
2	3	1	196111.
2	4	1	287804.
2	5	1	450650.
3	1	1	98832.
3	2	1	122813.
3	3	1	164929.
3	4	1	237343.
3	5	1	366101.

STEP	BETA	LAMBDA	MANUFACTURER'S COST
4	1	1	89690.
4	2	1	104238.
4	3	1	134161.
4	4	1	187644.
4	5	1	283818.

Free replacement policy is unrelated with the variation of warranty period step, then wananty period step is K=1 under stepdown warranty policy.

Table 3.2 FREE REPLACEMENT POLICY

BETA	LAMBDA	MANUFACTURER'S COST
1	1	116006.
2	1	161035.
3	1	231469.
4	1	347670.
5	1	552052.

Manufacturer's cost is not affeted by warranty period step variation in prorata policy. In Fig 3. 3 manufacturer's cost is showed.

Table 3.3 PRORATA WARRANTY POLICY

BETA	LAMBDA	MANUFACTURER'S COST
1	1	69334.
2	1	69159.
3	1	79532.
4	1	102021.
5	1	144604.

According to warranty period w and free replaerent period variation, w_1. manufacturer's cost is different in hybrid warranty policy.

As warraty period step is K=3, manufacturer's cost is calulated.

Table 3.4 HYBRID WARRANTY POLICY

BETA	LAMBDA	MANUFACTURER'S COST
1	1	89410.
2	1	98371.
3	1	117087.
4	1	152021.
5	1	216394.

Through the above table, if manufacturer's cost is compared stepdown wananty policy in warranty period step, K=3, with free replacement policy, hybrid and prorata policy. Manufacturer's cost is the most expensive in free replacement policy. By order of stepdown warranty, hybrid and prorata policy, manufacturer's cost is decreasing.

Ⅳ. Conclusion

In this paper, stepdown warranty policy is introduced to calculate manufacturer's cost and compared with existing warranty policy.

Free replacement policy is the most disadvantage one in warranty policy. And when manufacturer's cost is compared stepdown warranty policy with hybrid one, if warranty period step is increased stepdown warranty policy is lucrative in the manufacturer's point of views. Also prorata policy has a difficulty in actual application.

As a result of warranty cost analysis, stepdown warranty policy is more available than other warranty policies.

It is considerd that new warranty policies are contrived and customer's cost is analyzed in the later research direction.

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