

## The Effect Of Standard Limits And Fits On The Productivity Of Assembly Robots

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표준 Limits 및 Fits가 조립 로봇의 생산성에 미치는 영향

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### Abstract

This paper presents a methodology to enable the tolerances on mating parts of an assembly to be specified and be compatible to the precision of an assembly robot so as to achieve maximum system performance. The measure of performance is defined as the *Probability of Successful Assembly* (PSA). A typical loose fastener assembly, usually called *peg-in-a-hole* is investigated. The *Geometric Tolerancing System* is adopted to represent position tolerances of mating parts. Two models are presented by considering modifiers on a position tolerance, *Regardless of Feature Size* (RFS) and *Maximum Material Condition* (MMC). Using these models, it is analyzed how the *Standard Limits and Fits* recommended by ANSI influence the performance of an assembly robot. For this analysis, the Standard Limits and Fits are transformed to the representation scheme of the Geometric Tolerancing System. Due to low PSAs when the Standard Limits and Fits are taken into account, the effect of chamfers around a hole is also analyzed.

### Introduction

For the productivity of an assembly robotic system to be maximized, the characteristics of the parts to be assembled and the robot must be com-

patible. This is true when a new robot is specified to assemble parts which have already been designed. It is equally true when the parts are designed to be assembled by an existing robot system. It is common to make these specifications experimen-

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tally, "after the fact". This approach is ineffective and models are required to provide appropriate design guidelines.

The effects of the position error and/or the repeatability of the robot on robot system productivity have been considered in previous papers by several authors. The work in this paper is based upon the previous work by Kim and Knott[10,11].

An important consideration, in any assembly, is the functional characteristics of the mating parts. International Standards of Limits and Fits are in common usage[1,2,3,4] which attempt to reproduce conditions of fit, irrespective of the size of the mating parts. Previous researchers have considered the effect of tolerance on the productivity. These research efforts do not appear to have been related to a recognized system of Limits and Fits.

This paper presents a methodology to enable the limits and fits of mating parts to be specified and be compatible to the precision of an assembly robot so as to achieve maximum system performance. The geometry of mating parts is confined to a *peg-*

*in-a-hole* assembly shown in Figure 1. The measure of performance is defined as the *Probability of Successful Assembly (PSA)*. The *Geometric Tolerancing System* is adopted to represent position tolerances of mating parts. The basic concepts of the *ANSI Standard Limits and Fits* are examined and the terminology explained. Basic probabilistic models to estimate the productivity are then developed. The variables are the repeatability of the robot and the functional requirements of the assembly. Implications of *Standard Limits and Fits* on the productivity of an assembly robot are examined.

### Defintion of Robot Productivity

It is desirable that the center lines of the hole and the peg be coincidental when being assembled by a robot. Due to the random nature of the equipment and of the workpieces, deviations from this ideal condition occur. These deviations may be attributed to the peg or to the robot. The overall result however is that collisions of the parts may occur when assembly is attempted. The robot might be fitted with a sensor which allows the peg to be withdrawn and assembly attempted again. This collision and withdrawal will reduce the productivity of the robot.

Based upon this concept, the measure of the productivity of a robot is defined as the *Probability of Successful Assembly (PSA)*, which is the probability that the peg lies within the range of the hole or chamfer at the first attempt for assembly. It is assumed that when the peg lies within this range, successful assembly is achieved by passive or active compliance motions as in Remote Center Compliance (RCC). This assumption is reasonable

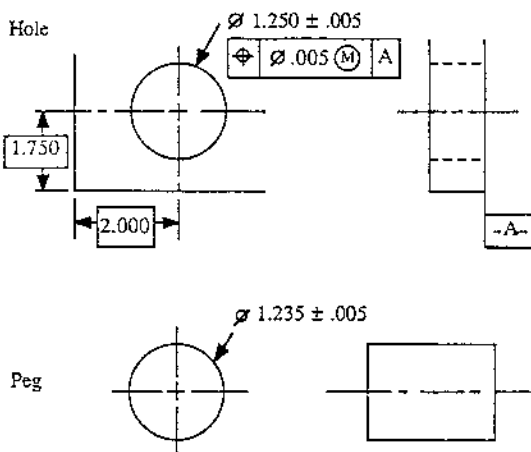


Figure 1. Geometry of the peg-in-a-hole assembly.

since the compliant positioning approaches have been well known to absorb position errors even with a few degrees and angular misalignment.

### Position Tolerance and Modifiers

A position tolerance is the total permissible variation in the location of a feature about its exact true position. In the geometric tolerancing system, the position tolerance is defined as the minimum hole diameter minus the maximum peg diameter. Suppose that the specifications of the peg and hole diameters are  $1.237 \pm .003$  and  $1.250 \pm .003$ , respectively. Then, the position tolerance corresponds to  $.007(1.247 - 1.240)$ . This calculation method assumes that the possibility of a zero interference-zero clearance condition of the mating part features at extreme tolerance limits[7]. The position tolerance expands further when it is used together with such a modifier as *Maximum Material Condition* (MMC). As the hole size deviates from

the MMC size of 1.247, the position of the hole is permitted to shift off its true position beyond the original tolerance zone to the extent of that departure. Therefore, the position tolerance expands up to .013 should the hole be produced to its high limit size of 1.253. Where *Regardless of Feature Size* (RFS) is applied to the controlled feature or its datums, the stated tolerance applies at any produced size of the feature or datum. There is no additional tolerance as with MMC.

### The System of Limits and Fits

To ensure the functional consistency between mating parts, designers use internationally accepted, published standards of Limits and Fits[1, 2, 3]. As there are minor differences in the standards, the discussions will be centered around the ANSI system. The ANSI standards recognize three types of fit. They are 1) Clearance, 2) Transition, and 3) Interference. These three types of fit are

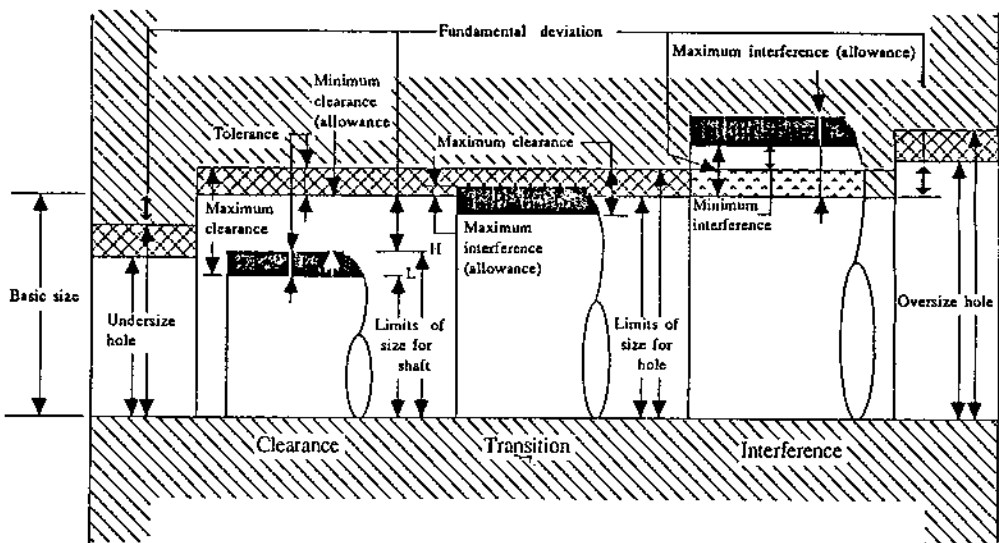


Figure 2. Types of Fit[4].

illustrated in Figure 2. Force will be required to assemble the mating parts with an Interference Fit and Transition Fit. Since the robot is not suitable for assembling the parts requiring force, this paper is limited to consideration of Clearance Fits.

Irrespective of the nominal size of the mating parts, the functional characteristics of the assembly is reproduced. This is achieved by describing the Fit through a two-part code. One part of the code refers to the hole and the other to the shaft. Each part of this code consists of a letter, followed by

one or more digits. An example of such a code is given in Figure 3. The nominal size of the mating

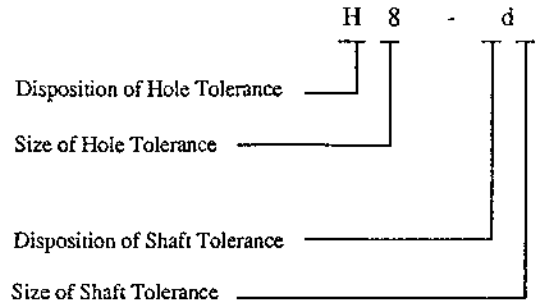
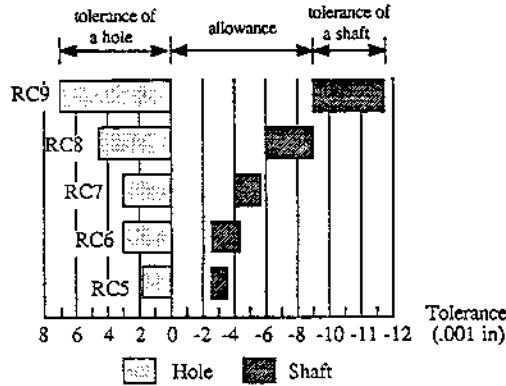
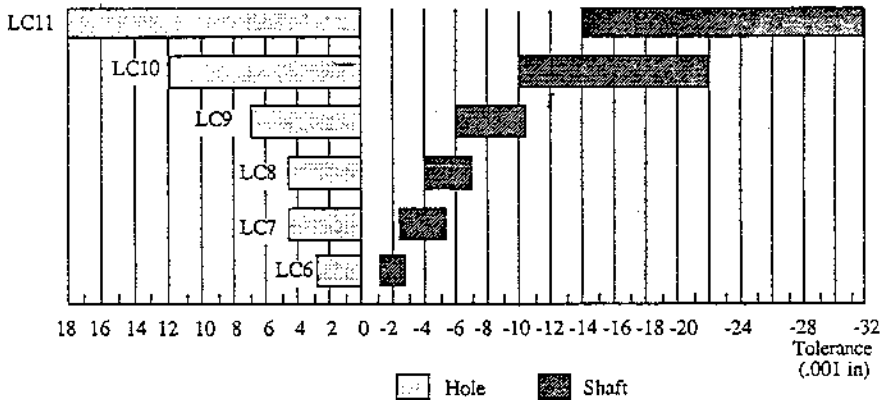


Figure 3. Example of code specifying a Type of Fit.



(a) Running and Sliding Clearance Fits(diameter=2.5 in)



(b) Locational Clearance Fits(diameter=2.5 in)

Figure 4. Graphical representation of clearance fits.

parts is then divided into ranges. These size ranges are then tabulated for each type and the tolerance on the resulting parts are specified. In this way complete reproducibility of functional fit is assured, irrespective of nominal size.

### Clearance and Locational Fits

Within the clearance group of fits, the ANSI standard recommends two types for general use. They are 1) Running and Sliding Clearance Fits (RC), and 2) Locational Clearance Fits (LC). These fits are represented graphically in Figure 4. Type RC Fits provide running fit with suitable lubrication, this running fit will be reproduced throughout the range of sizes covered by the standard. There are nine ranges of this type of fit. They are designated RC1, RC2, ..., RC9. With type LC Fits the mating parts are used for locating stationary mating parts. However, these parts can be freely assembled and disassembled. These types of fit range from snug fits, requiring accurate location, to medium clearance fits. They are used for ball bearing races to the loose fastener, where freedom of assembly is of prime importance.

### Limits for Holes and Shafts

Standard Limits and Fits use two basic variables. They are 1) Fundamental Tolerance and 2) Fundamental Deviation. The Fundamental Tolerance provides sixteen grades of tolerance for each size range. Each grade of Fundamental Tolerance is identified by the symbols IT. 1 through IT. 16. The Fundamental Tolerance is calculated using an equation relating to the size, reproducing condi-

tions. This variable specifies the range of tolerance on a mating part. The Fundamental Deviation establishes the location of the tolerance range on the mating parts. It is the position of the tolerance zone boundaries to the nearest zero line. There are 21 values of Fundamental Deviation, each identified by a letter. The letters are ABCDEFGHJK-MNPRSTUVXYZ. On shafts, the same lower case letters are used to identify the Fundamental Deviation.

### Estimation of PSA When RFS is Specified

#### Characteristics of Clearance

For a mating hole and shaft, the half clearance is identified as  $c$ . This is a random variable such that  $c = (d_1 - d_2) / 2$ . Since the diameters of the hole and shaft are distributed, the means of these diameters are  $\mu_{d_1} = D_1$  and  $\mu_{d_2} = D_2$ . The variances of these elements can be approximated by  $T_1 = k \sigma_{d_1}$  and  $T_2 = k \sigma_{d_2}$  where  $k$  is constant. It is usual to set  $k$  equal to 3. In this way 99.73% of the parts are within the tolerance range. The mean and variance of the half clearance can therefore be written as

$$\mu_c = (D_1 - D_2) / 2 \quad \text{and} \quad \sigma_c^2 = (T_1 + T_2) / 36 \quad \dots \dots (1)$$

### The Relationship Between Position Tolerance and Repeatability

The position Tolerance,  $Q$ , consists of two dimensional errors in the  $x$  and  $y$  axes. The repeatability is expressed by a  $\pm$  error in one direction. However the actual repeatability error is in two

directions, i. e., the x and y axes. If the variances in these two axes are not specified separately, it can be assumed they are equal. When this occurs, R becomes the radius of the repeatability error. Since these errors are two dimensional, the analysis may be based upon a bivariate normal distribution. Suppose that bivariate normal random variables (x, y) are mutually independent. The contour containing the confidence interval  $\alpha$  in the bivariate normal distribution [8, 9] is

$$\left(\frac{x-\mu_x}{\sqrt{k} \sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sqrt{k} \sigma_y}\right)^2 = 1 \dots\dots\dots(2)$$

where  $k = -2 \ln(1-\alpha)$ . The traditional value of  $\alpha$  equals 99.73%. This corresponds to a two-sided  $3\sigma$  limit for the univariate normal distribution. If this is used,  $\sqrt{k} = 3.44$  and the contour is circular. In this case, the two standard deviations can be obtained by  $\sigma_x = \sigma_y = R'/3.44$ , where  $R'$  is the radius of the contour. Suppose that the actual positions of the hole and the peg due to the position tolerance and repeatability, are  $(q_x, q_y)$  and  $(r_x, r_y)$ , respectively. The actual displacement between the peg and hole, s, is calculated by

$$s = \sqrt{(q_x - r_x)^2 + (q_y - r_y)^2} = \sqrt{s_x^2 + s_y^2} \dots\dots\dots(3)$$

where  $s_x = q_x - r_x$  and  $s_y = q_y - r_y$ . Given values of the position tolerance Q and the repeatability R, standard deviations of  $q_x, q_y, r_x,$  and  $r_y$  can be obtained as

$$\sigma_{q_x} = \sigma_{q_y} = \frac{Q}{2} \cdot \frac{1}{3.44} \text{ and } \sigma_{r_x} = \sigma_{r_y} = \frac{R}{3.44} \dots\dots\dots(4)$$

$s_x$  and  $s_y$  are distributed normally as  $(0, \sigma_{q_x}^2 + \sigma_{r_x}^2)$  and  $(0, \sigma_{q_y}^2 + \sigma_{r_y}^2)$ , respectively.

Therefore variances of  $s_x$  and  $s_y$  become

$$\sigma_{s_x}^2 = \sigma_{s_y}^2 = \left(\frac{Q}{6.88}\right)^2 + \left(\frac{R}{3.44}\right)^2 \dots\dots\dots(5)$$

When  $s_x$  and  $s_y$  are mutually independent and have means of zero and equal variances, s becomes a random variable of a *Rayleigh distribution*[5]

$$f_s(s) = \frac{s}{\sigma^2} \exp\left[-\frac{s^2}{2\sigma^2}\right] \dots\dots\dots(6)$$

where  $\sigma^2 = \left(\frac{Q}{6.88}\right)^2 + \left(\frac{R}{3.44}\right)^2$

The position tolerance in the geometric system is obtained by subtracting the diameter of the MMC peg from the diameter of the MMC hole[7]. Therefore,  $\sigma^2$  in equation (6) can be rewritten as

$$\sigma^2 = \left(\frac{D_1 - D_2 - T_1 - T_2}{6.88}\right)^2 + \left(\frac{R}{3.44}\right)^2 \dots\dots\dots(7)$$

### PSA with RFS Specified

Successful assembly can be achieved by the robot when the displacement between the hole and the peg is less than the half clearance. That is, when  $c > s$ . Thus, the PSA can be expressed as  $P(c > s)$  where  $c > 0$  and  $s > 0$ . The PSA can be derived as follows :

$$P(c > s > 0) = \int_0^c \left[ \int_0^c \frac{s}{\sigma^2} \exp\left(-\frac{s^2}{2\sigma^2}\right) ds \right] \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(c-\mu_c)^2}{2\sigma_c^2}\right) dc = \Phi\left(\frac{\mu_c}{\sigma_c}\right) - A \Phi\left(\frac{\sigma}{\sqrt{\sigma^2 + \sigma_c^2}} \frac{\mu_c}{\sigma_c}\right) \dots\dots(8)$$

where  $\Phi(\cdot)$  is a cumulative standard normal distribution and

$$A = \frac{\sigma}{\sqrt{\sigma^2 + \sigma_c^2}} \exp \left[ -\frac{\mu_c^2}{2(\sigma^2 + \sigma_c^2)} \right] \dots\dots\dots (9)$$

Since  $\Phi(\cdot)$  cannot be integrated in a closed form, the values can be obtained from a table of the cumulative standard normal distribution or by a numerical analysis technique such as the Gaussian Quadrature[6].

### Estimation of PSA When MMC is Specified

#### Characteristics of clearance

The half clearance on the case where MMC is specified is different from the RFS case. The half clearance which exists between any pair of mating parts will depend upon the actual diameters of the hole and the peg. The nominal diameter of the hole is  $D_1$  and the actual diameter is  $d_1$ . This actual diameter is a random normal variable. It is convenient, in developing the model, to consider a specific value of  $d_1$ . This value will be given the symbol  $\delta_1$ , where  $D_1 - T_1 < \delta_1 < D_1 + T_1$ . The half clearance between the hole and the peg can then be determined from  $c = (\delta_1 - d_2)/2$ . As  $\delta_1$  is a constant and  $d_2$  is a variable,  $c$  must be a random variable with mean and variance which can be obtained as follows :

$$\mu_c = (\delta_1 - D_2)/2 \text{ and } \sigma_c^2 = \frac{1}{4} \sigma_{d_2}^2 = \frac{1}{36} T_2^2 \dots (10)$$

$$P(c > s \mid d_1 = \delta_1)$$

$$= \int_0^s \left[ \int_0^c \frac{s}{\sigma^2} \exp \left( -\frac{s^2}{2\sigma^2} \right) ds \right] \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left( -\frac{(c-\mu_c)^2}{2\sigma_c^2} \right) dc = \Phi \left( \frac{\mu_c}{\sigma_c} \right) - A \Phi \left( \frac{\sigma}{\sqrt{\sigma^2 + \sigma_c^2}} \frac{\mu_c}{\sigma_c} \right) \dots (13)$$

### Relationship between Position Tolerance and Repeatability

when MMC is considered, the position tolerance increases effectively. The relationship which expresses this new position tolerance is

$$Q' = Q + \delta_1 - (D_1 - T_1) \dots\dots\dots (11)$$

where  $Q'$  = The modified position tolerance, due to the extra tolerance introduced as a result of using MMC,

$Q$  = Position tolerance of an MMC hole  
 $\delta_1 - (D_1 - T_1)$  = Difference between the actual diameter of the hole and the diameter of an MMC hole. This is usually referred to as a *bonus tolerance*.

The distance between actual positions of the hole and the peg is a Rayleigh random variable. Replacing  $Q$  in equation (6) with  $Q'$  in equation (11), constant  $\sigma^2$  of the Rayleigh distribution can be obtained as

$$\sigma^2 = \left( \frac{\delta_1 - D_2 - T_2}{6.88} \right)^2 + \left( \frac{R}{3.44} \right)^2 \dots\dots\dots (12)$$

### PSA with MMC Specified

Given  $d_1 = \delta_1$ ,  $c > 0$ ,  $s > 0$  and  $D_1 - T_1 < \delta_1 < D_1 + T_1$ , successful assembly occurs when the half clearance is greater than the displacement between the hole and the peg. The conditional PSA when  $d_1 = \delta_1$ , can be derived as follows :

$$\text{where } A = \frac{\sigma}{\sqrt{\sigma^2 + \sigma_c^2}} \exp \left[ -\frac{\mu_c^2}{2(\sigma^2 + \sigma_c^2)} \right] \dots (14)$$

The values of  $\mu_c$ ,  $\sigma_c^2$  and  $\sigma^2$  in equation (8) and (9) are different from those in equation (13) and (14). The PSA can be obtained by integrating the conditional PSA in terms of  $\delta_1$  as follows :

$$P(c > s) = \int_{D_1 - T_1}^{D_1 + T_1} P(c > s | \delta_1) f(\delta_1) d\delta \dots (15)$$

where  $f(\cdot)$  is a normal distribution function with  $(D_1, T_1^2/9)$ . If the error of 0.27% that the diameter lies outside the three sigma range is negligible, the integration range of equation (16) can be replaced by  $-\infty < \delta_1 < \infty$

### The Effect of Standard Limits and Fits on the PSA of Robot Assembly

The model development emphasizes the importance of the position tolerance on the PSA of a

robot assembly system. The ANSI standard on Limits and Fits does not use position tolerance. The geometric method of tolerancing has position tolerance as one of its main characteristics. The *allowance* used in the ANSI Standard on Limits and Fits is the minimum clearance. In geometric tolerancing, the *position tolerance* is defined as the diameter of an MMC hole minus the diameter of an MMC peg. These two definitions are similar. The models developed earlier can, therefore, be used to analyze the PSA of robotic assembly of parts designed with the ANSI Standard of Limits and Fits.

### Analysis of PSA of Assemblies Without Chamfered Parts

In this paper the analysis is based on a size range 1.97 to 3.15 inches. Further, the running and sliding clearance fits chosen were classes RC5 to

Table 1. Performance of a robot when Running and Sliding clearance Fits are considered.

Repeatability = .01 in.  
Nominal dimension of the hole = 2.500 in.  
Standard limits are in thousands of an inch.

	Class RC5		Class RC6		Class RC7		Class RC8		Class RC9	
	Hole H8	Shaft e7	Hole H9	Shaft e8	Hole H9	Shaft d8	Hole H10	Shaft c9	Hole H11	Shaft
Standard Limits (.001 in)	+1.8 -0	-2.5 -3.7	+3.0 -0	-2.5 -4.3	+3.0 -0	-4.0 -5.8	+4.5 -0	-6.0 -9.0	+7.0 -0	-9.0 -13.5
Nominal Dimension & Bilateral Tolerance	2.5009 ±.0009	2.4969 ±.0006	2.5015 ±.0015	2.4966 ±.0009	2.5015 ±.0015	2.4951 ±.0009	2.5023 ±.0023	2.4925 ±.0015	2.5035 ±.0035	2.4887 ±.0023
Allowance * (Position Tolerance of the Hole)	.0025		.0025		.004		.006		.009	
PSA	with RFS	.209	.296	.441	.724	.925				
	with MMC	.206	.289	.429	.696	.895				

\* The allowance between the hole and the shaft corresponds to the position tolerance of the hole.



Table 2. Performance of a robot when Locational clearance Fits are considered.

Repeatability = .01 in.  
Nominal dimension of the hole = 2.500 in.  
Standard limits are in thousands of an inch.

	Class LC6		Class LC7		Class LC8		Class LC9		Class LC10		Class LC11	
	Hole H9	Shaft f8	Hole H10	Shaft e9	Hole H10	Shaft d9	Hole H11	Shaft c10	Hole H12	Shaft h11	Hole H13	Shaft g12
Standard Limits (.001 in)	+3.0 -0	-1.2 -3.0	+4.5 -0	-2.5 -5.5	+4.5 -0	-4.0 -7.0	+7 -0	-6 -10.5	+12 -0	-10 -22	+18 -0	-14 -32
Nominal Dimension & Bilateral Tolerance	2.5015 ±.0015	2.4979 ±.0009	2.5023 ±.0023	2.4960 ±.0015	2.5023 ±.0023	2.4945 ±.0015	2.5035 ±.0035	2.4918 ±.0023	2.5060 ±.0060	2.4840 ±.0060	2.5090 ±.0090	2.4770 ±.0090
Allowance * (Position Tolerance of the Hole)	.0012		.0025		.004		.006		.010		.014	
PSA	with RFS	.176	.438	.576	.833	.993	.999					
	with MMC	.173	.423	.554	.797	.979	.994					

\* The allowance between the hole and the shaft corresponds to the position tolerance of the hole

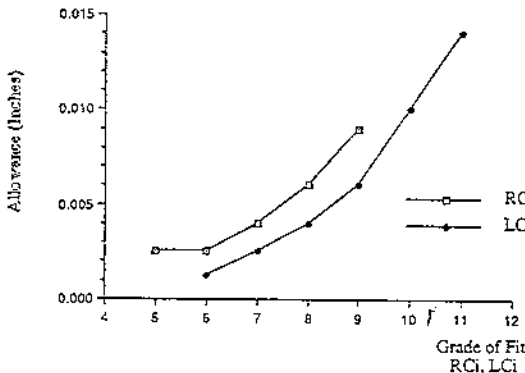


Figure 5. Comparison of allowance of different types and grades of fit.

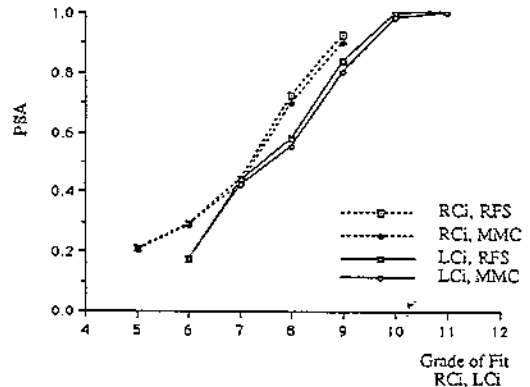


Figure 6. PSA for RCi and LCi types of fit with RFS and MMC modifiers.

RC9 and LC6 through LC11. The reason for this selection was the relatively large value of the allowance on these types of fit. The ANSI standard is based on a *unilateral basic hole system*. To use the proposed models the data obtained from the standard must be converted to a bilateral system. The repeatability of the robot has been assumed to be 0.01 inches. A nominal diameter of 2.500 inches has also been assumed. The proposed models have

been applied to determining the PSA with RFS and with MMC for the specified types of fit. The basic data and the results of this analysis are summarized in Table 1 and 2.

The allowance for the various grades of the two types of fit are shown graphically in Figure 5. This shows that as the grade of fit increases so does the allowance. Since the allowance and the position tolerance are synonymous, the position tolerance

is also dependent on grade of fit. Figure 6 shows the PSA for type RCi and LCi fits respectively. As could be expected, the PSA increases with an increase in the grade of fit. However, there does not appear to be a direct relationship.

Another approach determines the effect of the modifiers within a particular type of fit on the PSA. Here, the difference in PSA was calculated between the different modifiers for each type and grade of fit. In Figure 7, the results of this analysis are shown. It can be seen, the difference in the PSA due to the modifier is dependent on the grade of fit. This is true, even though no set relationship appears to exist.

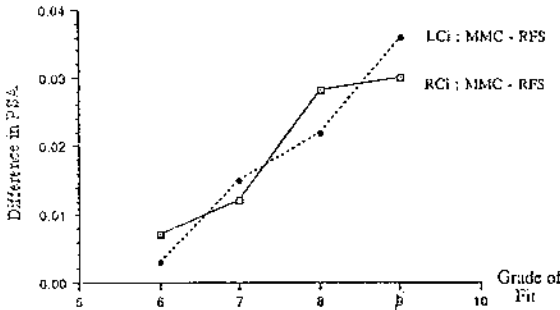


Figure 7. Effect of modifiers on PSA for RCi and LCi grades of fit.

Figure 6 shows the PSA falls below 0.800 for fits lower than RC8 and LC9. It is unlikely that this would be acceptable in the manufacturing environment. Where the tolerances are set by the functional requirements of the design, the system designer has two alternatives. One solution is to use a robot with an improved repeatability. The other solution is to provide chamfers on one of the mating parts.

### The Effect of Chamfering A Hole of an Assembly

Providing a chamfer on one of the mating parts of an assembly accommodates for lateral or angular misalignment. The models developed earlier are modified to provide a guideline for assigning the size of a chamfer to holes. The objective is again to maximize the PSA. It is assumed that the chamfer has the size and slope which allow the peg to be inserted successfully by compliance motion of the end effector.

#### Mathematical Model

Suppose that  $D_c$  is the chamfer size shown in Figure 8. Then the range which allows a peg to be inserted successfully into the hole is expanded to the boundary of the chamfer. The half clearance when RFS is specified is modified to

$$c_c = (D_c + \frac{d_1}{2}) - \frac{d_2}{2}$$

Therefore, the mean and variance of  $c_c$  are rewritten as

$$\mu_{c_c} = D_c + \frac{D_1 - D_2}{2} \quad \text{and} \quad \sigma_{c_c} = \frac{T_1^2 + T_2^2}{36} \dots \dots \dots (16)$$

where  $D_1 - D_2$  must be greater than zero, since only clearance fits are considered. Replacing  $\mu_c$  and  $\sigma_c^2$ , in equations (13) and (14) with  $\mu_{c_c}$  and  $\sigma_{c_c}^2$ , we can determine the PSA when RFS is specified.

When MMC is specified, the half clearance is

$$c_c = (D_c + \frac{\delta_1}{2}) - \frac{d_2}{2}$$

In this case the mean and variance become

$$\mu_c = D_c + \frac{\delta_1 - D_2}{2} \text{ and } \sigma_c^2 = \frac{T_2^2}{36} \dots\dots\dots (17)$$

By substituting  $\mu_c$  and  $\sigma_c^2$  in equations (13), (14) and (15) with  $\mu_c$  and  $\sigma_c^2$  in equation (17), the PSA when MMC is specified can be determined.

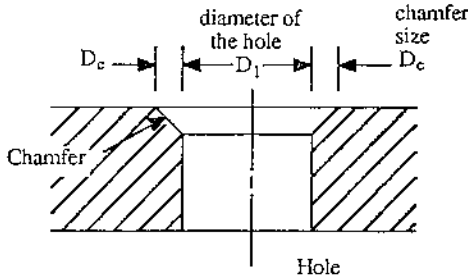


Figure 8. Defining the size of chamfer on a hole.

### Numerical example of the Effect of Chamfer Size

The results obtained for MMC and RFS, when calculating chamfer size, are expected to be similar. Therefore only the case of RFS will be considered here. The case where  $D_1 = 1.250$ ,  $D_2 = 1.234$ ,  $Q = 0.010$ , and  $R = 0.050$  is used as an example. The results of the analysis are summarized in Table 3. It is observed that PSAs do not vary so much with respect to the tolerance of the hole diameter  $T_1$ . This is due to relatively large values of  $Q$  and  $R$  compared to  $T_1$  and  $T_2$ . Therefore, it is desirable to analyze the results with respect to the chamfer size. The results when  $T_1 = 0.003$  are presented graphically in Figure 9. In practice, the size of chamfers of 0.005 to 0.0325 are common. This gives a PSA range from 0.40 to 0.97. To achieve a PSA of 1.00 the chamfer size has to be increased to 0.06.

Table 3. The effect of the chamfer  $D_c$  and the tolerances of diameters  $T_1$  and  $T_2$  when RFS is considered.

( $D_1 = 1.250$ ,  $D_2 = 1.234$ ,  $Q = .01$ ,  $R = .05$ )

$T_1$	$T_2$	PSA			
		$D_c = 0$	$D_c = .02$	$D_c = .03$	$D_c = .06$
.001	.005	.1403	.8400	.9667	1.0000
.002	.004	.1401	.8402	.9668	1.0000
.003	.003	.1400 *	.8402 **	.9669 **	1.0000 **
.004	.002	.1401	.8402	.9668	1.0000
.005	.001	.1403	.8400	.9667	1.0000

\* minimum PSA  
\*\* maximum PSA

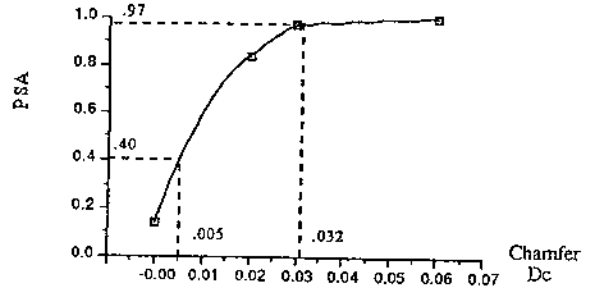


Figure 9. The effect of chamfer  $D_c$  on PSA when RFS is used

### Conclusion

The concept of the ANSI Standard on Limits and Fits has been used successfully for almost forty years. Where robotic assembly is being considered their use must be approached with caution. Here, the proposed equations predict the performance of an assembly robot and parts designed using the ANSI Standard. The numerical example used was restricted, but the approach is universal to the problem. The analysis showed that the performance of the robot assembly system was highly sensitive to the grade of fit used. This is true irrespective of the modifier which was being used. The actual performance level acceptable to any manufacturing

environment depends upon local requirements. However, it is unlikely that a PSA value of less than 0.800 would be acceptable. For desirable manufacture and assembly, proper chamfer sizes must be assigned to mating parts.

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## Appendix : Mathematical Notation

$c$  = A random variable of the half clearance between the hole and the peg.

$c_d$  = The difference between the nominal diameters of the hole and peg, i. e.,  $c_d = D_1 - D_2$   
 $D_1, D_2$  = Nominal diameters of the hole and the peg, respectively.

$d_1, d_2$  = Random variables of the hole diameter and the peg diameter, which are normally distributed ( $D_1, \sigma_{d1}^2$ ) and ( $D_2, \sigma_{d2}^2$ ), respectively.

$D_c$  = Chamfer size.

$\delta_1$  = A specific value of random variable  $d_1$

$Q$  = Position tolerance of the hole.

$q_x, q_y$  = Random variables of the position tolerance in the x and y axes, which are normally distributed ( $0, \sigma_{qx}^2$ ) and ( $0, \sigma_{qy}^2$ ), respectively.

$R$  = Repeatability of the robot expressed bilaterally as  $\pm R$ .

$r_x, r_y$  = Random variables of the repeatability of the robot in the x and y axes, which are normally distributed ( $0, \sigma_{rx}^2$ ), and ( $0, \sigma_{ry}^2$ ), respectively.

$s$  = Random variable of the distance between actual positions of the hole and the peg.