ALMOST COSYMPELECTIC MANIFOLDS WITH VANISHING CONTACT CONFORMAL CURVATURE TENSOR FIELD

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1. Introduction

The results of the study of a Sasakian manifold with vanishing contact Bochner curvature tensor had been listed in [5] and [6]. In [2] and [3], almost cosymplectic manifold with vanishing contact Bochner curvature tensor was discussed. In 1990, Kitahara, Matsuo and Pak [8,9] defined the so-called conformal curvature tensor field on a hermitian manifold which is conformally invariant, and nearly kaehlerian manifold with vanishing conformal curvature tensor field was discussed by present author [11]. Furthermore, Jeong, Lee, Oh and Pak [7] defined the so-called contact conformal curvature tensor field on Sasakian manifold, which is constructed from the conformal curvature tensor field by the Boothby-Wang's fibration [1]. In the present paper, we shall study almost cosymplectic manifold with vanishing contact conformal curvature tensor field and prove the following theorem:

Theorem A. An almost cosymplectic manifold with vanishing contact conformal curvature tensor field does not exist.

We shall be in $C^\infty$-category. Latin indices run from 1 to $2n + 1$. Einstein summation convention will be used.

2. Preliminaries

Let $(M, \phi, \xi, \eta, g)$ be a $(2n + 1)$-dimensional almost contact metric manifold, that is, $M$ is a manifold covered by a system of coordinate
neighborhoods \( \{ U; x^h \} \) and \( (\phi, \xi, \eta, g) \) an almost contact metric structure on \( M \), formed by tensors of type \((1,1),(1,0) \) and \((0,1) \) respectively, and a Riemannian metric \( g \) such that

\[
\begin{align*}
\phi^j _i \phi^h _i &= -\delta^j _i + \eta_j \xi^h , \phi^h _i = 0, \eta_i \phi^j _i = 0, \\
\eta_i \xi^i = 0, g_{ij} \phi^j _i \phi^i _i &= g_{ji} - \xi_j \xi_i , \xi_i = g_{ik} \xi^k 
\end{align*}
\]

(2.1)

On such a manifold we may always defined a 2-form by \( \phi(X,Y) = g(\phi X , Y) \) [10]. \((M, \phi, \xi, \eta, g) \) is said to be an almost cosymplectic manifold if the forms \( \phi \) and \( \eta \) are closed, i.e., \( d\phi = 0 \) and \( d\eta = 0 \), where \( d \) is the operator of exterior differentiation (cf. [4]). Let \( M \) be an \((2n+1)(n \geq 1)\)-dimensional almost cosymplectic manifold. Then we can consider the contact conformal curvature tensor field \( C_0 \) on \( M \) (the same definition as the contact conformal curvature tensor field in [7]).

\[
\begin{align*}
C_0_{kjih} = R_{kjih} + \frac{1}{2n} (g_{kh} R_{ji} - g_{jh} R_{ki} + R_{kh} g_{ji} \\
- R_{jik} g_{hi} - R_{khi} \eta_i + R_{jni} \eta_i - \eta_i R_{kj} \\
+ \eta_j h R_{ki} - \phi_{kh} R_{ji} + \phi_{jh} S_{hi} - S_{kh} \phi_{ji} \\
+ S_{ji} \phi_{ki} + 2 \phi_{kj} S_{ih} + 2 S_{kj} \phi_{ih} \\
+ \frac{1}{2n(n+1)} (2n^2 - n - 2 + \frac{(n+2)s}{2n}) (\phi_{kh} \phi_{ji} \\
- \phi_{kij} \phi_{jh} - 2 \phi_{kj} \phi_{ih}) \\
+ \frac{1}{2n(n+1)} (n + 2 - \frac{(3n+2)s}{2n}) (g_{kh} g_{ji} - g_{ki} g_{jh} \\
- \frac{1}{2n(n+1)} (4n^2 + 5n + 2 - \frac{(3n+2)s}{2n}) (g_{kh} \eta_j \eta_i \\
- g_{ki} \eta_j \eta_i + \eta_k \eta_i g_{jh} - \eta_k \eta_j g_{ih}),
\end{align*}
\]

where \( (\phi, \xi, \eta, g) \) denotes the almost cosymplectic structure, \( R_{kjih} \), \( R_{ji} \), and \( S \) are Riemannian curvature tensor, Ricci tensor and scalar curvature of \( M \), respectively, and \( S_{ji} = \phi^i _j R_{ii} \).

**Lemma 2.1.** ([10]) If \( M \) is an \((2n+1)\)-dimensional almost cosymplectic manifold, then it holds that

\[
\begin{align*}
R_{ji} \eta^i \eta' + |\nabla_j \eta'|^2 = 0,
\end{align*}
\]

(2.3)
Almost cosymplectic manifolds

where $\nabla_j$ denote the operator of covariant differentiation with respect to $g_{ij}$.

3. Proof of Theorem A

Assume that there is an $(2n+1)(n \geq 1)$-dimensional almost cosymplectic manifold with vanishing contact conformal curvature tensor field. Then, from (2.2) we have

$$\begin{align*}
(3.1) \quad R_{ji} &= -\frac{1}{2n}\{(2n-2)R_{ji} + 2g_{ji} - s\eta_j\eta_i + R^k_{ji}\eta_i\eta_k \\
&+ \phi^t_j S^i_{tt} + S^t_j\phi^i_{tt} + 2\phi_{ij}\phi^i_j + 2\phi_{ij}\phi^i_j\} \\
&+ \frac{1}{2n(n+1)}\{2n^2 - n - 2 + \frac{(n+2)s}{2n}\}(\phi^t_j\phi^i_j + 2\phi_{ij}\phi^i_j) \\
&- \frac{1}{n+1}\{n + 2 - \frac{(3n+2)s}{2n}\}g_{ji} \\
&+ \frac{1}{2n(n+1)}\{4n^2 - 5n - 2 + \frac{(3n+2)s}{2n}\} \\
&\times \{(2n-1)\eta_j\eta_i + g_{ji}\}.
\end{align*}$$

Thus we have

$$\begin{align*}
(3.2) \quad R_{ji}\eta^i\eta^i &= 2n.
\end{align*}$$

Hence, Theorem A has been proved from Lemma 2.1.

References


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