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Relative relocation of Tadzhikistan-Sinkiang events
타치키스탄 - 신강 지역 지진의 진앙에 대한 상대적 위치 결정

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Abstract/요 약

Sixty Tadzhikistan-Sinkiang events occurred from the year 1966 to 1978 were relocated using relative location method with one reference event technique. Since the area covered by the events is about 300 × 400 km, the region is subdivided into four. Each one has one reference event with accurate hypocenter location and travel time. The result shows average 0.3° change of distance.

1966년부터 1978년까지 타치키스탄-신강 지역에서 발생한 60개의 지진들의 진앙 위치를 잘 알려진 기준 지진을 이용한 상대적 위치 결정 방법을 사용하여 재배치 하였다. 지진들이 기록된 지역의 넓이가 약 300 × 400 km 정도가 되기 때문에 이 지역을 네 곳으로 분할 하였다. 각각의 지역들은 정확한 진앙 위치와 주시가 알려진 기준 지진들을 지니고있다. 계산 결과 기록된 진앙의 위치들이 평균 0.3도 이동되었다.

INTRODUCTION

It has been realized that calculated earthquake epicenters can be in error by much as 25-30 km. This fact was illustrated when the Aleutian arc nuclear explosion(Oct. 29, 1965) was mislocated by over 21 km when 272 well-distributed high quality P-wave arrival time observations were used with the standard single event location algorithm(Spence, 1973).

The most generally applicable technique for location of hypocenter is by use of the Geiger location technique applied to the P-wave arrival time. This method is inaccurate for shallow depths because P-waves propagated to teleseismic distance depart the source at almost vertical incidence. There are biases in depth estimation resulting from the use of incorrect travel time tables(Flinn, 1965 ; Evernden, 1969). One of the techniques suggested by Everden was to use the residuals from master events whose locations are known accurately. This is relative location method. He found that the master event had to be within 1° - 3° of the event of interest.

The data of Tadzhikistan-Sinkiang events collected from ISC bulletin covers the area extending from 38° N to 41° N in latitude, and from 72° E to 76° E in longitude. All these collected events occurred from the year 1966 to 1978. Relative location method with one reference events echique was to relocate the hypocenters.

A brief summary of Geiger's hypocenter location method and relative location method is presented for help of the interpretation of the calculated result.

GEIGER'S HYPOCENTER LOCATION METHOD

The classical method to determine earthquake hypocenter and origin time was first introduced by Geiger in 1972. It is usually called Geiger's method and involves two basic concepts in mathematics, i.e., iterative solution of nonlinear equation and least square method.

Let t_i be observed travel time at station i, and t_i^* be predicted travel time at station i. Total number of station should be greater than four. The time residuals at stations are

$$dt = \begin{pmatrix} t_1 - t_1^* \\ t_2 - t_1^* \\ \vdots \\ \vdots \\ t_N - t_N^* \end{pmatrix}$$

If the actual hypocenter parameter p and guess hypocenter parameter p^* are put as

$$p = \begin{pmatrix} \theta \\ \lambda \\ h \\ T \end{pmatrix}$$

$$p^* = \begin{pmatrix} \theta^* \\ \lambda^* \\ h^* \\ T^* \end{pmatrix}$$

the changes of hypocenter parameter are

$$\overline{dp} = \begin{pmatrix} \theta - \theta^* \\ \lambda - \lambda^* \\ h - h^* \\ T - T^* \end{pmatrix}$$

where θ, λ, h, T are colatitude, longitude, depth and origin time respectively.

Usually the origins of residuals come from timing errors, station location errors, earth model errors, and lateral variations in the crust and upper mantle.

Since travel time is not linear function of parameters, we need the first order approximation in Taylor series expansion of travel time in terms of hypocenter parameters.

$$t_i \approx t_i^* + \frac{\partial t_i}{\partial \theta^*}(\theta - \theta^*) + \frac{\partial t_i}{\partial \lambda^*}(\lambda - \lambda^*) + \frac{\partial t_i}{\partial h^*}(h - h^*) + \frac{\partial t_i}{\partial T^*}(T - T^*)$$

The partial derivative $\frac{\partial t_i}{\partial T^*}$ equals to unity, since travel time is linear in origin time. The relation above can be expressed in matrix form,

$$dt = A\ dp$$

where

$$A = \begin{pmatrix} \frac{\partial t_1}{\partial \theta^*} & \frac{\partial t_1}{\partial \lambda^*} & \frac{\partial t_1}{\partial h^*} & 1 \\ \frac{\partial t_2}{\partial \theta^*} & \frac{\partial t_2}{\partial \lambda^*} & \frac{\partial t_2}{\partial h^*} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial t_N}{\partial \theta^*} & \frac{\partial t_N}{\partial \lambda^*} & \frac{\partial t_N}{\partial h^*} & 1 \end{pmatrix}$$

The error involved in the first order linearization and observation is

$$E = dt - A\ dp$$

By minimizing the squared error $|E|^2 = E^T E$, we get the optimum condition for

$$\frac{\partial}{\partial p} E^T E = \frac{\partial}{\partial p}[dt^T dt - dt^T (A\ dp) - (A\ dp)^T dt + (A\ dp)^T (A\ dp)] = 0$$

$$-\frac{\partial}{\partial p}[dt^T (A\ dp)] - \frac{\partial}{\partial p}[(A\ dp)^T dp] + \frac{\partial}{\partial p}[(A\ dp)^T (A\ dp)] = 0$$

$$-dt^T A(\frac{\partial}{\partial p} dp) - (\frac{\partial}{\partial p} dp^T) A^T dt + (\frac{\partial}{\partial p} dp^T) A^T (A\ dp) + (A\ dp)^T A \frac{\partial}{\partial p} dp = 0$$

Let u be a unit column vector such as

$\frac{\partial}{\partial P} dp = u$ then

$- dt^T Au - u^T A^T dt + u^T(A^TA)dp + dp^T(A^TA)u = 0$

$(- A^T dt + A^TA\, dp)^T u + u^T(- A^T dt + A^TA\, dp) = 0$

$[u^T (-A^T dt + A^TA\, dp)]^T + u^T (-A^T dt + A^TA\, dp) = 0$

since for scalar $s^T = s$,

$2u^T (-A^T dt + A^TA\, dp) = 0$

Thus,

$A^TA\, dp = A^T dt$

This relation is normal equation in least square method. From this we get the desired solution,

$$dp = (A^T A)^{-1} A^T dt$$

The first step of the procedure is to set a guessed parameter p^*, and to calculate matrix A at point p^*. Next, t_i^* at each station i is calculated using velocity structure model. We get residuals dt from the data t_i and estimated t_i^*. The first improvement dp is calculated from the relation,

$$dp = (A^T A)^{-1} A^T dt.$$

For the next guess, the improved one p^* + dp is used as new p^*. If the same procedures above are iterated until the predertermined minimum value of dp is satisfied, the final $p^* = p^* + dp$ is the desired solution.

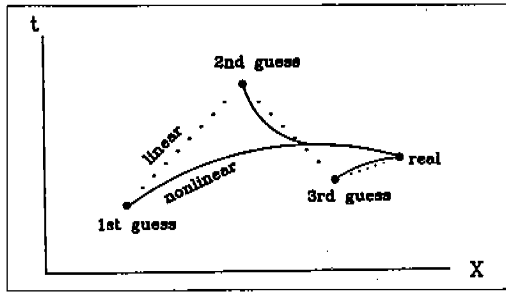

Fig. 1 The schematic diagram for Geiger's nonlinear inversion method. The inversion processes are iterated until the predetermined minimum value of dp is satisfied.

The result of least square method is a special case of generalized inverse problem, which has eigenvector with zero eigenvalue in data space and no such eigenvector in model space(Aki, 1979).

In order to decide confidence of solutions the sum of weighted time residuals are put in the form of κ^2 distribution,

$$\kappa^2 = \sum_{i=1}^{n} (t_i - T_i)^2 / \sigma_i^2$$

where t_i is observed travel time, T_i is assumed or calculated travel time. and σ_i is standard deviation of t_i. Original n degrees of freedom are reduced to(n-4) degrees of freedom by minimizing the sum(Aki, 1979). Using statistical tables we can see if the normalized sum falls in the range of predertermined value of confidence.

The pitfalls in Geiger's method are that velocity structure is required, that at least four stations are needed, and that these stations must be located such that the earthquake hypocenter is surrounded. If all the events are outside of the station network, some columns of the matrix A become dependent or identical. Lateral heterogeniety in earth's structure makes the method fail.

RELATIVE LOCATION METHOD

The disadvantage of Geiger's method can be improved by use of relative location method. The reference event used in this method should have pretty accurate hypocenter parameters and travel times. The hypocenter parameters and travel times of reference event are used in calculation of time residuals and matrix A in stead of using guess hypocenter. Since the information on velocity structure model can be calculated from the known parameters and travel times of reference event, we do not need to assume velocity structure model. The reference event should be very close to each event to be relocated.

The calculation is similar to Geiger's method case.

Taking first order Taylor series approximation of travel times, we get

$$dt = A\,dp$$

where

$$dt = \begin{pmatrix} t_1 - t_{R1} \\ t_2 - t_{R2} \\ \vdots \\ t_N - t_{RN} \end{pmatrix},\ dp = \begin{pmatrix} \theta - \theta_R \\ \lambda - \lambda_R \\ h - h_R \\ T - T_R \end{pmatrix}$$

$$A = \begin{pmatrix} \frac{\partial t_1}{\partial \theta_R} & \frac{\partial t_1}{\partial \lambda_R} & \frac{\partial t_1}{\partial h_R} & \frac{\partial t_1}{\partial T_R} \\ \frac{\partial t_2}{\partial \theta_R} & \frac{\partial t_2}{\partial \lambda_R} & \frac{\partial t_2}{\partial h_R} & \frac{\partial t_2}{\partial T_R} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial t_N}{\partial \theta_R} & \frac{\partial t_N}{\partial \lambda_R} & \frac{\partial t_N}{\partial h_R} & \frac{\partial t_N}{\partial T_R} \end{pmatrix}.$$

After solving least square problem, we get

$$dp = (A^T\ A)^{-1} A^T\, dt$$
$$p = p_r + dp$$

Only one iteration is needed since the location of the reference events is known.

The advantage of this method is that the earth model errors and travel time errors can be removed, by use of time residuals with respect to reference event.

RESULTS OF COMPUTATION AND DISCUSSION

Arrival times, locations and magnitudes of about sixty events were copied from ISC bulletin. The area covers from 38° N to 41° N in latitude and from 72°E to 76° E

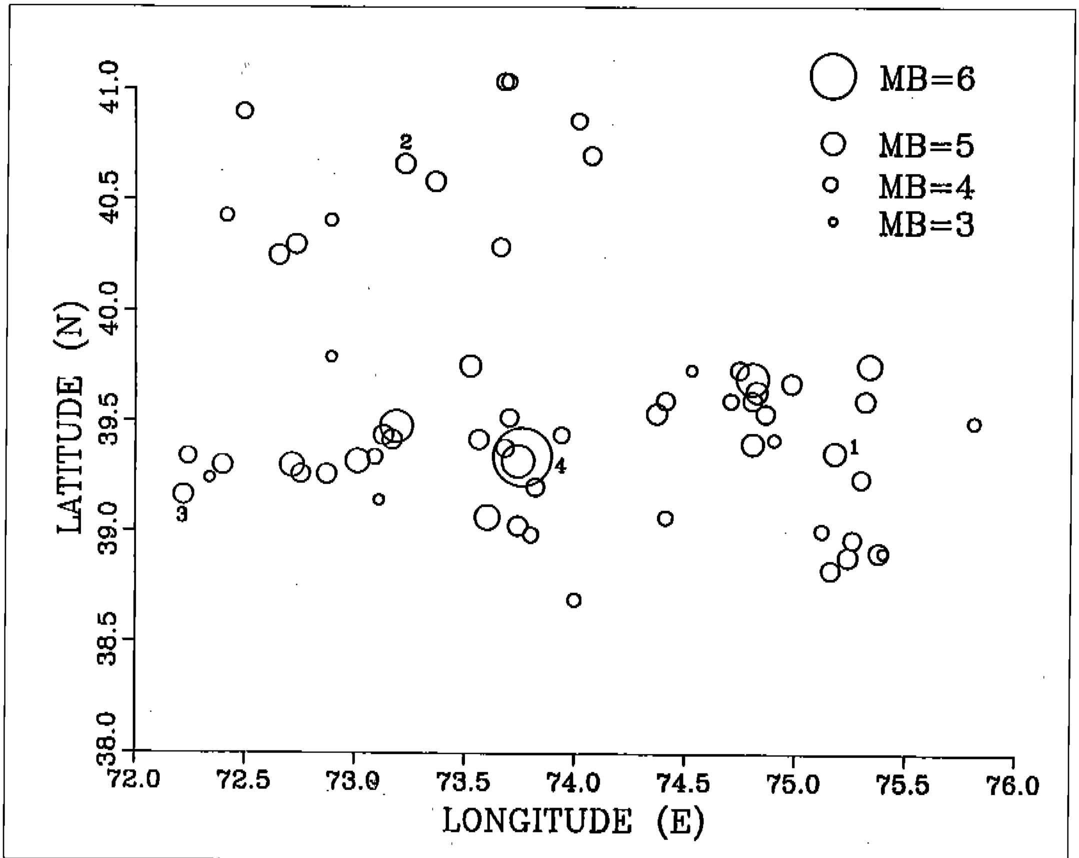

Fig. 2 Original hypocenter locations of Tadzhikistan-Sinkiang events from the year 1966 to 1978. The size of circle indicates magnitude of each event. The four numbered circles represent reference events.

longitude.

Fig. 2 shows the spatial plot of original data. The size of symbol indicates magnitude of each event. Four events with accurate locations and travel times are selected as references events. Each of the reference events is numbered from 1 to 4. The region is divided into four parts, each one of which falls in roughly half degree of reference event.

P phase for the travel time data was used and the standard deviation of travel time was put as 0.1 . From six to seventeen stations for reading arrival times of each event were available for calculation.

Fig. 3 shows the result of relocations. The position of center of each symbol indicates the new location and the arrow indicates the length and direction of shift from the original to new location of an event. The position of the other end of the arrow is old location. The average change of distance is 0.3 degree, and the maximum change is up to 3.0 degrees of distance. This may come from the errors of too small number of data, and travel time

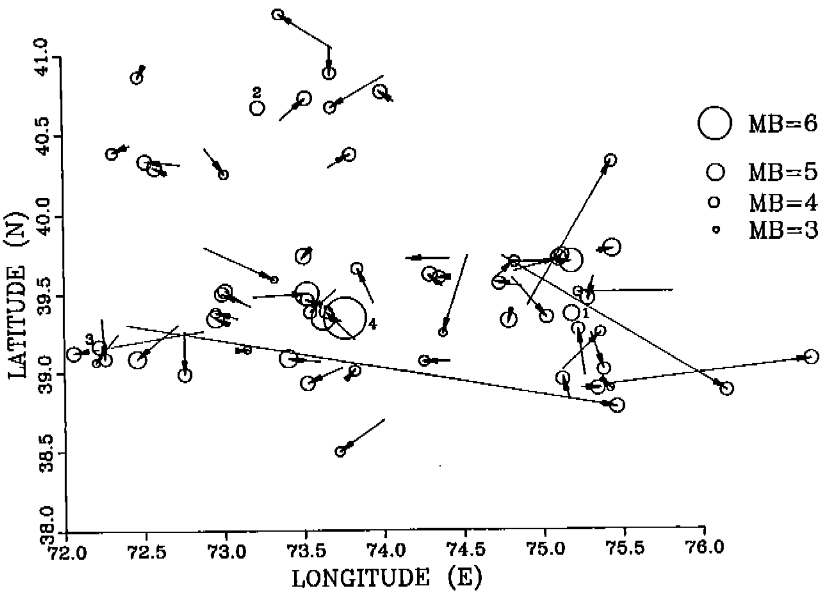

Fig. 3 Relocated hypocenters of events in Fig. 2. The center position of each circle indicates the new location. The arrows indicate the length and direction of shift from the original to new locations of an event. The maximum change of distance is up to 3.0 . The average change of distance is 0.3 .

reading errors.

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