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A Study on the Hybrid Position Control of the Liquid-Slop System

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Liquid-Slop 시스템의 하이브리드제어에 관한 연구

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초 록

이 논문에서는 Hoop속에 Ball이 구르는 간단한 전기 기계 구동 장치를 고려 하였다. 이 장치의 구조는 간단하지만 원유수송선과 같은 대량 액체를 고속 수송할 때 발생되는 "liquid slop"문제를 해석하고 제어하는데 좋은 모델이 된다. 본 연구에서는 liquid slop dynamics를 언급 하였고, 이러한 시스템을 제어하기 위하여 재래식 PID, state-feedback 알고리즘의 유용성을 검토하였고 하이브리드 제어방식을 제안하였다. 하이브리드 제어방식이 다른방식보다 성능면에서 다소 우세함을 알 수 있었다.

1. INTRODUCTION

One of the consequences of the trend toward the high speed transportation of bulk liquids is that the infleunce of the cargo upon the vehicle must fully accounted for. This has always been true in the static sense, but in recent years the dynamical interaction between the material being transported and its container has grown in importance. In fact, the behaviour of a vehicle during a manoeuvre is very often a joint function of liquid cargo and vehicle dynamics. This problem, referred to colloquially as "liquid slop", is known to be a problem in high speed road and rail transport of bulk liquids, maritime transpotation especially in oil tankers, and liquid fuelled missiles, etc.

Any arrangement which involves the rapid movement of large quantities of fluid and an oscillating load such as a moving crane or body of liquid can exhibit the characteristic oscillations which are associated with "liquid slop". The dynamical behaviour of liquid slop is recognized as causing control problem in a wide range of applications[1].

The initial purpose of this paper is to describes that the ball and hoop system demonstrates the phenomenon of fuel slosh in liquid-fuelled missiles and to research dynamics slop quenching techniques. This system is extremly rich in dynamics in a manner which allows simple physical demonstration of complex transimission zeros, nonminimum phase behaviour, pole and zero assignments.

In paticular, if a cylindrical vessel is considered, then to a first approximation, the essential dynamical character of liquid movement in the cylinder (Fig. 1 a) is captured by the motion of a body rolling inside a hoop (Fig. 1 b). The vehicle motion is introduced by allowing the hoop to rotate under the action of a direct drive servo-motor, while the liquid motion is modeled by the oscillation of a ball rolling in the inner periphery of the hoop.

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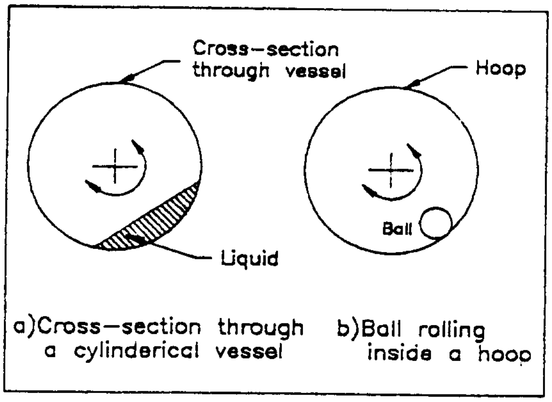


Fig. 1 Principle of Liquid Slop Simulation

In the design of dynamic system, the major concern is to obtain the most desirable response to a given input. In the case of DC motor regulating system, the objective is for the system always to be maintained at its equilibrium state. When subjected to a disturbance the system is expected to return to its equilibrium state as quickly and as smoothly as possible.

Last part of this paper shows the ball and hoop operating as feedback control system with the aim of actively quenching ball oscillations during hoop angular position manoeuvres by using a hybrid control algorithm.

2. SYSTEM MODELLING

Consider Fig. 2 which shows a ball and hoop system with the relevant variable annotated. Assume that hoop is driven by a pure torque source T(t) and that the coordinates which define the dynamical behavior of the system are θ and y, where θ is the angular position of the hoop with respect to a datum point A and y is the postion of the ball on the inner periphery of the hoop, measured with respect to the datum point A, respectively.

In the figure, ψ represents the slop angle of the ball, R and r are the radius of the hoop and the rolling radius of the ball (recall that the disk inner periphery may be grooved to act as a guide for the ball), V is the liner velocity of the ball, and ϕ is the angular rotation of the ball, respectively.

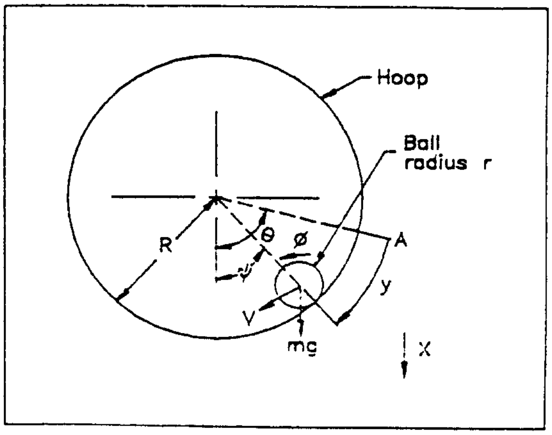


Fig. 2 Ball and Hoop Model

Cleary y and θ completely and independently specify the system motion and can be used as generalized coordinates in a variational model of the ball and hoop system. The system Lagrangian is

$$L = [I_a(\dot{\theta})^2 + I_b(\dot{\phi})^2 + mV^2] / 2 \quad (1)$$

Rewriting in terms of the generalised coordinates we have

$$L = [I_a(\dot{\theta})^2 + I_b(\dot{y}/r)^2 + m(CR - r)(\dot{\theta} - \dot{y}/R)^2] / 2 \quad (2)$$

where I_a is the moment of inertia of the hoop; I_b, moment of inertia of the ball; and m, mass of the ball. In addition, the system co-content J is associated with the rolling friction coefficient of the ball(b_b) and the rotational coefficient of the motor assembly(b_m).

Thus

$$J = [b_b(\dot{y}/r)^2 + b_m(\dot{\theta})^2] / 2 \quad (3)$$

The generalised inputs are, for coordinate θ, the torque $T_\theta(t)$ given by

$$T_\theta(t) = T(t) - mg(\partial x / \partial\theta) \quad (4)$$

where x is the vertical displacement of the ball given by

$$x = -(R - r)(1 - \cos(\theta - y/R)) \qquad (5)$$

Hence the Lagrange's equation for the θ coordinate is

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} + \frac{\partial J}{\partial \dot{\theta}} = T_0(t) \qquad (6)$$

Lagrange's equation for the y coordinate is

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{y}}\right] - \frac{\partial L}{\partial y} + \frac{\partial J}{\partial \dot{y}} = Fy(t) \qquad (7)$$

where Fy(t) is the generalised force given by

$$\begin{aligned} Fy(t) &= mg(\partial x / \partial y) \\ &= (mg(R - r)/R)\sin(\theta - y/R) \qquad (8) \end{aligned}$$

Thus, the following equations of motion for the ball and hoop arise :

$$\begin{aligned} [I_a + m(R-r)^2]\ddot{\theta} + b_m\dot{\theta} - (m(R-r)^2/R)\ddot{y} \\ = T(t) - mg(R-r)\sin(\theta - y/R) \qquad (9) \end{aligned}$$

$$\begin{aligned} [I_b/r^2 + m(R-r)^2]\ddot{y} + (b_b/r^2)\dot{y} - (m(R-r)^2/R)\ddot{\theta} \\ = (mg(R-r)/R)\sin(\theta - y/R) \qquad (10) \end{aligned}$$

Equations (9) and (10) describe the motion of the ball and hoop in full. The equation can be simplified in any one of a number of ways. First, if the ball has relatively small mass compared with I_a, then Eq. (9) becomes that of an inertial load I_a with viscous damping b_m, driven by the input torque T(t).

Thus

$$I_a\ddot{\theta} + b_m\dot{\theta} = T(t) \qquad (11)$$

Eq. (10) can be linearized by assuming that the angle (θ-y/R) is relatively small giving

$$\begin{aligned} [I_b/mr^2 + ((R - r)/R)^2]\ddot{y} + (b_b/mr^2)\dot{y} + \\ (g(R - r)/R^2)y = ((R - r)^2/R)\ddot{\theta} + \\ (g(R - r)/R)\dot{\theta} \qquad (12) \end{aligned}$$

Further simplification occurs in the coefficients without altering the dynamical structure if the subtituting the following relations is made :

$I_b = 2mrb^2/5$, $R \gg r$, $rb \approx r$

where r_b is radius of the ball.

With these alterations (12) becomes

$$\begin{aligned} (7/5)\ddot{y} + (b_b/mr^2)\dot{y} + (g/R)y \\ = R(\ddot{\theta} + (g/R)\theta) \qquad (13) \end{aligned}$$

Taken together (11) and (13) are linearized, decoupled equations of motion, and have the transfer function block diagram representation of Fig. 3(a). An alternative formulation for the equation of motion takes the angles θ and ψ as coorinates, where ψ is the slop angle and replaces the variable y as the system output. For dynamical analysis and control studies of fluid slop, ψ is the relevent output to consider. Substituting for y in (13) from the relation $Y = R(\theta - \psi)$ gives

$$\begin{aligned} (7/5)\ddot{\psi} + (b_b/mr^2)\dot{\psi} + (g/R)\psi \\ = (2/5)\ddot{\theta} + (b_b/mr^2)\theta \qquad (14) \end{aligned}$$

The transfer function form of Eq. (14) is presented in Fig. 3(b).

The interesting equation is Eq. (14) since this contains the oscillatory modes of ψ which are associated with liquid slop. These transfer functions are utilized to slop quenching control problem.

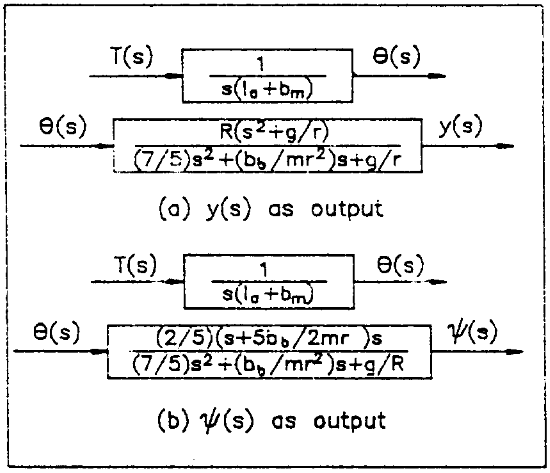


Fig. 3 Transfer Function Block Diagram

3. CONTROL ALGORITHMS

Most DC servo motor drives are operated as closed-loop position control system. Generally, external position loop and an internal current loop are the most commom feedback techniques. A simple proportional gain in the position loop may not be sufficient to provide a precise control. This may result in a high overshoot and also an undesirable steady-state error in position. Therefore some kind of compensation technique has to be employed to improve the performance of the control. In this paper, a hybrid control scheme is proposed. For the purpose of comparison, three control algorithms are considered : the classical PID control algorithm, the state feedback algorithm, and the hybrid control (PID+state feedback) algorithm. Static and dynamic performance of three control schemes are studied in terms of system time specification. Computer simulation results obtained from the ball-hoop system using the control schemes described below are given and discussed. The transfer fuctions of the ball-hoop system are given in the Fig. 3.

3.1 PID Control

The PID control algorithm is widelly used in industrial systems of any order, even in plants whose transfer function has been completely defined. Practical design reasons dictate the use of a low-pass filter to eliminate undesirable frequency noise. In this case, the mathematical model of the PID control algorithm becomes

$$G_c(s) = K[1/T_i*s+(1+T_d*s)/(1+a*T_d*s)] \quad (15)$$

where K represents the proportional gain, T_d is the derivative time, T_i is the integarl time, and a is the tameness constant. [5]

Tuning should be done with respect to the process model. For processes modelled by a plant delay with an integration, Zeigler and Nichosl(1942) have derived direct formulas giving the values of K, T_i, T_d that minimize the performance index.

Using the frequency-domain approach, E polak and D. Q. Mayne[2] have solved the tuning problem and then Z. S. Wang and A. Seireg[3] proposed a method which is based on a time-domain formulation. The method is deduced from analytically integrating the quadratic function by using the modal matrix. However the parameter optimization of control systems has not been as fully explored yet.

In this study the parameters of PID controller were evaluated in the time-domain approach. The results are K=3000, T_i=10, T_d=0.03, a=0.1.

3.2 State-Feedback Control

According to optimal control theory[4], the objectives of the control of dynamic systems can be achiveed by implementing an optimal control law through a state-variable feedback controller. That is, for a given dynamic system

$$\dot{\underline{x}}(t) = [A]\underline{x}(t)+[B]\underline{u}(t) \quad \underline{x}(0) = x_o \quad (16)$$

$$\underline{y}(t) = [C]\underline{x}(t) \quad (17)$$

find a control vector

$$\underline{u}(t) = -[K]\underline{x}(t) \quad (18)$$

that minimizes a quadratic integral performance index

$$I=\int_0^{\infty}\{\underline{x}^T(t)[Q]\underline{x}(t)+\underline{u}^T(t)[R]\underline{u}(t)\}dt \quad (19)$$

where $x(t)\in R^n$ is a state vector; $u(t)\in R^m$ is a control vector; $y(t)\in R^1$ is an output vector. [A] is an n x n state matrix; [B] is an n x m input matrix; [C] is an 1 x n output matrix; [K] is an m x n feedback matrix; and [Q] is an n x n weighting matrix for the state vector x(t), which is symmetric semi-definite. [R] is an m x n weighting matrix for the control vector u(t), which is symmetric and positive definite. It is well known that if all the states are accessible, the optimal control is given by

$$\dot{\underline{u}}(t) = -[\dot{k}]\underline{x}(t) \quad (20)$$

where

$$[\dot{k}] = -[R]^{-1}[B]^T[P] \quad (21)$$

where [P] is an n x n positive definite symmetric matrix, which can be obtained from solving the algebraic Riccati matrix equation

$$[A]^T[P]+[P][A]-[P][B][R]^{-1}[B]^T[P] = -[Q] \quad (22)$$

Once [P] is obtained, $[\hat{K}]$ can be analytically determined from Eq. (21).

$[\hat{K}]$ is the optimized state-feedback gain matrix.

In the Ball-Hoop system, the state matrices are represented as follows:

$$[A]=\begin{bmatrix} -1.333 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 86.667 & -1.2 & -112 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$[B]^T = [\ 1\ \ 0\ \ 0\ \ 0\ ],$$

$$[C]^T = [\ 0\ \ 0\ \ 0\ \ 2.4\ ],$$

$$\underline{x}^T = [\ \dot{\theta}\ \ \theta\ \ \dot{\phi}\ \ \phi\ ], \qquad y = \phi$$

where $\dot{\theta}, \theta$ are the hoop angular velocity and position, respectively and $\dot{\psi}, \psi$ are the bll slop angular velocity and position, respectively.

When the weighting matrix [Q] and [R] corresponding to x(t) are given as

$$[Q] = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[R] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$[\dot{K}]$ can be determined as follows:

$$[\dot{K}]^T = [\ 31.33\ \ 32.44\ \ -0.02\ \ -1.04\ ]$$

[K] con be determined as follows :
$[K]^T$=[31.33 32.44 -0.02 -1.04]

The major difficulty of solving optimal parameter problem for the state feedback control is predescribing the weighting matrices [Q] and [R]. In this system hoop angle(θ) and angular velocity ($\dot{\theta}$) are weighed much more than the other states by the trail-and-error investigation.

3.3 Hybrid Control

This hybrid control means combination of PID and state-feedback control. The block diagram of a hybrid control system is depicted in Fig. 4.

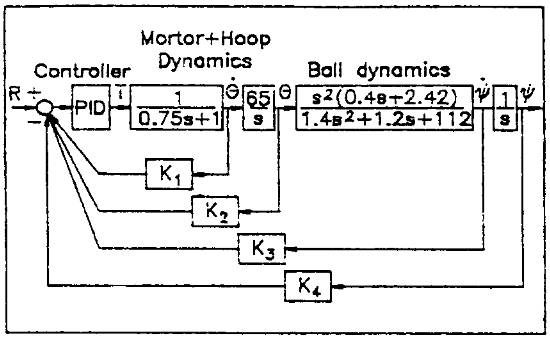
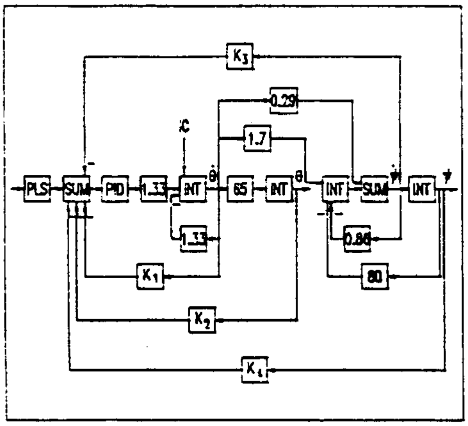


Fig. 4 A Hybrid Control Block Diagram For The Ball-Hoop System

4. COMPUTER SIMULATION

A number of general purpose simulation packages, ie, MATLAB, CC, TUTSIM, SIMNON, etc, exist today which will handle a wide variety of control problems. Hovever, their generality leads to some inefficiency and difficulty in using them. TUTSIM[5] is a simulation program for engineering design and optimization of continuous dynamic systems. This TUTSIM is a block diagram oriented simulation language. It has the convenience of the analog computer and the speed and accuracy of the digital computer. Since the program can be coded in the form of the block diagram, each state of the dynamic system can be calculated. The other programs are difficult to do it. Thus, in this study the time responses of the control variable(T), the hoop position (θ) and ball slop position (ψ) are simulated by the TUTSIM. The simulation block diagram of a hybrid control system is shown in Fig. 5. In the figure, each block respresents a mathematical operation.



Fgi. 5 TUTSIM Block Diagram For Simulation

TUTSIM block diagrams may be written from the equations term by term, or sometimes by direct inspection of the real system. It is one of the advantages of the TUTSIM program.

5. RESULTS AND DISCUSSION

Fig. 6 represents the impulse response of open -loop control system when the system is subjected to impluse distrubance. In Fig. 6, curve 1 represents the hoop angle in radians and curve 2 is the ball slop angle in radians. As it is shown, the ball is oscillating with the frequency of 1.33 Hz.

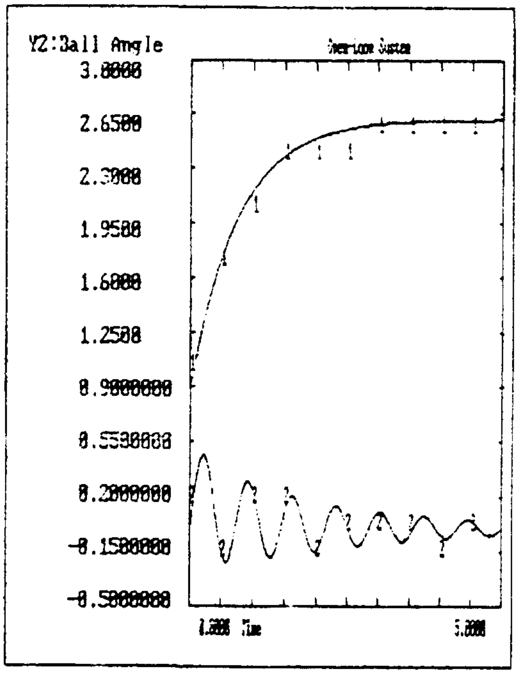


Fig. 6 Impulse Response of Open-Loop System

In order to reduce the oscillatory motion, we should apply a control law to the system. When the PID control algorithm is utilized to the ball-hoop system, the impluse response of the system is represented as Fig. 7. In the figure curve 1 indicates the control variable (torque), curve 2 represents the hoop angle, and curve 3 is the ball angle.

Fig. 7 shows that the amplitude of oscillatory motion of the ball slop angle (curve 3) is drastically reduced compared to the open-loop system but the oscillation is still remained.

Fig. 8 shows the impluse response of the system by using the state feedback control algorithm. The oscillating motion of the ball slop angle is damped out significantly, and the amplitude of the first peak becomes smaller than the one of PID case. Therefor, it is said that the state feedback algorithm performs better than the PID in general.

Finally the hybrid control algorithm is applied to the system. The impluse response of the system is depicted in Fig. 9. It shows that the settling time of the ball slop angle by the hybrid control algorithm is faster than the state feedback algorithm, and the peak amplitude is also reduced more than the others. When suddenly the disturbance occurs to the hoop, the ball should be settled down as soon as possible in some sense. For this requirement, it is said that the hybrid control algorithm would be better.

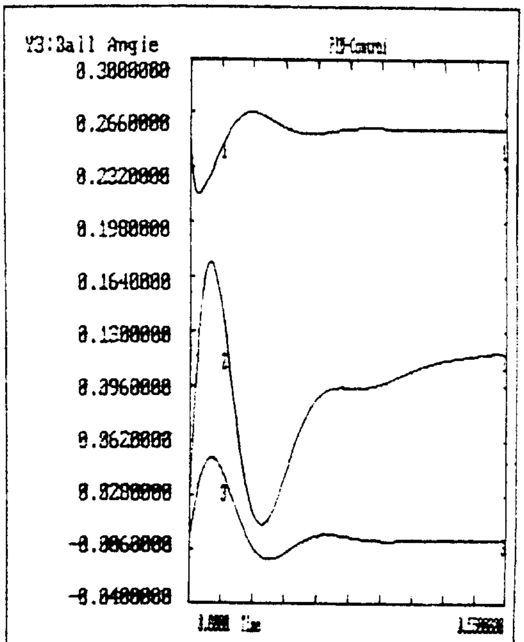


Fig. 7 Impulse Respons of PID Control System

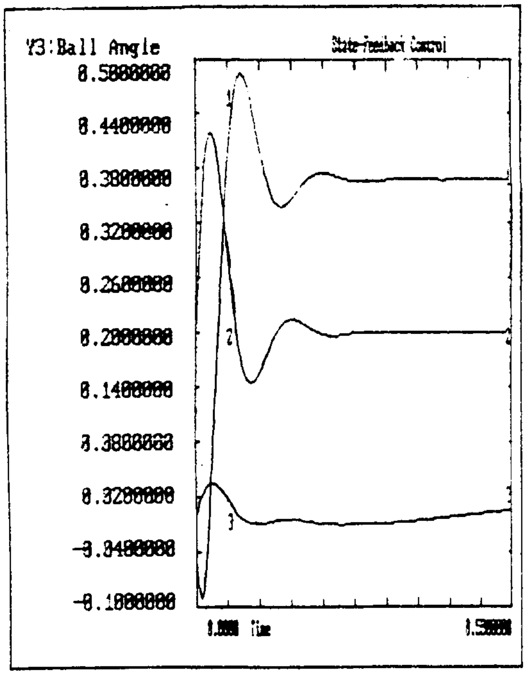


Fig. 8 Impulse Respons of State Feedback Control System

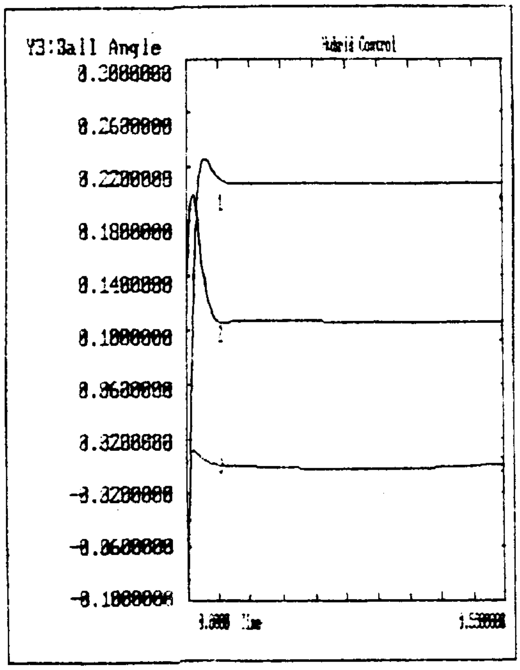


Fig. 9 Impulse Respons of Hybrid Control System

6. CONCLUSION

This study has described the dynamics and control of a liquid slop problem. The action of the ball rolling on the hoop inner periphery is presented as a liquid slop sumulator.

To reduce the undesirable chattering motion of the ball subjected to the impulse disterbance, the conventional control algorithms such as PID and state-feedback were applied to the system. Finally the proposed hybrid control algoritm has been compared with the conventional algorithms.

The results by TUTSIM simulation shows that the proposed hybrid control acts much better than the others. Even if the hybrid control performs satisfactory in simulation, the implementation would be faced to difficulty. This is so because the hybrid control action requires too much torque to drive the motor at the initial stage (Fig. 9). Therefor, further investigation study will be carried out using LQG/LTR method for the easy implementation.

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