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Strain Analysis using Fourier Transform Grid Method and Its Image Processing

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퓨리에 변환 격자법과 화상 처리를 이용한 스트레인 해석

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초 록

진동하는 구조물을 설계할 때에는 그 구조물 중의 Strain이나 응력이 최대가 되는 장소나 시각을 알 필요가 있다. 지금까지의 Strain 해석에는 Strain gauge 등과 같은 접촉법이 많이 이용되고 있다. 더우기, 접촉법으로 대변형 진동을 하는 물체의 Strain을 해석하는 것은 곤란하다. 최근에는 비접촉법으로 Strain 분포를 해석하기 위해 화상처리를 이용한 계측이 행하여지고 있다. 이들의 Strain 분포를 측정하는 광학적인 방법으로는 격자법, Moire법, 홀로 그랩픽 간섭법 등이 있다. 특히 대변형이나 대Strain을 해석하는 데에는 격자법이 많이 이용되고 있는데, 종래의 격자법은 Data를 처리하는 데에 많은 시간과 노력이 소요되고 작업도 매우 복잡하며, Data의 수도 제한이 되어서 구조물의 분포의 해석 정도에 큰 영향을 미치게 된다. 본 논문에서는 스테레오법을 이용해서 2차원 격자를 붙인 시료표면의 각 점의 3차원 좌표를 계측하고, 또 Fourier 변환 격자법을 적용하여 촬영된 2차원 격자의 화상에서 위상치를 구한다. 그리고 물체의 변형 전후의 대응 관계의 화상에서 3차원 형상과 Strain 분포를 해석하는 방법을 제안한다. 이 방법을 이용하면 진동하는 구조물의 3차원 변위분포, Strain 분포를 정도 좋게 해석할 수가 있다.

1. INTRODUCTION

In an optimal design of structure, it is very important to know the shape and strain distribution of an object. It is, however, difficult to measure the shape and strain of tender objects or much deformed objects using contact methods. Therefore the grating method using image processing which is not contact method has been introdued. Some grating methods using image prosessing were proposed to analyze either shape or strain of objects. In the conventional grating methods using image processing technique, the position of a grating lines is measured in an integer of pixels.

For interpolating data between grating lines, Sciammarella et al[1]. have developed a method of analyzing the phases of mismatched fringes by using one-dimensional Fourier transform. Moreover, Takeda and Mutoh[2] have presented the Fourier transform profilometry. Morimoto et al[3-6]. have developed a new moire method by using the first harmonic of the Fourier spectrum of the deformed grating image to analyze the strain distribution on the surface of the object. This method is called the Fourier transform grid method(FTGM). For analysis of the strain on the surface of the object, a two-dimensional grating pattern has been made on the surface of the object.

Using the FTGM, the positions of the grating lines are measured in decimal pixels by

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analyzing the phase distribution so that the accuracy is higher.

In this paper, the three-dimensional shape and surface strain distribution of a vibrating rubber plate are measured using a stereoscopic method. The two-dimensional grating is drawn on the surface of the specimen. The gratings are recorded from two different directions by two CCD cameras and recorded by two video recorders. Each pair of corresponding points in the two grating images observed from two different directions are matched using the two-dimensional Fourier transform grid method. After that, using the stereoscopic method, the 3D shape and surface strain of the specimen are calculated. Consecutive images make it possible to analyze the dynamic behavior of vibrating objects.

2. THEORY OF FOURIER TRANSFORM GRID METHOD

In order to analyze a two-dimensional grating image, a cross-grating with orthogonal lines as shown in Fig. 1 is used. The brightness intensity function of a two-dimensional cross-grating can be expressed as the product of two single-grating intensity functions. One single-grating normal to the x-axis denoted as x-grating, and another single-grating normal to the y-axis denoted as y-grating, are utilized for displacement measurement in the x-and y-directions, respectively. The intensity function g(x, y) of the cross-grating in the Fourier series expansion is

$$g(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} i_{m,n}(x,y)\exp\{j2\pi m\omega_x x + j2\pi n\omega_y y\} \quad (1)$$

where $i_{m,n}$ is the coefficient of the harmonic of the order (m, n), m and n are integers, j is the imaginary unit, w_x and w_y are frequencies of the x-and y-gratings, respectively. The Fourier transform of g(x, y) is expressed in the following equation.

$$G(\Omega_x,\Omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\exp\{-j2\pi(\Omega_x x, \Omega_y y)\}dxdy = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} I_{m,n}(\Omega_x - m\omega_x, \Omega_y - n\omega_y)$$

where Ω_x and Ω_y are in the x-and y-directional frequencies respectively. Figure 2 shows a schema of the two-dimensional Fourier spectrum of the grating. Each circle shows the region where the harmonic of the order (m, n) exists. In order to seperate into the x and y terms containing the coresponding dispalcement component, respectively, only the first harmonic is extracted and shifted. If each of the first harmonic in the x-and y-directions is not overlapped by the other harmonics, it can be extracted, respectively. Then the one-dimensional analysis in the x-direction. The (1, 0) order harmonic indicated with oblique lines in Fig. 2 is extracted by filtering, and its inverse Fourier transform is computed as the following equation.

$$i_{1,0}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_{1,0}(\Omega_x - \omega_x + \Omega_y - \omega_y)\exp\{-j2\pi(\Omega_x x + \Omega_y y)\}d\Omega_x d\Omega_y = C_{1,0}\exp\{j\theta_x(x,y)\} \quad (3)$$

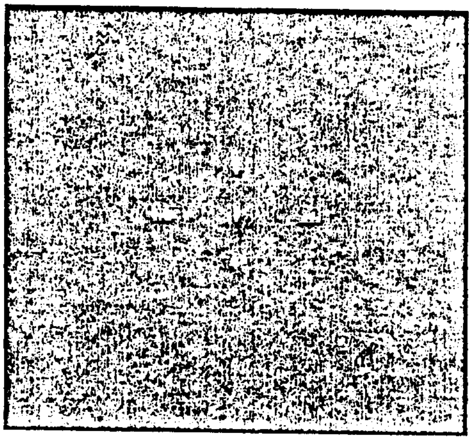
The real and imaginary parts of Eq. (3) are

$$\mathrm{Re}\{i_{1,0}(x,y)\} = C_{1,0}\mathrm{COS}\{\theta_x(x,y)\}$$
$$\mathrm{Im}\{i_{1,0}(x,y)\} = C_{1,0}\mathrm{SIN}\{\theta_y(x,y)\} \quad (4)$$

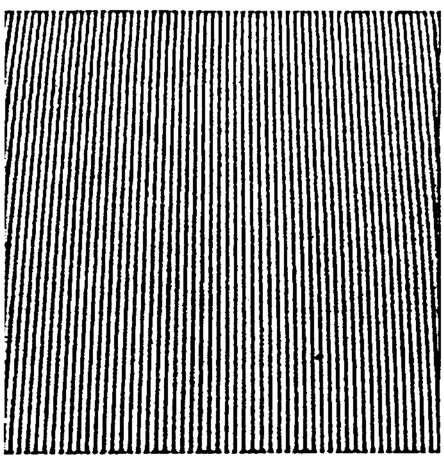
Each equation shows a sinusoidal fringe pattern. The image of this equation shows only the x-gration. The phase distribution $\theta_x(x, y)$ is obtained by calculating

$$\theta_x\{(x,y)\} = \arctan\frac{\mathrm{Im}\{i_{1,0}(x,y)\}}{\mathrm{Re}\{i_{1,0}(x,y)\}} \quad (5)$$

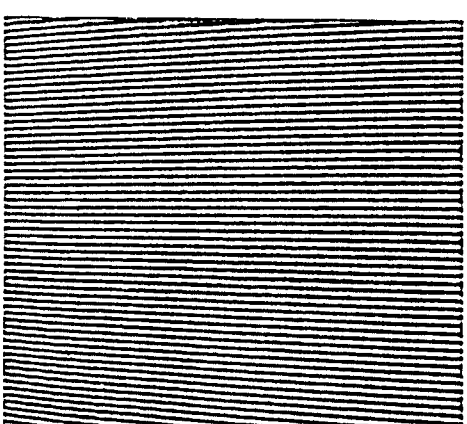
Similary, using the y-directional first hamonic $I_{0,1}(x, y)$ the phase distribution $\theta_y(x, y)$ of the y-graying is obtained. The position of the point (x, y) corresponding to a certain phase (θ_x, θ_y) can be calculated in a decimal pixel unit by a two-dimensional interpolation based on the continuity of phase distribution. Since the phases obtained by Eq. (5) are confined to the range of $-\pi$ to π the phases are adjusted so as to



(b) Fourier spectrum of 2D grating



(c) Real part of (1, 0) order harmonic



(d) Real part of (0, 1) order harmonic

Fig. 4 FTGM analysis of 2D grating on rubber plate

the same phase vector exist on both the left and right images. These points can be regarded as the same point on the specimen. The correspondence of the left and right images can be determined by searching the points which have the same phase vector on the both left and right images. The coordinates of these points which have same phase vector are calculated not in integer but in real number using the liner interpolation.

By analyzing the two-dimensional phase distributions in both images recorded by camera 1 and camera 2, the exact positions of the intersection on each image are obtained. From each coordinate which is obtained by this procedure, the 3D coordinate of the point on the specimen can be calculated based on the stereoscopic method. From the 3D coordinates of the whole surface in the analyzable area on the specimen, the shape of the specimen and the distribution of the strain can be calculated. The three-dimensional positions of grid points are determined by Eq. (6). The positions of the intersection of the grating are stored in a database. By using the above methods, the shape of the plate before deformation is obtained by the position of each intersection of the grating on the object. After deformation, the shape of the vibrating object is meaured by the same method. Furthermore the shape and strain distribution of the vibrating specimen can be dynamically alalyzed from the images at each time step for one period. By analyzing positions of the object before and after deformation, the shape and strain distribution can be calculated. The results of the three-dimensional shape and strain ε distribution obtained at times of 0sec and 1.07sec are shown in Fig. 5 and Fig. 6, respectively. The matching of the corresponding points between before and after deformation can be also automated by comparing the phases.

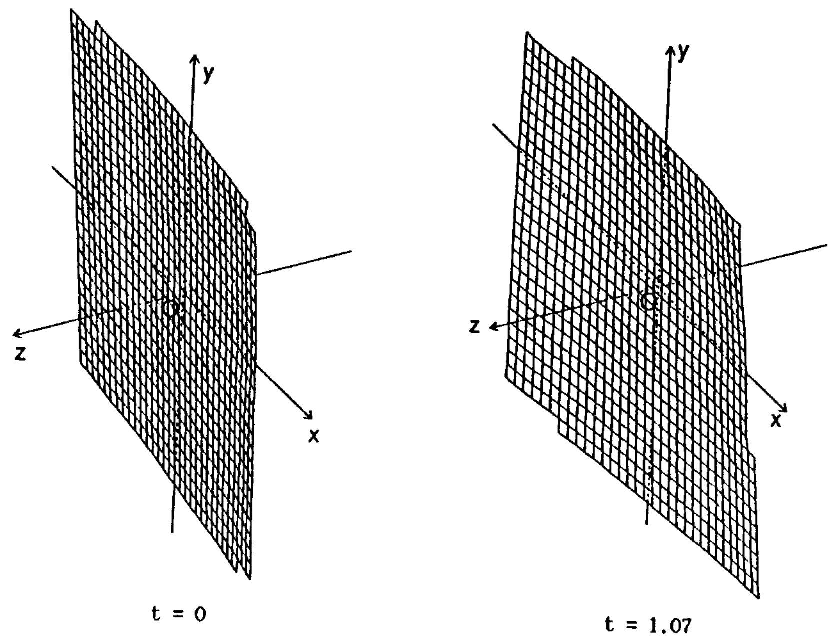


Fig. 5 Results of calculating 3D shape

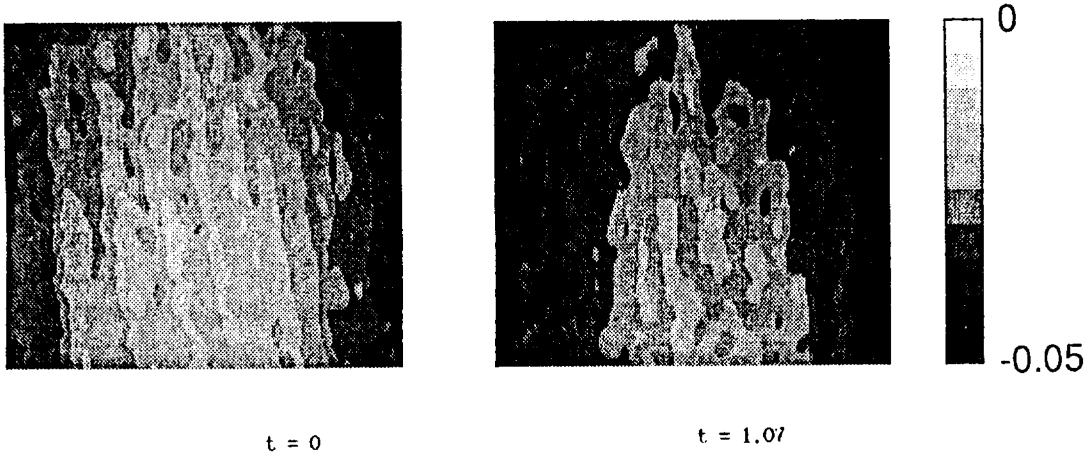


Fig. 6 Surface strain distributions

4. CONCLUSIONS

In order to analyze the shape and strain on the surface of vibrating objects, we have previously proposed the Fourier transform grid method to analyze the three-dimenstional shape and surface strain dirstribution of standstill objects by analyzing the two-dimensional grating images recorded with two cameras. The three-dimensional coordinates of the surface points of an object have been claculated using the stereoscopic method. Using the Fourier transform grid method, the two-dimensional grating can be easily seperated to x-and y-grating. By be continuous by adding or subtracting 2π. The phase distribution in the image input from the CCD camera at a different angle can be computed in the same way. Then, the corresponding points of the different images can be found out by interpolating the phase distribution.

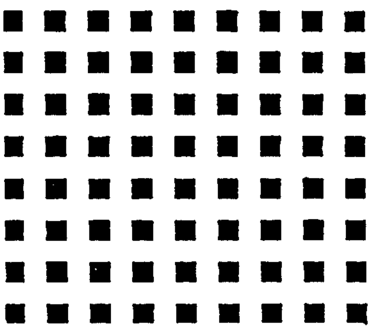


Fig. 1 Part of 2D grating

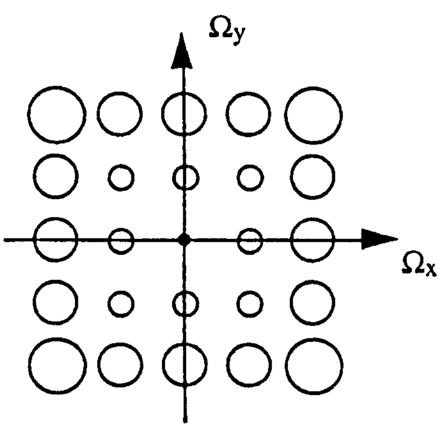


Fig. 2 Schema of Fourier spectrum of 2D grating

3. EXPERIMENT AND ANALYSIS

In order to measure 3D shape and strain distribution of a vibrating rubber plate, the meaurement system shown in Fig. 3(a) was developed based on the stereoscopic method. Figure 3(b) shows the geometrical relation of the position of the specimen and two cameras. Two-dimensional grating lines are drawn on the object for matching the corresponding points of the two images recorded from dirfferent directions in the stereoscopic method. These grating lines can also be used as the corresponding points before and after deformation. Figure 3 shows the views from the direction of the y-axis.

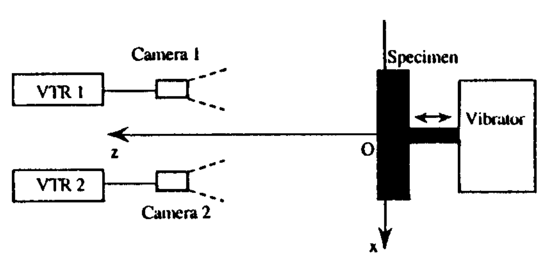
Now the specimen is assumed to exist at the place shown in Fig. 3. The coordinate of a point P on the specimen can be calculated from the gemetrical relationship. C_1 and C_2 are the position of camera 1 and camera 2, respectively. The camera 1 and camera 2 are assumed to be parallel with the z-axis and to be toward the negative z-direction. In the views from camera 1 and camera 2, P is photographed as if it existed at P_1 and P_2 on the x-y plane, respectively. It is easy to determine the correspondence of the coordinates on the image and the x-y coordinates on the x-y plane. Therefore the 3D coordinates of P_1 and P_2 are immediately obtained from the images photographed by camera 1 and camera 2. The 3D position (x, y, z) of each grid point is calculated using the following equations :

$$x = z_P(x_{C1} - x_{P1})/z_{C1} + x_{P1}$$

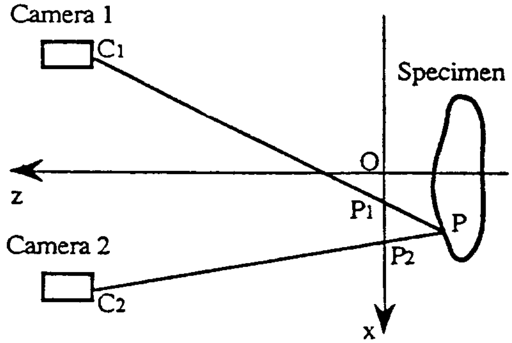
$$y = z_P(y_{C1} - y_{P1})/z_{C1} + y_{P1} \qquad (6)$$

$$z = \frac{-x_{P1} + x_{P2}}{(x_{C1} - x_{P1})/z_{C1} - (x_{C2} - x_{P2})/z_{C2}}$$

where the coordinate components of the point C_1, C_2, P_1 and P_2 are expressed in x, y and z with the suffixes C_1, C_2, P_1 and P_2, respectively.



(a) Measurement system



(b) Geometrical relation of the system

Fig. 3 Shape and strain measurement system for 3D object

The distance of the measured point on the video images is accurately by the FTGM techniques, and then the three-dimensional position of each point on the object is analyzed by Eq. (6).

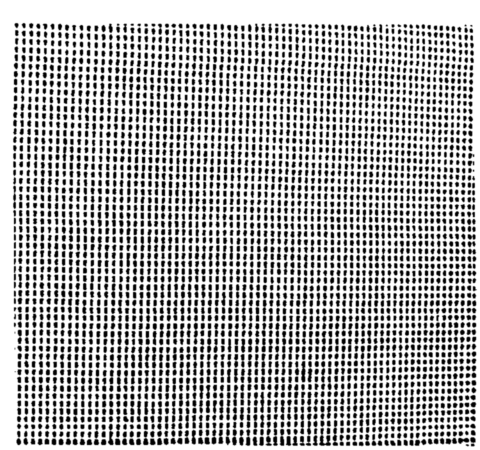
By using this measurement method, the position of each intersection of the gration on the object is obtained both before and after the deformation. In this paper, the strains are used to simplify the treatment. The strains are defined as follows :

$$\varepsilon_x = \frac{du}{dx}$$
$$\varepsilon_y = \frac{dv}{dy} \qquad (7)$$
$$\gamma_{xy} = \frac{du}{dx} + \frac{dv}{dy}$$

where u and v are the components of displacement in the x-and y-directions respectively, ε_x and ε_y are the x-and y-directional normal strains, respectively, and γ_{xy} is the shear strain.

Let us show the way to analyze shape and strain from the two images obtained in this experiment. The 3D shape and strain of a vibrating rectangular rubber plate is measured by the method metioned above. The specimen is a rubber plate whose size is 450×40×10mm. The upper end of the specimen is fixed and the lower end is statically pulled in the perpendicular direction to the original rubber surface. A two-dimensional grating pattern with a pitch 1mm is drawn on the plate before deformation The specimen is set with inclining 30 degree from z-axis. A portion of the deformed grating is recorded with two CCD cameras. The region for the measurement is the rectangular between 100 and 20mm from the upper end of the specimen. The pitch between two grating lines in each image is about 6 pixels. Figure 4(a) shows the grating image recorded by camera 1. Each image obained from cameras is transformed to the Fourier spectrum by calculating the Fourier transform. The Fourier spectrum of the grating image is shown in Fig. 4(b). The cross grating can be divided to the one-dimensional gratings for each x-and y-direction by extractings the first harmonic of each direction and by calculating the inverse Fourier transform. These gratings have a real part and an imaginary part. The real part of the inverse Fourier transform of the (1, 0) harmonic and the (0, 1) harmonic are shown in Fig. 4(c) and Fig. 4(d), respectively.

The phase distributions of each direction can be obtained from these gratings by Eq. (5). A point is selected as the datum point of the phase distribution, which has the same phases in each of the x-and y-directions, between the left and right images. The point which have



(a) 2D grating image

improving the Fourier transform grid method program in order to perform the automated and high-speed analysis of the FTGM, the three-dimensional shape and surface strain distributions of a vibrating object changing with time are measured. Moreover, by calculating and interpolating phase distribution, the decimal pixel unit measurement and automated matching is performed.

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