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Comparison of Parameter Estimation Methods for Time Series Models in the Presence of Outliers[1]

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ABSTRACT

We propose an iterated interpolation approach for the estimation of time series parameters in the presence of outliers. The proposed approach iterates the parameter estimation stage and the outlier detection stage until no further outliers are detected. For the detection of outliers, interpolation diagnostic is applied, where the atypical observations are interpolated. The modified GM-estimate which replace atypical observations by the one-step-ahead predictor instead of downweighting is also proposed. The performance of the proposed estimation methods is compared with other robust estimation methods by simulation study. It is observed that the iterated interpolation approach performs reasonably well in general, especially for single AO case and large ϕ in absolute values.

1. Introduction

Outliers in time series can affect the model identification, the parameter estimation, and the forecasting. Since Fox(1972) discussed two types of outliers, the additive outliers(AO) and the innovational outliers(IO), many authors studied the effects of different types of outliers on those procedures. Fox(1972) developed a likelihood ratio test for detecting outliers in autoregressive(AR) models. This test was extended by Chang(1982), Tsay(1988), and Chang, Tiao and Chen(1988) to autoregressive integrated moving average(ARIMA) models. Bruce and Martin(1989), Lee(1990), Ledolter(1990) and Ryu et al.(1992) proposed outlier detection procedures based on the deletion method which mainly use the innovational variance. Denby and Martin(1979), Martin et al. (1979, 1980, 1981,

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1990), and Bustos and Yohai(1986) discussed a class of estimates(M, GM, RA and S) that are robust toward outliers. Tsay (1986) and Chuang and Abraham(1989) discussed model building strategies in the presence of outliers and Ledolter(1987) studied the impacts of outliers on the forecasts.

A simulation study to compare the parameter estimation methods in time series with outliers was performed by Chuang and Abraham(1989). In their comparison, the RA-estimate was not considered and the outlier types were restricted to the AO and IO cases. Though AO and IO are frequently studied, many authors suggested the use of consecutive AO's, see Bruce(1989), Bruce and Martin(1989), Lee(1990), and Ryu (1991). In this paper we review the various parameter estimation methods including the RA-estimate and propose an iterative approach based on the interpolation diagnostic by Ryu et al.(1992) and a modification of GM-estimate. The performances of various estimation methods are compared through a simulation study where a single AO and consecutive AO's are imposed on the simulated series.

Let $\{Z_t\}$ denote an outlier-free time series generated by an ARMA model

$$\phi(B)Z_t = \theta(B)a_t,$$

where $\phi(B) = 1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p$, $\theta(B) = 1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q$, and B is a backshift operator such that $BZ_t = Z_{t-1}$. The polynomials $\phi(B)$ and $\theta(B)$ are assumed to have all roots outside the unit circle and have no factors in common and a_t is a sequence of independent identically distributed normal random variables with mean 0 and variance σ^2.

Suppose that outliers occur at $t=T, T+1, \cdots, T+k-1$ in a series $\{y_t\}$. In the consecutive AO's model

$$y_t = Z_t + \sum_{i=0}^{k-1} w_i I_t(T+i)$$

$$\phi(B)Z_t = \theta(B)a_t \ ,$$

where

$$I_t(T) = \begin{cases} 0 \ , & t \neq T \\ 1 \ , & t = T \end{cases}$$

is an indicator for the occurrence of outliers and w_i is the magnitude of disturbance at $t=T+i$.

2. Estimation Methods in the Presence of Outliers

Estimation methods considered in this paper may be classified into two types. One is the robust approach such as M-estimates, GM-estimates, and RA-estimates, where the outliers are not identified but are downweighted using some kind of robustifying loss function. Another type is the iterative approach such as the procedure proposed here and the iterative maximum likelihood procedure proposed by Chang et al.(1988), where the outlier detection stage and the parameter estimation stage are repeated until no further outliers are identified.

2.1 M-estimates

Denby and Martin(1979) suggested the use of an M-estimate $\hat{\phi}_M$, defined by the minimization problem

$$\min_{\phi} \sum_{t=1}^{n-1} L(y_{t+1} - \phi y_t) \ ,$$

where $L(\cdot)$ is a symmetric robustifying loss function in AR(1) model. Alternatively, $\hat{\phi}_M$ is a solution of the M-estimate equation

$$\sum_{t=1}^{n-1} y_t \psi(y_{t+1} - \hat{\phi}_M y_t) = 0 \ ,$$

where $\psi(\cdot) = L'(\cdot)$ is a bounded function with $t\psi(t) \geq 0$ and usually $\psi'(0) = 1$.

2.2 GM-estimates

The M-estimate is highly robust in terms of efficiency for the IO model, but has an asymptotic bias difficulty for the AO model. This is due to the unboundedness of the predictor variable y_t . To overcome this difficulty, Denby and Martin(1979) proposed a GM-estimate $\hat{\phi}_{GM}$, as a solution of the generalized M-estimate equation

$$\sum_{t=1}^{n-1} g(y_t) \psi(y_{t+1} - \hat{\phi}_{GM} y_t) = 0 \ ,$$

where $g(\cdot)$ is bounded and $tg(t) \geq 0$, in AR(1) model. The M-estimate and GM-estimate for higher order AR models can be obtained analogously and are referred to Martin(1979). One of the most frequently used ψ functions is the redescending Tukey's bisquare function

$$\psi_{B,C}(u) = u(1-u^2/c^2)^2 \ , \ \ 0 \leq |u| \leq c \ .$$

The M-estimate and GM-estimate may be conveniently computed using the iterated weighted least squares(IWLS) techniques.

2.3 Modified GM-estimates

Though the GM-estimate is consistent under a perfectly observed autoregressive model, they do not completely use the structure of time series when downweighting observations, see Bustos and Yohai(1986) for discussion. We propose a modified GM-estimate where instead of downweighting the observations we use the predictor from the structure when the residuals are large, i.e., instead of $g(y_{t+1})$ we use $\hat{y}_t(1)$, the one-step-ahead predictor of the observation using data up to and including period t if $(y_{t+1} - \hat{\phi}_{GM} y_t)/\hat{\sigma} > C$ where C is the "efficiency-tuning" constant for various choices of ψ function, for details, see Denby and Martin(1979).

Our approach is similar to the Iterated Robust Estimate(IRE) proposed by Martin, Samarov and Vandaele(1981). The IRE uses the GM-estimate as the initial estimate and, using the filtering approach, replace the outliers. After outliers are replaced they obtain the LS estimate instead of the GM-estimate.

2.4 RA-estimates

Bustos and Yohai(1986) introduced a robust estimate based on robustified residual autocovariances (RA-estimate). Let $e_t(\Pi) = \theta^{-1}(B)\phi(B)Z_t$, $\Pi = (\phi_1, \cdots, \phi_p, \theta_1, \cdots, \theta_q)'$ be the residuals and $\gamma_i(\Pi) = \sum_{t=p+1}^{n-i} e_{t+i} e_t$ be the main part of the estimate of residual autocovariances. By robustifying $\gamma_i(\Pi)$, i.e., by replacing the $\gamma_i(\Pi)$ by

$$\gamma_i(\Pi) = \sum_{t=p+1}^{n-i} \eta(e_{t+i}/\hat{\sigma},\ e_t/\hat{\sigma}),\ i=1,2,\cdots,n\ ,$$

we get the class of robust estimates. The RA-estimates are defined by the following equations:

$$\sum_{h=0}^{n-j-p-1} s_h \gamma_{h+j}(\Pi) = 0,\ \ 1 \le j \le p$$

$$\sum_{h=0}^{n-j-p-1} t_h \gamma_{h+j}(\Pi) = 0,\ \ 1 \le j \le q \qquad (*)$$

$$\sum_{t=p+1}^{n} \psi(e_t/\hat{\sigma}) = 0\ ,$$

where s_i and t_i are defined by

$$\phi^{-1}(B) = \sum_{i=0}^{\infty} s_i B^i\ ,\ \ \theta^{-1}(B) = \sum_{i=0}^{\infty} t_i B^i\ ,$$

respectively and $\hat{\sigma}$ is computed using

$$\hat{\sigma} = Med(|e_{p+1}|,\cdots,|e_n|)/.6745\ .$$

Two ways of choosing η are

$$\eta_M(u,v) = \psi(u)\psi(v)\ ,\ \ \textit{Mallow type}$$

$$\eta_H(u,v) = \psi(u,v)\ ,\ \ \textit{Hampel type}\ .$$

In the case of $\eta(u,v) = \psi(u)\psi(v)$, the solution $\hat{\Pi}$ of eq(*) is the LS estimate when the series is given by z_t^* , where

$$a_t^* = \psi(a_t(\hat{\Pi}/\hat{\sigma})\hat{\sigma}$$

and

$$z_t^* = \hat{\phi}^{-1}(B)\ \hat{\theta}(B) a_t^*\ .$$

2.5 Iterated Interpolation Approach

Ryu et al.(1992) suggested an outlier detection procedure based on the innovational variance estimate. Adopting their procedure in the outlier detection stage, we propose an iterated interpolation approach.

The procedure begins with modeling the original series assuming no outlier. Then the outlier detection stage and the parameter estimation stage will be followed as in Chang et al.(1988), who regarded outliers as being generated by dynamic intervention models and suggested an iterative procedure of identifying and adjusting the effect of outliers in the estimation of the parameters. In our approach, to detect outliers we use the interpolation diagnostic suggested by Ryu et al.(1992) instead of the intervention approach. Once an observation is identified as an outlier, it is replaced by an interpolator and the parameters are reestimated in the estimation stage. This is iterated until no further outliers are detected.

The interpolation diagnostic for AR(p) is based on the innovational variance estimate

$$DI_k(T) = \sum_{t=p+1}^{n-p} (y_t^* - \hat{y}_t^*)^2 \ ,$$

where

$$y_t^* = \begin{cases} \tilde{y}_T(t), & t = T, T+1, \cdots, T+k-1 \ , \\ y_t \ , & \text{otherwise} \ , \end{cases}$$

T the time point of possible outlier occurrence, $\tilde{y}_T(t)$ the interpolator obtained following Pourahmadi(1989), and $\hat{y}_t^*$ the one-step-ahead predictor based on the interpolated series. For example, in the AR(1) case, the single observation can be interpolated by

$$\tilde{y}_T(t) = \phi y_{T-1} + \phi(y_{T+1} - \phi^2 y_{T-1})/(1+\phi^2) \ .$$

The interpolation diagnostic adopts the deletion approach by Peña (1990), Bruce and Martin(1989), Lee(1990), and Ledolter(1990).

Our procedure is closely related to the EM algorithm approach by Chuang and Abraham(1987) in that the outliers are considered as missing and estimated following EM algorithm. But their procedure is based on the estimates of the ARMA coefficients instead of the innovational variance estimate. It has been observed that the deletion procedure based on the innovational variance is more sensitive to outliers than are the procedure based on the autoregressive and moving average coefficient estimates, see Bruce and Martin(1989) and Ledolter(1989). Therefore, we do not consider their procedure in this paper.

3. A Simulation Study

Estimation methods mentioned in Section 2 are compared through a simulation study. For the simulation we generate data from AR(1), with sample size n=100. All the procedures are repeated 500 times. In our study we consider a single AO and consecutive AO's. T=50 is chosen as the time position for a single AO, T=50 and 51 for 2 consecutive AO's, and T=50, 51, and 52 for 3 consecutive AO's. The magnitudes of outliers considered are 5σ and 9σ but we report the 5σ case only, since the 9σ case is almost the same as the 5σ case.

The following approaches are considered for the simulation.

(1) Conditional Least Squares(CLS) estimate

(2) M-estimates(M) in Section 2.1 using Huber's ψ with tuning constant 1.345 and Tukey's bisquare ψ with tuning constant 4.685

(3) For GM-estimates(GM) in Section 2.2, Huber's ψ with tuning constants 1.5 and 1.0 and Tukey's ψ with 6.0 and 3.9 are used following Denby and Martin(1979)

(4) For modified GM-estimates(GMM) in Section 2.3, the same ψ and tuning constants as in GM are used.

(5) For RA-estimates(RA) in Section 2.4 the Mallow type with tuning constants 1.65 and 4.58 are used. We do not consider the Hampel type following the result of Bustos and Yohai(1986).

(6) The Iterated Interpolation Approach(INT) with 75% cutoff values.

All the estimates except CLS are computed by IWLS. The same convergence criterion used by Chuang and Abraham(1989) was also used as a stopping rule. The scale estimate σ was estimated by the median of the absolute deviations of the observations from their sample median divided by 0.6745. Mean(MEAN), mean squared errors(MSE), efficiency(EFF), and relative bias(RELB) are calculated. The simulation results for a single, 2, and 3 consecutive AO's case are reported in Table 1-3, respectively.

4. Summary and Conclusions

A simulation study was conducted to compare the performance of the estimaion methods, especially for time series containing consecutive AO's. Though the present study covers only the case of consecutive AO's having same magnitudes w_i of disturbance, some understanding about the behavior of the estimates are obtained from the results of simulation study.

From Tables 1-3 the followings can be observed:

(1) CLS and M have relatively severe asymptotic bias difficulties in almost all the cases involving a single as well as consecutive AO outliers. In many cases, especially for $\phi<0$, the RELB's of CLS and M increase as the number of consecutive AO's increases. For example, when $\phi_1=-0.3$ CLS yields RELB of 0.1969, 0.8453, and 1.3126 in 1, 2, and 3 consecutive AO's sereis respectively, while M yields 0.1507, 0.1569, and 0.1720. In each case, the largest RELB's among the estimates besides CLS and M are 0.0457, 0.0367, and 0.0222, respectively.

(2) The MSE's and RELB's of M are noticeably smaller than those of CLS. Further refinements are observed in GM and GMM in almost all cases. For example, when $\phi_1=-0.3$ GM(GMM) yields RELB of 0.0045(0.0057), 0.0042(0.0367), 0.0059(0.0078) while M yields 0.1507, 0.1569, and 0.1720. The MSE's of GM(GMM) are 0.0115 (0.0117), 0.0116(0.0130), and 0.0119(0.0115) while M yields 0.0119, 0.0117, and 0.0139, respectively. In comparison between GM and GMM, GMM performs slightly better than GM in EFF as well as RELB for a single AO case. For consecutive AO's cases, the two methods perform well, especially for $\phi>0$ and lead to nearly same values in terms of EFF as well as RELB.

(3) Iterated interpolation approach(INT) perform well, in general, in this simulation study. For a single AO case, INT performs better than any other estimate methods considered in this study. For consecutive AO's cases, no estimation methods perform noticeably better than the others. In terms of EFF, INT shows good performance, especially when $|\phi|$ is relatively close to 1. In general, the EFF of INT is relatively close to the EFF of the best case. For example, when $\phi=-0.3$ and 3 consecutive AO's are present, the EFF of INT

is 13.797 which is fourth among the estimates while the largest EFF is 14.839(RA).

(4) The performance of RA is reasonably good in this simulation study. However, RA has some degrees of difficulties in EFF when $\phi(>0)$ becomes close to 1. For example, the EFF's of RA for $\phi=0.9$ are 1.991, 1.472, and 1.233 for 1,2, and 3 consecutive AO's series, respectively. Meanwhile, the EFF's of the best case are 3.304, 2.219, and 1.979. For the case of $\phi=0.6$, similar behavior can be observed.

(5) Since the consecutive AO's series can be approximated by an IO (Ryu et al., 1992), CLS performs well for small positive ϕ, e.g. 0.3 and 0.6, as was indicated by Martin(1980), see Table 2 and 3. This is the same phenomena reported in Chang et al.(1988).

The above simulation study leads us to conclude that INT performs reasonably well compared with other methods in almost all cases while GM and GMM do well especially for $\phi>0$. Though GMM performs well in some cases, further study might be needed to improve GMM by considering better way of dealing with y_t in the M-estimate equation $\sum_{t=0}^{n-1} y_t\psi(y_{t+1}-\phi y_t)=0$, which can use the structure of time series. In our simulation study we do not compare Chang et al.'s procedure since their procedure is not intended to identify consecutive AO's and do not work well in the consecutive AO's case, Ryu et al.(1992).

Table 1. Means, Mean Squared Errors, Efficiencies and Relative Biases of the AR(1) Parameter (Single AO, w_T=5.0 at T=50)

φ	METHOD	MEAN	MSE	EFF	RELB
-0.9	CLS	-.8280	.0125	1.000	.0800
	M	-.8717	.0057	2.218	.0315
	GM	-.8733	.0053	2.379	.0296
	GMM	-.8740	.0053	2.371	.0289
	RA	-.8746	.0050	2.506	.0282
	INT	-.8820	.0041	3.083	.0200
-0.6	CLS	-.5037	.0172	1.000	.1606
	M	-.5511	.0117	1.479	.0816
	GM	-.5887	.0075	2.293	.0189
	GMM	-.5897	.0077	2.239	.0179
	RA	-.5856	.0074	2.333	.0240
	INT	-.6025	.0070	2.478	.0042
-0.3	CLS	-.2409	.0130	1.000	.1969
	M	-.2548	.0119	1.089	.1507
	GM	-.3014	.0115	1.123	.0045
	GMM	-.3017	.0117	1.111	.0057
	RA	-.2960	.0101	1.281	.0133
	INT	-.3137	.0116	1.116	.0457
0.3	CLS	.2288	.0160	1.000	.2372
	M	.2318	.0152	1.049	.2272
	GM	.2799	.0143	1.113	.0669
	GMM	.2800	.0144	1.106	.0665
	RA	.2713	.0146	1.092	.0956
	INT	.2897	.0159	1.006	.0344
0.6	CLS	.4890	.0231	1.000	.1851
	M	.5257	.0185	1.249	.1238
	GM	.5661	.0131	1.756	.0565
	GMM	.5663	.0132	1.743	.0562
	RA	.5608	.0137	1.679	.0653
	INT	.5828	.0126	1.833	.0286
0.9	CLS	.8297	.0094	1.000	.0782
	M	.8776	.0061	1.539	.0248
	GM	.8733	.0038	2.475	.0296
	GMM	.8743	.0038	2.471	.0286
	RA	.8796	.0047	1.991	.0227
	INT	.8817	.0029	3.304	.0204

Table 2. Means, Mean Squared Errors, Efficiencies and Relative Biases of the AR(1) Parameter (2Consecutive AO's w_T=5.0 at T=50,51)

φ	METHOD	MEAN	MSE	EFF	RELB
-0.9	CLS	-.7352	.0411	1.000	.1831
	M	-.8688	.0070	5.870	.0347
	GM	-.8738	.0050	8.290	.0291
	GMM	-.8862	.0040	10.339	.0153
	RA	-.8795	.0044	9.358	.0228
	INT	-.8759	.0048	8.606	.0268
-0.6	CLS	-.3191	.0894	1.000	.4681
	M	-.5514	.0115	7.790	.0810
	GM	-.5889	.0075	11.960	.0185
	GMM	-.6050	.0075	11.903	.0083
	RA	-.5962	.0079	11.352	.0064
	INT	-.5878	.0078	11.401	.0204
-0.3	CLS	-.0464	.0732	1.000	.8453
	M	-.2529	.0117	6.250	.1569
	GM	-.3013	.0116	6.292	.0042
	GMM	-.3110	.0130	5.647	.0367
	RA	-.3046	.0121	6.042	.0154
	INT	-.2992	.0114	6.444	.0027
0.3	CLS	.3524	.0101	1.000	.1748
	M	.2703	.0154	.654	.0989
	GM	.2810	.0146	.688	.0633
	GMM	.2808	.0147	.686	.0639
	RA	.2672	.0142	.709	.1093
	INT	.2755	.0154	.652	.0817
0.6	CLS	.5557	.0087	1.000	.0738
	M	.5686	.0155	.564	.0523
	GM	.5692	.0134	.652	.0513
	GMM	.5691	.0134	.651	.0515
	RA	.5540	.0138	.635	.0767
	INT	.5638	.0139	.627	.0604
0.9	CLS	.8396	.0072	1.000	.0671
	M	.8856	.0047	1.533	.0160
	GM	.8766	.0036	1.990	.0260
	GMM	.8760	.0037	1.976	.0266
	RA	.8780	.0049	1.472	.0244
	INT	.8766	.0033	2.219	.0260

Table 3 Means, Mean Squared Errors, Efficiencies and Relative Biases of the AR(1) Parameter (3 Consecutive AO's w_T=5.0 at T=50,51,52)

φ	METHOD	MEAN	MSE	EFF	RELB
-0.9	CLS	-.6518	.0826	1.000	.2758
	M	-.8708	.0060	13.874	.0325
	GM	-.8731	.0052	15.925	.0299
	GMM	-.8726	.0054	15.222	.0305
	RA	-.8752	.0052	15.826	.0276
	INT	-.8785	.0045	16.704	.0239
-0.6	CLS	-.1746	.1944	1.000	.7090
	M	-.5489	.0115	16.972	.0852
	GM	-.5887	.0078	25.070	.0189
	GMM	-.5805	.0080	24.434	.0326
	RA	-.5889	.0077	25.337	.0184
	INT	-.5832	.0082	23.841	.0280
-0.3	CLS	.0938	.1653	1.000	1.3126
	M	-.2484	.0139	11.856	.1720
	GM	-.3018	.0119	13.939	.0059
	GMM	-.3023	.0115	14.315	.0078
	RA	-.3001	.0111	14.839	.0002
	INT	-.2933	.0120	13.797	.0222
0.3	CLS	.4375	.0258	1.000	.4583
	M	.3121	.0242	1.069	.0402
	GM	.2827	.0145	1.783	.0576
	GMM	.2825	.0146	1.772	.0583
	RA	.2845	.0145	1.782	.0517
	INT	.2909	.0184	1.406	.0303
0.6	CLS	.6024	.0060	1.000	.0040
	M	.6100	.0180	.335	.0166
	GM	.5734	.0131	.458	.0443
	GMM	.5735	.0132	.456	.0442
	RA	.5647	.0134	.450	.0588
	INT	.5767	.0130	.462	.0388
0.9	CLS	.8466	.0060	1.000	.0594
	M	.8915	.0038	1.564	.0094
	GM	.8807	.0037	1.633	.0215
	GMM	.8845	.0049	1.226	.0172
	RA	.8791	.0049	1.223	.0233
	INT	.8805	.0030	1.979	.0217

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