

# Throughput Performance of Common Spreading Code and Transmitter-Oriented CDMA Packet Radio Networks

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## 單一擴散코드 및 送信機別 코드분할 多重접속(CDMA) 패킷 라디오 네트워크들의 Throughput 性能

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### ABSTRACT

The performance of common spreading code scheme employing multiple-capture receiver is compared to that of transmitter-oriented (T/O) code division multiple access (CDMA) scheme in view of the possibility of collision-free transmissions and the effect of secondary multiple-access interference. For performance comparisons, secondary multiple-access interference is characterized for the common code scheme and the T/O CDMA scheme that assures perfectly collision-free transmissions. Throughput performance is then evaluated for these two schemes with direct-sequence spread-spectrum/differential-phase-shift-keying (DS-SS/DPSK) data modulation and forward-error-control coding (BCH codes) in the presence of an additive white Gaussian noise (AWGN). It is shown that when the number of radios is relatively large, the maximum normalized throughput is greater for the common code scheme than for the T/O CDMA scheme at a moderate signal-to-noise ration (SNR).

### 要 約

Collision-free 傳送 가능성과 2차 多重접속 干涉의 영향면에서 多重캡처 受信機를 사용한 單一擴散코드 방식의 性能이 送信機別 (T/O) 코드분할 多重접속 (CDMA) 방식의 性能과 比較된다. 性能比較를 위해, 單一코드 방식과 완전한 collision-free 傳送을 보장하는 T/O CDMA 방식에 대해 多重접속 干涉의 특징이 敍述된다. 白色 가우시안 雜音하에서 直接시퀀스 差動 位相편이키잉 데이터 變調와 forward 에러訂正 符號化를 (BCH 코드) 사용한 위의 두 방식에 대해 throughput 性能이 評價된다. 라디오의 數가 비교적 많은 경우에, 적당한 信號對 雜音比(SNR)에서 單一코드 방식이 T/O CDMA 방식보다 最大 throughput을 增加시킬 수 있음을 確認하였다.

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論文番號: 92-94(接受1992. 1. 24)

## I. INTRODUCTION

Up to date, there have been a lot of works on the single capture performance [DaGr80, PoSi87, SoGe91] for the spread-spectrum networks which employ receiver-oriented transmission policies, that is, all transmissions to an intended receiver are using the same SS code to compete for its attention. However, it is possible, for two or more transmissions to succeed on the same code even though they arrive at the receiver overlapped in time, that refers to the multiple-capture capability [Kim90]. For this reason, we here address a multiple-capture issue for a common code network with somewhat limited star topology, in which all radios use the same SS code for spectral spreading. Besides, we consider a conventional CDMA network for performance comparisons with the common code network.

In a centralized SS packet radio network, to achieve multiple simultaneous successful transmissions, we may consider three different approaches, namely, the common spreading code, random code assignment, and transmitter-oriented CDMA schemes. First, in the common code scheme, all radios use a common spreading code for their packet transmissions. Hence all transmitted packets are strongly correlated if they arrive at the receiver with time offset less than a few chip times, depending on the real implementation of the receiver. In that case, these packets will be destroyed because of uncorrectable number of errors caused by primary multiple-access interference [SoGe91]. But if the time offset between any two transmissions is sufficient to enable the receiver to lock on to them, there will be collision-free transmissions to be successfully captured with high probability.

For the random assignment scheme, each radio randomly chooses one of distinct spreading codes when he has a packet to send, in which the number of distinct codes is considerably smaller than the number of radios in the network, that was first proposed and compared with the common

code scheme employing the SS multiple-capture receiver [KiSc91]. On the other hand, in the T/O CDMA scheme, each radio is assigned a unique spreading code from a set of distinct spreading codes with low cross-correlation. Hence the packets transmitted by different radios will be on different code channels and not collide with each other. In this case, if AWGN is assumed negligible, packet errors result only from secondary multiple-access interference.

For overall comparisons of the three transmission schemes mentioned above, we summarize their characteristics, advantages and disadvantages to be expected in Table 1.

In this paper, we will investigate throughput performance for the common spreading code and T/O CDMA schemes with DS-SS/DPSK data modulation and forward error-control coding (BCH codes). Here the common code scheme employs a SS multiple-capture receiver to allow two or more collision-free transmissions to be successfully captured even though concurrent transmissions using the same code are overlapped in time. For evaluation of throughput, secondary multiple-access interference is characterized for these two schemes which gradually affects throughput as the number of radios increases [WeHuBa81].

## II. SYSTEM AND NETWORK MODELS

A centralized SS packet radio network is considered in which a finite number  $K$  of mobile radios communicate with a single central node in a slotted random-access mode so that different radios can synchronize their transmissions at the packet level. In the common code network, all radios adopt a common spreading code for transmission of a particular packet, and hence the central node needs only one receiving code for packet demodulation. However, in the transmitter-oriented CDMA network, each radio has its own spreading code for transmission, so the central node should have a list of all transmitting codes to be able to listen to several of them.

Table 1. Overall comparisons of the three transmission schemes available in the centralized SS packet radio networks.

Contents	Common spreading code scheme	Random spreading code assignment scheme	Transmitter-oriented CDMA scheme
Number of codes to be used	1	More than one, but much less than K	K
System complexity	Small	Medium	Large
Possibility of collision-free transmission	Collision occurs with relatively high probability	Collision occurs with relatively small probability	No collision and perfect collision-free transmission
Effect of multiple-access noise	Relatively small due to the best common spreading code selection	Relatively medium due to the possibility of good code selection	Relatively large due to the limited possibility of good code selection
Advantages	Good performance under low traffic conditions with small system complexity	Overall good performance with medium system complexity	Good performance under medium traffic conditions
Disadvantages	High packet loss due to collisions under heavy traffic conditions	Transmitter's burden due to the random code assignment strategy	Poor performance under heavy traffic conditions and large system complexity

We consider two different types of spreading code sequences, that is, one is an auto optimal phase m-sequence with least sidelobe energy (AO/LSE) to be used for the common code scheme, while the other is a random sequence of infinite length with chip-by-chip independence used for the T/O CDMA scheme. It was reported [PuRo79] that the AO/LSE m-sequence is optimal with respect to the peak correlation parameters and the mean square correlation parameters, and the m-sequence is expected to be a good one as the common spreading sequence. As the number of radios in the T/O CDMA network increases, it is difficult to accurately model spreading sequences used by different radios for performance evaluations. Fortunately, the random sequences lead to a close approximation for those CDMA systems which require sets of distinct spreading sequences with low cross-correlation.

A multiple-capture issue was restricted to a centralized network with star topology, because it appeared intractable to exactly model the cap-

ture phenomenon without avoiding the near-far problem. With such a limited topology, it is assumed that the received signal power can be normalized for all radios by using the power control. The capture mechanism consists of two phases: in the first phase, the receiver searches for the presence of all transmitted packets by processing the header attached to the data packet. After the header detection process is successfully completed, in the second phase, we initiate the data demodulation process. For simplicity, in the common code scheme, we assume that the first phase is successfully completed if a packet does not collide with adjacent packets. By contrast, in the T/O CDMA scheme, we are absent from the collision problem and assume the first phase to be always completed.

We adopt a simplified network model in which all radios are identical and transmission processes for different radios are assumed to be statistically independent. In addition, we assume the heavy traffic condition so that there is always a packet for transmission at the beginning of every slot.

The effect of acknowledgements is ignored, assuming a perfect and instantaneous acknowledgement channel is available.

### III. THROUGHPUT PERFORMANCE

#### 1. Common Code Scheme

In order for multiple simultaneous transmissions to succeed on a common code, a DS-SS /DPSK multiple-capture receiver is designed at the central node in which the equivalent complex base-band model is depicted in Fig. 1. This central receiver makes use of a high rate sampling technique with the code-matched filter rather than code acquisition and tracking as in conventional detection. Here decision statistics is sampled at a high rate in which the sampling rate is equal to the chip rate or even a higher rate, and compared it with a zero threshold for differential decoding. The decoded output is gated into a data file that consists of a number of data sequences, each containing either noise or useful signals. In the header detection process, we are able to eliminate data sequences from only noise signals and also achieve synchronization at the packet-level for useful data sequences. Thereafter, each data sequence is simply decoded on a packet (or code-

word) basis by using a forward-error-correction scheme.

In the common code scheme, we encode each bit of all transmitted packets by multiplying one period of a common AO /LSE m-sequence so that concurrent transmissions interfere with each other at the bit-time level. Then collision occurs between desired and interfering packets when their relative bit-time offsets are within the order of a chip time( $T_c$ ). Given the  $m$  simultaneous transmissions in a slot, the probability of packet success  $P_s(m)$  can be expressed by

$$P_s(m) = P_{cf}(m)P_s(m|\xi) + [1 - P_{cf}(m)]P_s(m|\bar{\xi}) \quad (1)$$

where  $\xi$  denotes the event of a desired packet being collision free and  $P_{cf}(m) \triangleq \Pr\{\xi\}$ . It is assumed that a packet is collision-free if the relative bit-time offsets of the packet from its adjacent packets are larger than the minimum time offset  $\Delta = T_c(1 + \frac{1}{2\lambda})$  depending on the sampling rate  $\lambda$ , the number of samples per  $T_c$ . For a given  $\lambda$ , we then always have a collision-free sampling time that gives rise to the largest correlation value of a collision-free signal. The average contribution to the packet success by collided packets is expected very small and ignored, instead we de-

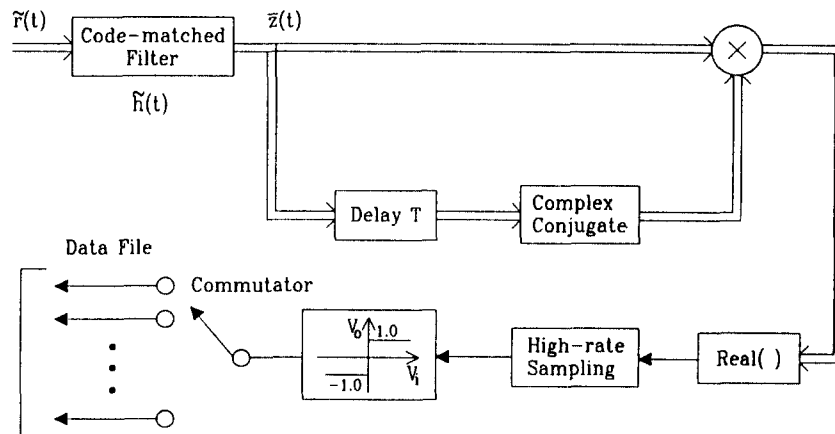


Fig. 1. DS-SS /DPSK multiple-capture receiver.

rive a conservative packet success probability as

$$P_s(m) \cong P_f(m) P_s(m|\xi). \quad (2)$$

We denote  $\tau_k$  as the time of arrival of the  $k$ -th radio's at the central node, assuming that without loss of generality,  $\{\tau_k\}$  are independent and uniformly distributed over  $[0, T_b]$  (a bit time). For evaluation of system performance at the link-level, we should be able to derive the conditional distribution of the number of collision-free packets, given  $m$  simultaneous transmissions. In a continuous manner, it is not tractable to exactly analyze the packet collisions so that we approximate  $\{\tau_k\}$  to be uniformly distributed among the set of  $L$  discrete times, equally spaced over  $[0, T_b]$  with spacing  $\frac{\Delta}{2}$ .

By the combinatorial analysis in [Kim91], it can be shown that

$$\begin{aligned} P_{F|M}(f|m) &\triangleq \Pr\{F=f \text{ collision-free packets} \\ &\quad M=m \text{ simultaneous transmissions}\} \\ &= \frac{1}{L^m} \sum_{k=0}^{f+k} (-1)^k \binom{f+k}{f} \binom{m}{f+k} \sum_{r=1}^{f+k} \\ &\quad (f+k-1)! L \binom{f+k}{r} \binom{L-2(f+k)-1}{r-1} \\ &\quad [L-2(f+k)-r]^{m-f-k} \end{aligned} \quad (3)$$

where  $L=2N/(1+1/2\lambda)$  for a given  $\lambda$ . In order to validate theoretical evaluation of  $P_{F|M}(f|m)$ , we can refer to the comparisons between the theoretical and simulation results in [Kim 91] when a noncoherent scheme is employed for header detection.

Because of the symmetry, the probability of a desired packet being collision-free, given the  $m-1$  interfering packets in the common channel, has the expression

$$P_f(m) = \frac{1}{m} \mathbf{E}\{F|M=m\}$$

$$= \frac{1}{m} \sum_{f=1}^m f P_{F|M}(f|m). \quad (4)$$

Let  $\Delta\tau_i$  denote the distance of the bit epoch time  $\tau_k + lT_b$  (an integer  $l$ ) from the nearest sampling instant that yields the largest correlation value of the  $i$ -th desired signal. If we define  $\epsilon_i = \frac{\Delta\tau_i}{T_b}$ , we find that  $\epsilon_i$  is uniformly distributed over  $[0, \frac{1}{2\lambda}]$ . For a differential system with hard decisions, if the multiple-access interference can be assumed as Gaussian noise, we obtain the probability of data bit error, conditioned on  $(\epsilon_i, m)$ , in Appendix :

$$\begin{aligned} P_b(\epsilon_i, m|\xi) &\cong \frac{1}{2} \left( 1 - \frac{\bar{\sigma}_r^2(m)}{\sigma_r^2(m)} \right) \\ &\quad \exp \left[ \frac{[1 - \epsilon_i [1 - \bar{\rho}_c(1)]]^2}{-2\sigma_r^2(m)} \right] \end{aligned} \quad (5)$$

where the second-order moments can be evaluated as

$$\begin{aligned} \sigma_r^2(m) &= \left( \frac{2E_b}{N_o} \right)^{-1} + \frac{(m-1)}{3(N-2)} \sum_{l=1}^{N-2} \\ &\quad [\bar{\rho}_c^2(l) + \bar{\rho}_c^2(l+1) + \bar{\rho}_c(l)\bar{\rho}_c(l+1)] \quad (6) \\ \bar{\sigma}_r^2(m) &= \frac{2(m-1)}{3(N-2)} \sum_{l=1}^{N-2} [\bar{\rho}_c(l)\bar{\rho}_c(N-l) + \frac{1}{4} \\ &\quad [\bar{\rho}_c(l)\bar{\rho}_c(N-l-1) + \bar{\rho}_c(l+1)\bar{\rho}_c(N-l)]] \end{aligned} \quad (7)$$

in which  $\frac{E_b}{N_o}$  is the bit signal-to-thermal-noise ratio, and the discrete partial autocorrelation function  $\bar{\rho}_c(l)$  is defined by

$$\bar{\rho}_c(l) = \begin{cases} \frac{1}{N} \sum_{j=0}^{N-1-l} c_j c_{j+l}, & 0 \leq l \leq N-1 \\ 0, & l \geq N \end{cases}$$

in which  $\{c_j\}$  is the AO/LSE  $m$ -sequence.

As the forward-error-control coding, we employ a BCH coding in connection with interleaving, in which the coding rate  $\gamma$  is bounded by

$$\gamma \leq 1 - \frac{t}{L_p} \quad (8)$$

where  $t$  is the number of correctable errors and  $L_p$  is the packet (or codeword) length, assumed  $L_p = 1024$ . If we assume independent bit errors at the decoder input, the conditional probability of packet success can be estimated as

$$P_S(\mathbf{m}|\xi) \cong \mathbf{E}_{\epsilon_i} \left\{ \sum_{e=0}^t \binom{L_p}{e} P_b^e(\epsilon_i, \mathbf{m}|\xi) [1 - P_b(\epsilon_i, \mathbf{m}|\xi)]^{L_p - e} \right\} \quad (9)$$

Since channel traffic can be modeled as a binomial random variable with parameters  $K$  and  $\delta$  (transmission probability), the average number of packet successes per slot becomes

$$\beta_c = \sum_{m=1}^K m P_S(m) f_M(m) \quad \text{packets / slot} \quad (10)$$

$$\cong \sum_{m=1}^K m P_{cf}(m) P_S(m|\xi) f_M(m) \quad (11)$$

where  $f_M(m) = \binom{K}{m} \delta^m (1-\delta)^{K-m}$  for  $m \leq K$ . For the overall system performance incorporating the effect of coding rate  $\gamma$ , the throughput  $\beta_c$  is normalized to be

$$\bar{\beta}_c = \gamma \cdot \beta_c \quad (12)$$

On the other hand, to gain insights into the impact of bursty traffic and the random access protocol on throughput, we assume the general arrival model for channel traffic as follows: the composite traffic consists of newly transmitted packets and retransmitted packets after transmitted packets were received in error, in which the new packets are transmitted in a slot with probability  $\delta_0$ , while the retransmissions occur in a slot with probability  $\delta_r$ .

First, given  $f$  of  $m$  simultaneously transmitted packets are collision free, we specify the conditional distribution of packet successes which is defined by

$$P_{S|f,M}(s|f, \mathbf{m}) \cong \Pr\{S=s \text{ packet successes} |$$

$F=f$  collision-free packets,

$M=m$  simultaneous transmissions.

Assuming the multiple-access interference is independent from packet to packet, we obtain the approximation to the conditional distribution of packet successes

$$P_{S|M}(s|\mathbf{m}) \cong \sum_{f=0}^m P_{S|f,M}(s|f, \mathbf{m}) P_{F|M}(f|\mathbf{m}) \quad (13)$$

$$= \sum_{f=0}^m \binom{f}{s} P_S^s(\mathbf{m}|\xi) [1 - P_S(\mathbf{m}|\xi)]^{f-s} P_{F|M}(f|\mathbf{m}), \quad (14)$$

Next, the analysis for the general arrival model is based on the approach used in [Rayc81, PoSi87]. For derivation of the steady-state composite arrival distribution  $f_M(m)$ , we need to obtain the stationary probability  $\pi_R(n)$  from the solution of

$$\pi_R = \pi_R P$$

where a random variable  $R$  denotes the number of packets for retransmission in a slot and  $P$  is the state transition matrix of the Markov chain according to  $R=n$ , with one-step transition probabilities

$$p_{nl} \cong \Pr\{R(j+1)=l | R(j)=n\}$$

where  $j$  indicates the sequence number of slot.

Let  $m_0$  and  $m_r$  denote the number of newly transmitted packets and retransmitted packets in the  $j$ -th slot, respectively, given the  $n$  packets for retransmission in the slot. Since all transmitted packets are equally likely to be collision-free, by the symmetry condition and referring to [PoSi87], the one-step transition probability is given by

$$p_{nl} = \sum_{m_0, m_r}^{K-n, \max(0, l-n)} B(m_0, K-n, \delta_0)$$

$$\sum_{m_l=\max(0, n-l)}^n B(m_l, n, \delta_r) P_{S|M}(n+m_0-l | m_0+m_l) \quad (15)$$

where  $B(q, k, \delta) \triangleq \binom{k}{q} \delta^q (1-\delta)^{k-q}$  for  $q \leq k$ . If the set of elements  $\{p_{ni}\}$  are used for the state transition matrix  $P$ , the stationary probabilities  $\{\pi_R(n)\}_{n=0}^K$  result from the solution of  $\pi_R = \pi_R P$ .

Finally, the steady-state throughput  $\beta_c$  in case of  $\delta_0 \neq \delta_r$  can be calculated directly by

$$\beta_c = \sum_{m=1}^K \sum_{s=1}^m s P_{S|M}(s | m) \sum_{n=0}^K f_{M|R}(m | n) \pi_R(n) \quad (16)$$

packets / slot

where the conditional arrival distribution  $f_{M|R}(m | n)$  is found in [PoSi87]. The normalized throughput  $\bar{\beta}_c$  is then computed from (12).

## 2. Transmitter-Oriented CDMA Scheme

In the transmitter-oriented CDMA scheme, we can employ a multi-receiver to allow multiple simultaneous transmissions to succeed on different codes, that refers to the multi-code capability. In this scheme, it is desirable to adopt different spreading sequences for each bit of a given packet's by segmenting a single long sequence, giving a low cross-correlation value among segmented sequences. Then it is assumed that we can acquire perfect synchronization at the packet and symbol-levels by using a programmable matched-filter [CaBrSt76] for each code channel.

In the literature, it is known that if we model sets of different spreading sequences as random binary sequences and invoke the Gaussian assumption on the multiple-access interference, the probability of data bit error for a differential system with hard decisions, conditioned on  $m$ , is given by the expression

$$P_b(m) = \frac{1}{2} \exp \left[ \frac{1}{-2\sigma_r^2(m)} \right] \quad (17)$$

where the second-order moment  $\sigma_r^2(m)$  becomes

$$\sigma_r^2(m) = \left( \frac{2E_b}{N_0} \right)^{-1} + \frac{m-1}{3N} \quad (18)$$

Similarly, when the BCH codes are employed for the forward-error-control coding, the probability of packet success can be computed using the conservative formula

$$P_s(m) \cong \sum_{e=0}^l \binom{L_p}{e} P_b^e(m) [1 - P_b(m)]^{L_p - e} \quad (19)$$

Under the channel traffic with binomial distribution, the average unnumber of packet successes per slot  $\beta_t$  is simply found in (10), and then the normalized throughput  $\bar{\beta}_t$  in (12). In the general arrival model of  $\delta_0 \neq \delta_r$ , if we assume independent packet-error events for different code channels, the conditional distribution of packet successes is shown to be

$$P_{S|M}(s | m) = \binom{m}{s} P_s^s(m) [1 - P_s(m)]^{m-s} \quad (20)$$

Here we substitute (20) into (15) and (16) to derive the steady-state throughput  $\beta_t$  and also the normalized throughput  $\bar{\beta}_t$ .

## IV. RESULTS

At the link-level, numerical results on throughput are provided for both coded and uncoded DS-SS / DPSK systems, in which the AO / LSE  $m$ -sequence of period  $N=63$  is chosen as the common spreading code, that is generated by the polynomial  $g(x) = x^6 + x^4 + x^3 + x + 1$  with initial sequence (1,1,0,0,0,1), and the coding rate  $\gamma$  is set to one of  $\gamma = 1.0, 0.89, 0.78, 0.67, 0.57$ .

Figs. 2-4 show the maximum normalized throughputs that can be achieved by the use of forward-error-control coding as a function of the number of radios  $K$  for various sampling rates  $\lambda$  and signal-to-noise ratios  $\frac{E_b}{N_0}$ . At the SNR of 8 dB, the common code scheme with rates  $\lambda=5, 10$  outperforms the transmitter-oriented CDMA scheme as  $K$  increases, since an increase of thermal noise and the multiple-access interference

caused by larger sidelobes in the T/O CDMA scheme degrades severely the data bit-error rate. We also find that for the case of SNR=10dB,

using the common code scheme with rate  $\lambda=10$  results in higher throughput compared to the T/O CDMA scheme. But as SNR increases

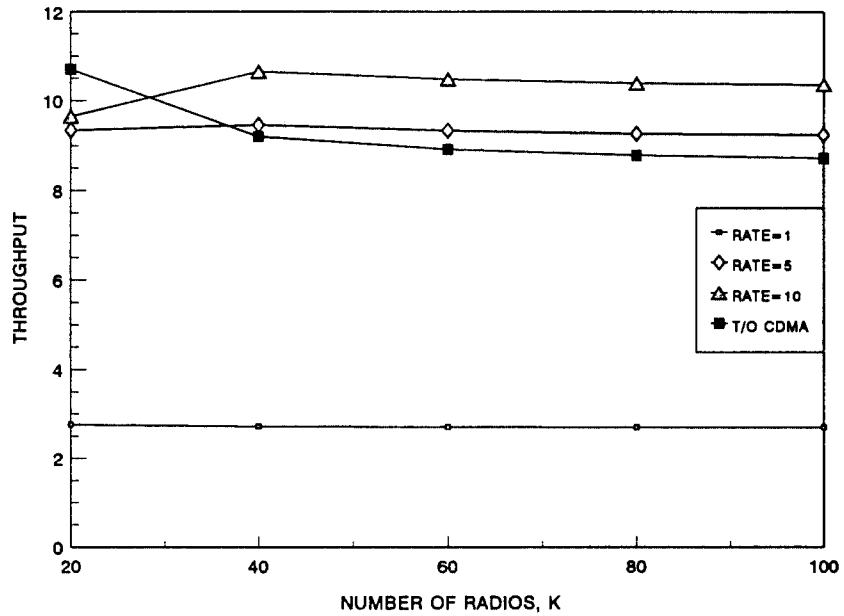


Fig. 2. Maximum normalized throughput versus K for coded systems when SNR = 8dB

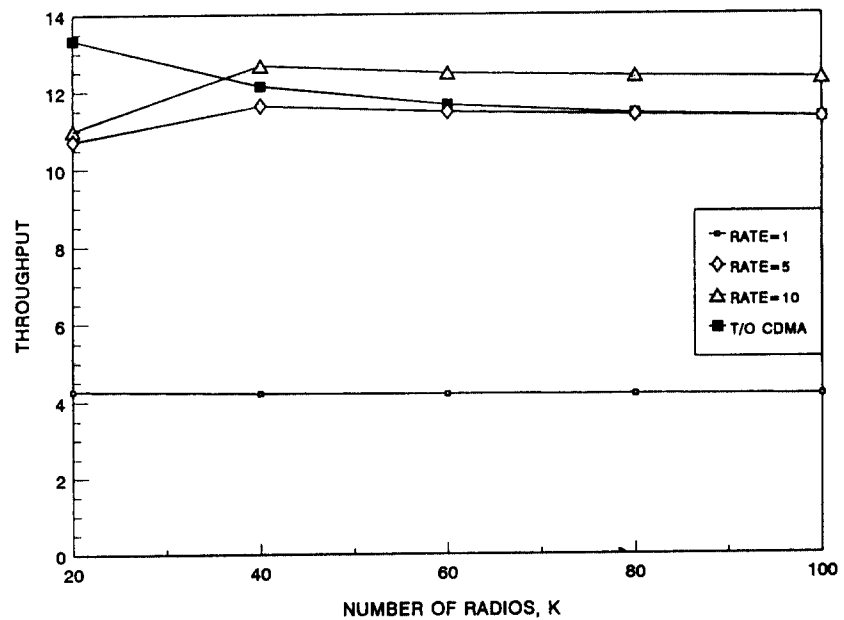


Fig. 3. Maximum normalized throughput versus K for coded systems when SNR = 10dB



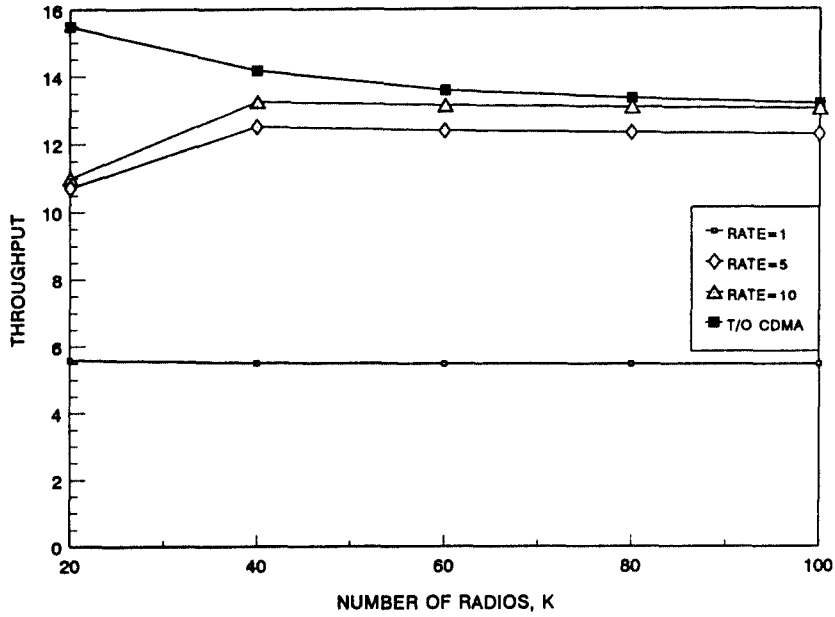


Fig. 4. Maximum normalized throughput versus K for coded systems when SNR = 12dB

further, the combined effect of thermal noise and multiple-access interference is relatively less sig-

nificant so that the T/O CDMA gives a slightly better performance at the SNR of 12dB.

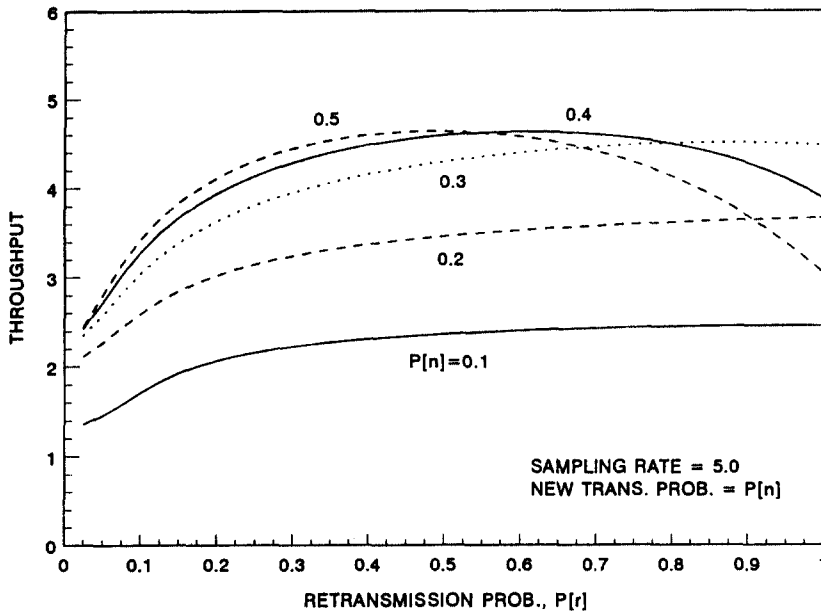


Fig. 5. Throughput versus P[r] for common code scheme when K=20, SNR=10dB

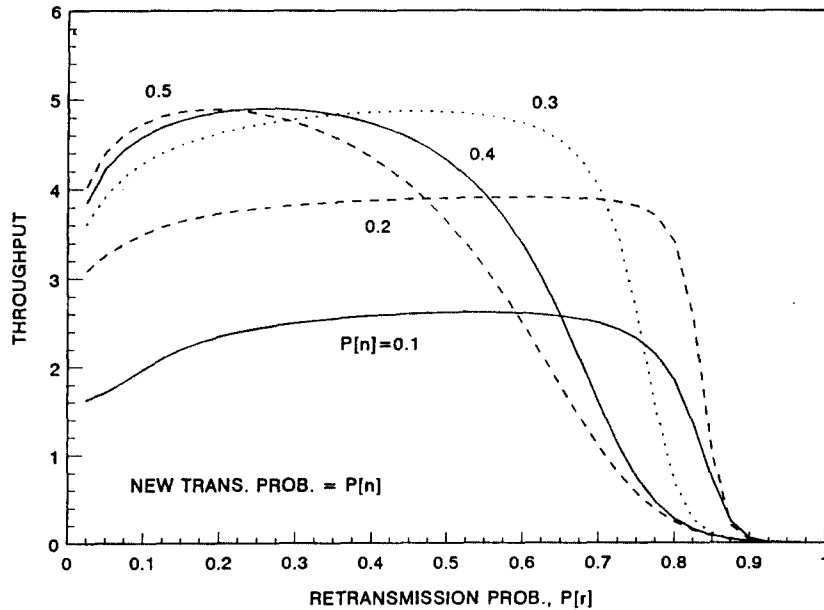


Fig. 6. Throughput versus  $P[r]$  for T/O CDMA scheme when  $K=20$ ,  $SNR=12dB$

Under the general arrival model, Fig. 5 shows throughput  $\beta_r$  as a function of the retransmission probability  $\delta_r$  for the common code scheme when  $K=20$ ,  $SNR=10dB$ ,  $\lambda=5$ ,  $\gamma=1.0$ . We achieve a nearly flattened maximum throughput over the middle range of  $\delta_r$ . This is because we expect a gradual increase of the multiple-access interference because of smaller sidelobes of the AO/LSE m-sequence. In Fig. 6, we plot  $\beta_r$  versus  $\delta_r$  for the T/O CDMA scheme when  $K=20$ ,  $SNR=12dB$ ,  $\gamma=1.0$ . For this specific uncoded case, the common code scheme is superior to the T/O CDMA scheme by 2dB in terms of the maximum throughput.

### V. CONCLUSION

We have seen that the common code scheme with high rate sampling performs better than the transmitter-oriented CDMA scheme in a moderate range of  $SNR=8-10dB$ . For the common code scheme, we have disadvantages such as the possi-

bility of packet collisions and the loss of detected signal power because of imperfect bit sync. However, as the number of radios in the network increases, the effect of secondary multiple-access interference in the T/O CDMA scheme overwhelms these unfavorable factors, because the cross-correlation properties of a set of distinct spreading sequences is getting worse in proportion to  $K$ , while the AO/LSE m-sequence as the common spreading code has the optimum autocorrelation properties and assures higher multiple-capture capability. In addition, the common code scheme under the general arrival model shows 2 dB performance gain with respect to the T/O CDMA scheme without forward-error-control coding.

### Appendix

The Data Bit-Error Probability  $P_b(\epsilon_r, m|\xi)$  :

For the common DS-SS/DPSK signaling, the complex envelope of the received signal can be

expressed by

$$\bar{r}(t) = \sum_{k=1}^m \sum_{l=1}^{l_k-1} \sqrt{2P} d_l^{(k)} c(t - \tau_k - lT_b) \exp(j\theta_k) + \bar{n}(t) \quad (21)$$

where  $P$  is the received signal power,  $d_l^{(k)}$  is the  $\kappa$ -th radio's differentially-encoded binary data sequence, each data taking values of  $\pm 1$  with equal probability and mutually independent,  $\theta_k$  is the  $\kappa$ -th radio's unknown signal phase, uniformly distributed over  $[0, 2\pi]$ .  $c(t)$  is the common spreading code waveform consisting of an AO/LSE  $m$ -sequence with period  $N = T_b/T_c$  in which the chip waveform is a rectangular pulse of duration  $T_c$ .  $\bar{n}(t)$  is a zero-mean complex AWGN with power spectral density  $2N_0$

The normalized output of the code-matched filter with impulse response  $\bar{h}(t) = \frac{1}{2} c(T_b - t)$  is given by

$$\begin{aligned} \bar{z}(t) &= \frac{1}{T_b} \sqrt{\frac{2}{P}} \int_{t-T_b}^t \bar{r}(u) \bar{h}(t-u) du \\ &= \sum_{k=1}^m \frac{1}{T_b} [d_{lk} f(\bar{\tau}_k) + d_{l,k+1} f(T_b - \bar{\tau}_k)] \exp(j\theta_k) + \hat{n}(t) \end{aligned} \quad (22)$$

where  $l_k = \lfloor \frac{t - \tau_k - T_b}{T_b} \rfloor$ ,  $\bar{\tau}_k = (\tau_k + T_b - t) \bmod T_b$ , the continuous-time partial autocorrelation function  $f(\tau)$  is defined by

$$f(\tau) = \int_0^\tau c(t) c(t - \tau + T_b) dt, \quad (23)$$

and the complex Gaussian noise  $\hat{n}(t)$  has the form

$$\hat{n}(t) = \frac{1}{T_b \sqrt{2P}} \int_{t-T_b}^t \bar{n}(u) c(T_b - t + u) du \quad (24)$$

in which  $\text{var}\{\text{Re}\{\hat{n}(t)\}\} = \text{var}\{\text{Im}\{\hat{n}(t)\}\} = \frac{N_0}{2E_b}$  and  $\text{cov}\{\text{Re}\{\hat{n}(t)\}, \text{Im}\{\hat{n}(t)\}\} = 0$ .

For differential decoding of a desired  $i$ -th collision-free packet, the receiver forms the decision statistics  $\text{Re}\{\bar{z}(t)\bar{z}^*(t - T_b)\}$  and compares it with a zero threshold at  $t = \tau_i + lT_b - \epsilon_i T_c$ . Here

we require that  $\theta_i$  is constant over the duration of two adjacent data bits for DPSK communication. If the interference components of  $\bar{z}(t)$  and  $\bar{z}(t - T_b)$  can be modeled as complex Gaussian random variables, the probability of data bit error  $P_b(\epsilon_i, \mathbf{m}|\xi)$  can be written approximately as [Steif66] [Steif66]

$$P_b(\epsilon_i, \mathbf{m}|\xi) \cong \frac{1}{2} \left( 1 - \frac{\tilde{\sigma}_r^2(m)}{\sigma_r^2(m)} \right) \exp \left[ \frac{\eta_r^i(\epsilon_i) + \eta_s^i(\epsilon_i)}{-2\sigma_r^2(m)} \right] \quad (25)$$

where  $(\eta_r(\epsilon_i), \eta_s(\epsilon_i))$  are the in-phase and quadrature components of the  $i$ -th desired signal in  $\bar{z}(t)$  with  $d_i$  ignored, which is given by

$$\eta_r^i(\epsilon_i) + \eta_s^i(\epsilon_i) = [1 - \epsilon_i [1 - \bar{\rho}_c(1)]]^2, \quad (26)$$

and the second-order moments are defined by

$$\sigma_r^2(m) = \text{var}\{\text{Re}\{\bar{z}(t)\}\} \mid \epsilon_i, d_{l_{i+j}} \quad (j = 0, 1)$$

$$\tilde{\sigma}_r^2(m) = \text{cov}\{\text{Re}\{\bar{z}(t)\}, \text{Re}\{\bar{z}(t - T_b)\}\} \mid \epsilon_i, d_{l_{i+j}} \quad (j = -1, 0, 1)$$

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