Quantity Discounts Using A Joint lot Size Model under Learning Effects -Multiple Buyers Case

- 통합로트량 결정모형을 이용한 가격할인 모형 - 복수구매자의 경우 -

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요 지

최근 제조공정상의 학습효과를 고려한 통합로트량 결정모형이 개발되었다. 본 논문은 기존의 단일 구매자에서 복수구매자의 모델로 확장된 가격할인 모형을 다룬다. 이 모형에서는 복수구매자의 발주간격은 가장 짧은 구매자의 발주간격에 정수배라는 가정하에서 모형이 개발되었다. 구매자의 계수변화로 인한 민감도 분석이 되었다. 소개된 모형의 효과를 보이기 위해 수치예제를 이용하였다.

INTRODUCTION

The manufacturer's production policy is influenced by the buyer's orders. In case of multiples buyers each buyer places orders based on their Economic Order Quantity (EOQ) and the manufacturer pursues an Economic Production Quantity (EPQ) for the buyers' requirements. Instead of determining lot size policies independently, if multiple buyers and manufacturer cooperate and determine a joint economic ordering policy, they would achieve considerable savings.

In this paper we develop a joint buyer/manufacturer inverntory model for multiple buyers. It examines the production costs under a range of learning curve ratios and levels of learning retention caused by break in production. A quantity discount schedule is proposed based on joint total variable costs of multiple buyers and manufacturer. A sensitivity analysis examines the effect on lot size and joint total cost of the existing single buyer when additional buyers are involved.

LITERATURE REVIEW

Lal & Staelin [6] studied the case of multiple buyers as homogenous groups with respect to order quantity and annual demand. They estimated the holding and setup cost of each buyer with respect to demand and order quantity. Their work does not consider the situation where the manufacturer faces numerous buyers, each having a different ordering policy. In joint replenishment problems (JRP) and economic lot scheduling problems (ELSP) [1,2,3,4,5,8], it is usually assumed that the order interval of all products are integer multiple of the shortest interval. A similar assumption (substituting the buyer for the product) is used in this paper.

JOINT INVENTORY MODEL FOR MULTIPLE BUYERS' CASE

COST FUNCTION

* Buyer's Total Variable Cost (TCB)

The total variable cost of all buyers is the sum of the total cost of each buyer.

$$TCB(n') = \sum_{i=1}^{m} TCB(n/OF_i)$$

$$= \sum_{i=1}^{m} \{ n / OF_i [(\tfrac{1}{2} q_i^2 / D_i) HCB_i + OCB_i]\} \qquad (1)$$

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where n' : number of initial setups : $\sum_{i=1}^{m} n/OF_i$

- i : Buyer 1, 2, ... , m.
- q_i : Order quantity of buyer i.
- D_i : Annual demand of buyer i.
- OF_i : Order frequency of buyer i.
- n : Number of period.
- HCB_i : Inventory holding cost of buyer i per unit.
- OCB_i : Ordering cost of buyer i per order.

* Manufacturer's Total Variable Cost (TCM)

$$TCM(n) = n\ SCM + \sum_{j=1}^{n} \tfrac{1}{2}\ t_j\ SUM_j\ HCM + K \sum_{j=1}^{n} t_j \qquad (2)$$

where

$$SUM_j = \sum_{j=1}^{m} q_{(i,j)}$$

- $q_{(i,j)}$: demand of buyer i in period j.
- K : Constant to convert time to cost.
- SCM : Setup cost of manufacturer per setup.
- HCM : Inventory holding cost of manufacturer per unit.
- t_i : Production time in period j.

* Joint Total Variavle Cost (JTC)

The joint total variable cost is sum of the total variable cost of the buyers and manufacturer.

$$JTC(n) = TCB(n') + TCM(n) \qquad (3)$$

FORMULATION OF GENERAL EQUATIONS

We present an algorithm using ICA(7) to solve Eq.3. To apply the algorithm the following general equations are needed:

Notation

- c_{1i} : Number of combined EOQ in immediate porceeding period having a setup for buyer i.
- c_{2i} : Number of combined EOQ in current period for buyer i.
- θ : $c_{1i} + c_{2i}$
- Γ' : $j - OF_i * c_{1i}$
- B_1 : $\sum_{j=1}^{k-1} SUM_j$
- Y_1 : Production time required for the first unit.
- $PS_{(i,j)}$: Production cost saving with proposed Production for buyer i in period j.
- $EHCB_{(i,j)}$: Extra inventory holding cost of buyer with increased order for buyer i in period j.
- $EHCM_{(i,j)}$: Extra inventory holding cost of manufacturer with proposed production for buyer i in period j.

Table 1. Joint lot size and Joint Incremental Cost for Multiple Buyers' Case

ITERATION		1	2	3	4	5	6	7	8	9	10	TOTAL V. COST
1	B1 $Q_{(1,j)}$	100	100	100	100	100	100	100	100	100	100	
	$JIC_{(1,j)}$	L	- 22*	44	- 5	49	4	52	11	55	15	1000
	B2 $Q_{(2,j)}$	45	0	45	0	45	0	45	0	45	0	
	$JIC_{(2,j)}$	L	L	36	L	41	L	42	L	44	L	428
	M SUM_j	145	100	145	100	145	100	145	100	145	100	5616
BUYER'S TOTAL VARIABLE COST : 1428 JOINT TOTAL COST : 7044												
2	B1 $Q_{(1,j)}$	200	0	100	100	100	100	100	100	100	100	
	$JIC_{(1,j)}$	L	L	159	- 5*	49	4	52	11	55	15	1050
	B2 $Q_{(2,j)}$	45	0	45	0	45	0	45	0	45	0	
	$JIC_{(2,j)}$	L	L	45	L	41	L	42	L	44	L	428
	M SUM_j	245	0	145	100	145	100	145	100	145	100	5544
BUYER'S TOTAL VARIABLE COST : 1478 JOINT TOTAL COST : 7022												
3	B1 $Q_{(1,j)}$	200	0	200	0	100	100	100	100	100	100	
	$JIC_{(1,j)}$	L	L	387	L	167	4	52	11	55	15	1100
	B2 $Q_{(2,j)}$	45	0	45	0	45	0	45	0	45	0	
	$JIC_{(2,j)}$	L	L	42	L	48	L	42	L	44	L	428
	M SUM_j	245	0	245	100	145	100	145	100	145	100	5489
BUYER'S TOTAL VARIABLE COST : 1528 JOINT TOTAL COST : 7017												

NOTE: R = 1225 UNITS/YEAR, K = \$5/HOUR, LCR = 90%, α = 10%, Y_1 = 1.87 HRS/UNIT, SCM = \$50/SETUP, HCM = \$10/UNIT/YEAR, Y_1 = 1.87 HRS/UNIT, BUYER 1 EOQ = 100 UNITS, BUYER 2 EOQ = 45 UNITS.

M : MANUFACTURER
B1 : BUYER 1
B2 : BUYER 2
SUM_j: MANUFACTURER PRODUCTION QUANTITY IN PERIOD j
TOTAL V. COST : TOTAL VARIABLE COST

$$EHCB_{(i,j)} = [\tfrac{1}{2}\, q_{2(i,j)}/D_i)\ (\theta^2 - c_{1i}^2 - c_{2i}^2)]\ HCB_i \qquad (4)$$

$$t_k = Y_i\ [\ (SUM_k + q_{(i,j)} * c_{2i} + \alpha * B_i)^{1-b} - (\alpha * B_i)^{1-b} \qquad (5)$$

where $k = j - OF_i * c_{1i}$

$$t_k = Y_i\ [\ (SUM_k + \alpha * (B_i - q_{(i,j)} * c_{2i}))^{1-b} - (\alpha * (B_i + q_{(i,j)} * c_{2i}))^{1-b}\ ] \qquad (6)$$

where $k = j - OF_i * c_{1i} + 1,\ \ldots\ ,\ j-1$

$$t_k = Y_i\ [\ (SUM_k - q_{(i,j)} * c_{2i} + \alpha * (B_i + q_{(i,j)} * c_{2i}))^{1-b} - (\alpha * (B_i + q_{(i,j)} * c_{2i}))^{1-b}\ ] \qquad (7)$$

where $k = j$

$$EHCM_{(i,j)} = (EHCM1 + EHCM2 + EHCM3) * HCM \qquad (8)$$

$$EHCM1 = \tfrac{1}{2}\ (SUM_k + q_{(i,j)}) * t_k - \tfrac{1}{2}\ SUM_k * t'_k \qquad (9)$$

where $k = j - OF_i * c_{1i}$

$$EHCM2 = \tfrac{1}{2}\ (SUM_k + q_{(i,j)}) * t_k - \tfrac{1}{2}\ SUM_k * t'_k \qquad (10)$$

where $k = j - OF_i * c_{1i} + 1,\ \ldots\ ,\ j-1$

$$EHCM3 = \tfrac{1}{2}\ (SUM_k + q_{(i,j)}) * t_k - \tfrac{1}{2}\ SUM_k * t'_k \qquad (11)$$

where $k = j$

$$PS_{(i,j)} = K \sum_{k=r}^{j} (\ t_k - t'_k\ ) \qquad (12)$$

Algorithm

STEP 1. Take the demand schedule as sum of each buyer's demand and assign combined EOQ number to 1 in period having a setup for each buyer.

STEP 2. Compute total variable cost of each buyer and total variable cost of all buyers using Eq.(1).

STEP 3. Compute production time in each period and compute the manufacturer's total variable cost (TCM(n)) using Eq.(2).

STEP 4. Compute the joint total cost for initial requirements by adding both total variable cost

$JTC(n') = TCB(n') + TCM(n)$

STEP 5. The joint incremental cost for the first period and for the periods with joint lot size equal to zero is assigned a large positive value L.

STEP 6. Compute the extra inventory holding cost ($EHCB_{(i,j)}$) using Eq.(3) and incremental cost of buyer i in period j ($ICB_{(i,j)}$) which is

$ICB_{(i,j)} = - OCB_i + EHCB_{(i,j)}$

STEP 7. For the incremental cost of manufacturer ($ICM_{(i,j)}$) for buyer i in period j, compute as follows

* proposed production time (t_k) where $k=j-OF_i, \ldots$ j with Eq.(5) through (7).
* extra inventory holding cost ($EHCM_{(i,j)}$ with Eq.(8)
* production saving ($PS_{(i,j)}$) with Eq.(12).

$ICM_{(i,j)} = - SCM + PS_{(i,j)} + EHCM_{(i,j)}$ where, $SUM_j = \sum_{i=1}^{n} q_{(i,j)}$

$PS_{(i,j)} + EHCM_{(i,j)}$ otherwise

STEP 8. Compute the joint incremental cost ($JIC_{(i,j)}$) for buyer i period j.

$JIC_{(i,j)} = ICB_{(i,j)} + ICM_{(i,j)}$

Repeat STEP 6 through 8 for period 2 through period n for first iteration. From second iteration those steps are repeated for proceeding and succeeding periods of the period identified with minimum incremental cost in the previous iteration.

STEP 9. Find the lowest joint incremental cost.

STEP 10. If all joint incremental costs are greater or equal to zero , then stop algorithm. This is best joint lot size. Otherwise continue.

STEP 11. Combine the sum of the buyer's demand, add the number of combined EOQ and compute the joint total cost.

$$JTC(n'-1) = JTC(n') + JIC_{(i,j)}$$

go to STEP 5.

This algorithm for solving the joint inventory model was programmed in Fortran 77 and run on a PC.

EXAMPLE 1

For the multiple buyer case we have modified the Example 1 in single buyer case [7]. used by Nanda and Nam [7]. We have added an another buyer whose order interval is twice that of the single buyer with an EOQ of 45 units. The proposed ICA algorithm was applied to determine the joint lot size for multiple buyer's case and the results are shown in Table 1 and the computations for each step are as follows:

STEP 1. The demand schedule (sum of each buyer) is $SUM_1 = 145$, $SUM_2 = 100, \ldots, SUM_{10} = 100$ units, assign combined EOQ number of 1 in the period having a setup for each buyer.

STEP 2. The total variable cost of each buyer using Eq.(1) is

$$TCB(10) = 10[(½\ 100^2 / 1000) * 10 + 50] = 1000$$

$$TCB(5) = 5[(½\ 45^2 / 225) * 9.5 + 42.75] = 428$$

The total variable cost of all buyers is

$$TCB(15) = 1000 + 428 = 1428$$

STEP 3. Compute existing production time for initial requirements at each period gives following

$$t'_1 = 1.87 * (145)^{.848} = 127.26$$

$$t'_2 = 1.87 * [(100 + .1 * 145)^{.848} - (.1 * 145)^{.848}] = 86.10$$

Similary, $t'_3=117.10$, $t'_4= 80.99$, $t'_5=112.18$, $t'_6=77.72$, $t'_7=108.65$, $t'_8=75.29$, $t'_9=105.88$, $t'_{10}=73.75$.

Total variable cost with 10 setup using Eq.(2) is

$$TCM(10) = 10 * 50 + (\sum_{j=1}^{10} ½\ t'_j * SUM_j) * .0048 + 5 * \sum_{j=1}^{10} t'_j = 5616$$

STEP 4. Joint total cost for initial requirements is

$$JTC(15) = TCB(15) + TCM(10) = 1428 + 5616 = 7044$$

STEP 5. $JIC_{(1,1)} = L$, $JIC_{(2,1)} = L$, $JIC_{(2,2)} = L$, ... , $JIC_{(2,10)} = L$, STEP 6. Incremental cost for buyer 1 in period 2,

* Extra holding cost of buyer using Eq.(4) is

$EHCB_{(1,2)} = [½ * (100^2 / 1000) (2^2 - 1 -1)] * 10$

$= 100$

where, $c_{11} = 1$, $c_{21} = 1$, $\theta = 2$

* Incremental cost is

$ICB_{(1,2)} = - 50 + 100$

$= + 50$

STEP 7. Incremental cost of manufacturer for buyer 1 in period 2,

* proposed production time using Eq.(5) through (7) is

$t_1 = 1.87 * [(145 + 100*1)^{.848}] = 106.17$

$t_2 = 0.0$

* extra inventory holding cost using Eq.(8) is

$EHCM_{(1,2)} = (EHCM1 + EHCM2) * .0048$

$= 51.79$

$EHCM1 = ½ [(145 +100) * 198.54 - 145 * 127.26]$

$= 15094.8$

$EHCM2 = ½ [(- 100 * 86.10] = - 4305$

* production saving using Eq.(12) is

$$PS_{(1,2)} = 5 * \sum_{k=1}^{2} (t_k - t'_k) = - 74.09$$

where $j=2$, $OF_1=1$, $c_{11}=1$, $c_{12}=1$, $SUM_1=145$, $SUM_2=100$.

* Incremental cost of manufacturer is

$ICM_{(1,2)} = - 50 - 74.09 + 51.79 = - 72$

STEP 8. Joint incremental cost for buyer 1 in period 2 is

$JIC_{(1,2)} = ICB_{(1,2)} + ICM_{(1,2)}$

$= 50 - 72$

$= - 22$

Similarly, $JIC_{(1,3)} = + 44$, $JIC_{(1,4)} = - 5$, $JIC_{(1,5)} = + 49$, $JIC_{(1,6)} = + 4$, $JIC_{(1,7)} = + 52$, $JIC_{(1,8)} = + 11$, $JIC_{(1,9)} = + 55$, $JIC_{(1,10)} = + 15$, $JIC_{(2,3)} = + 36$, $JIC_{(2,5)} = + 41$, $JIC_{(2,7)} = + 42$, $JIC_{(2,9)} = + 44$.

STEP 9. The minimum JIC identified is $JIC_{(1,2)} = - 22$

STEP 10. There exist negative value of joint incremental cost.

STEP 11. Combine the sum of buyer's demand ($SUM_1=245$, $SUM_2=0$, ...), combined EOQ number is 2,0,1,1',1,1,1,1,1,1' for buyer 1 and count mumber of buyer 2 is not changed and compute total cost

$JTC(14) = JTC(15) + JIC_{(1,2)}$

$= 7044 - 22 = 7022$

STEP 5. $JIC_{(1,2)} = L$

$JIC_{(2,10)} = L$,

STEP 6. Incremental cost for buyer 1 in period 3,

* Extra holding cost of buyer using Eq.(4) is

$EHCB_{(1,3)} = [½ * (100^2 / 1000) (3^2 - 2^2 -1)] * 10$

$= 200$

where, $c_{11} = 2$, $c_{21} = 1$, $\theta = 3$

* Incremental cost is

$ICB_{(1,3)} = -50 + 200$

$= +150$

STEP 7. Incremental cost of manufacturer for buyer 1 in period 3,

* proposed production time using Eq.(5) through (7) is

$t_1 = 1.87 * [(245 + 100*1)^{.848}] = 265.41$

$t_2 = 0.0$

$t_3 = 1.87 * [(145 - 100*1 + .1 * (245 + 100))^{.848}]$

$- (.1 * (245 + 100))^{.848}] = 38.78$

* extra inventory holding cost using Eq.(8) is

$EHCM_{(1,3)} = (EHCM1 + EHCM3) * .0048$

$= 66.45$

$EHCM1 = \frac{1}{2} [(245 + 100) * 265.41 - 245 * 198.54]$

$= 21462.08$

$EHCM3 = \frac{1}{2} [(145 - 100 * 38.78 - 145 * 117.10]$

$= -7617.20$

* production saving using Eq.(12) is

$$PS_{(1,3)} = 5 * \sum_{k=1}^{3} (t_k - t'_k) = -57.25$$

where $j=3$, $OF_1=1$, $c_{11}=1$, $c_{12}=1$, $SUM_1=245$, $SUM_2=0$, $SUM_3=145$.

* Incremental cost of manufacturer is

$ICM_{(1,3)} = -57.25 + 66.45 = +9$

STEP 8. Joint incremental cost for buyer 1 in period 3 is

$JIC_{(1,3)} = ICB_{(1,3)} + ICM_{(1,3)}$

$= 150 + 9 = 159$

Similarly, $JIC_{(1,5)} = +49$, $JIC_{(2,3)} = +45$, $JIC_{(2,5)} = +41$.

Table 1 shows that from second iteration onward the joint incremental cost remains unchaged except for the proceeding and succeeding periods of the period identified with minimum joint incremental cost in the previous iteration.

STEP 9. The minimum JIC identified is $JIC_{(1,4)} = -5$.

STEP 10. There exist negative value of joint incremental cost.

STEP 11. Combine the sum of buyer's demand ($SUM_1=245$, $SUM_2=0$, $SUM_3=245$, $SUM_4=0$, ...), the combined EOQ number is 2,0,2,0,1,1,1,1,1,1,1 for buyer 1 and combined EOQ number of buyer 2 is not changed and compute total cost

$JTC(13) = JTC(14) + JIC_{(1,4)}$

$= 7021 - 5 = 7016$

Repeat algorithm until the minimum of joint incremental cost is positive. The best solution is attached in the 3rd iteration. The lot size of buyer 1 is 200 units every other period for first 2 lots and others are 100 units every period with a total cost of $ 1100. The lot size of buyer 2 is 45 units every other period with a total cost of $428. The joint total cost of the buyers and manufacturer is $7017.

QUANTITY DISCOUNTS

We determined the joint lot size which resulted in minimizing the total variable costs of multiple buyers and manufacturer in the algorithm above. If the joint lot size for all buyers is adopted, rather than individual buyer EOQ, the extra cost of all buyers is

$$ECBA = TCB(n'^{*}) - TCB(n') \qquad (13)$$

where n'^{*} : minimum cost of setup for multiple buyers case.

n' : number of initial setups : $\sum_{i=1}^{m} n/OF_i$

The saving of manufacturer for all buyers is

$$SMA = TCM(n') - TCM(n'^{*}) \qquad (14)$$

We know that under all situation the buyer's extra cost can make a persuasive argument in favor of adopting the joint lot size in conjunction with an appropriate quantity discount schedule. Adoption of the joint lot size can benefit both buyer and manufacturer without the need for cost sacrifice on part of either party.

Two quantity discounts (QDM) for multiple buyers can be proposed as in the case of the single buyer model .

$$QSM_{min} = ECBA \;/\; \sum_{i=1}^{m} D_i = [\; TCB(n'^{*}) - TCB(n') \;] \;/\; \sum_{i=1}^{m} D_i \qquad (15)$$

$$QSM_{max} = SMA \;/\; \sum_{i=1}^{m} D_i = [\; TCB(n') - TCB(n'^{*}) \;] \;/\; \sum_{i=1}^{m} D_i \qquad (16)$$

Adapting the joint lot size results in an extra cost incurred by all buyers (ECBA), which is \$100 (\$1100 - 1000) from Table 1, but saving associated with the manufacturer(SMA) is \$127 (\$5616 - 5489). Thus all buyer losses are more than offset by the manufacturer's gain. The net saving realized by the joint inventory policy is \$27. If the manufacturer offers a minimum quantity discount (QSM_{min}) of 8 cents per unit, the \$100 increase of all buyers (ECBA) is offset by saving (\$ .0.8*1225) of the purchase cost. If the manufacturer offers a maximum quantity discount (QDM_{max}) of 10 cents per unit, the \$127 of the entire savings of the manufacturer(SMA) is transferred to all the buyers.

SENSITIVITY ANALYSIS OF PARAMETER CHANGE

The objective of this analysis is to examine how the lot size and joint total cost of the existing buyer are affected by involving multiple buyers under different levels of learning retention. For each problem solved the same set of parameters are assumed in single buyer's case as with the multiple (one additional) buyers case as follows:

LCR = 90%, K = \$25/hour (year = 2080 hours), P_r = 600 units/year,

SCM = \$30/setup, HCM = \$5/unit/year, OCB = \$9/order,

HCB = \$6/unit/year.

In addition the differences are :

D_1 = 300, D_2 = 600, EOQ_1 = 30 units.

Two kinds of the buyer 2 are involved : one has half and the other has twice the order interval of buyer 1.

In Figure 1 the successive lot size of buyer 1 decrease when the order interval of buyer 2 is short. The change of lot size is from 100 to 30 units with a short order interval. With a long order interval of buyer 2 the lot size is changed from 150 to 60 units. Buyer 1 has been affected by the order policy of buyer 2 and lot size of buyer 1 affects the trend of decreasing the lot size when buyer 2 has a short interval.

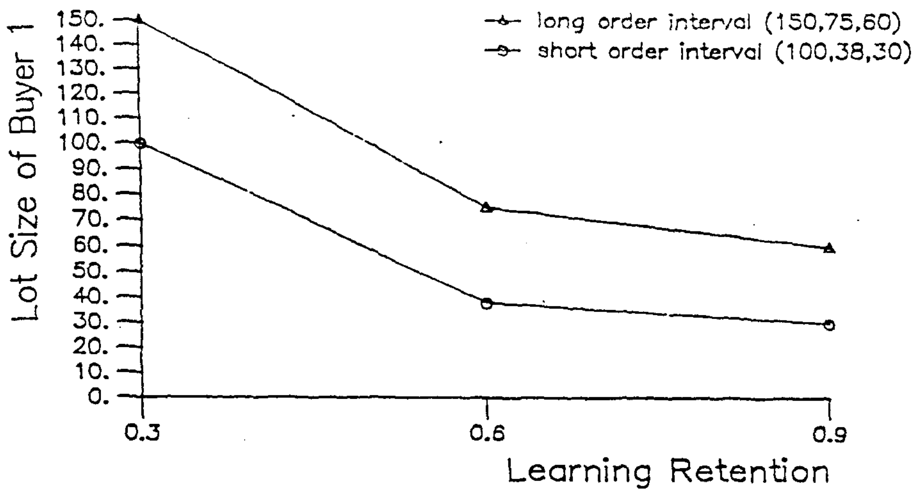

Figure 1. Effect of Buyer on Lot Size

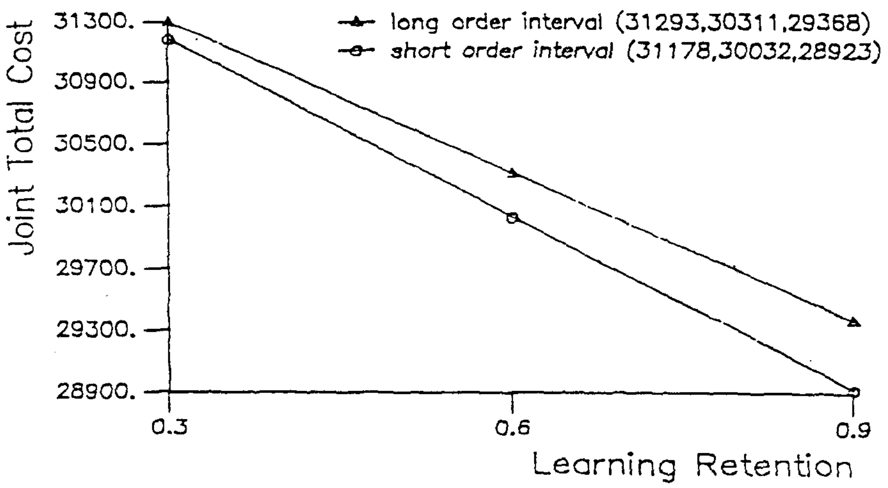

Figure 2 Effect of Buyer on Joint Total Cost

In Figure 2 the joint total cost also decrease when the order interval of the buyer 2 is short. The difference of joint total cost is $2255 with a short order interval. With long order interval there is a $1925 cost difference in joint total cost.

As seen from the sensitivity analysis, one buyer's order policy is impacted by the order policy of the other buyer. If one of the buyers has a short order interval with a small order quantity (e.g., as result of JIT & FMS), it can force the other buyers to move in the direction of a smaller lot size.

CONCLUSION

A joint inventory model was developed for multiple buyers incorporating the effects of learning, learning retention (after production breaks), and quantity discounts. The effects of learning retention on the joint total cost is most sensitive to change in a buyer's order interval..

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