工業經營學會誌 第15卷 第26輯 1992年 11月 119

Multivariate Exponentially Weighted Moving Average(EWMA) Process Control and Statistical Process Monitoring in the Process Industry
- 장치산업에서 다변량 EWMA 공정제어와 통계적 공정감시 -

김 복만*
최 성운**

요 지

본 논문은 장치산업에서 적용되는 다변량 EWMA 공정제어와 통계적 공정감시 통합시스템을 제안한다. 본 논문에서 제안한 통합시스템은 자동공정제어(APC)의 예측, 조정기능과 통계적 공정감시(SPM)의 이상점 발견 및 제거등의 각각의 장점을 이용하였다. 기존의 다변량 EWMA연구는 데이타간의 독립성을 가정하였으나 본 논문은 데이타간의 종속적인 형태인 IMA(1,1)모델을 대상으로 하였다.

1. Introduction

The people responsible for SPM and those responsible for APC are usually from different departments and have different technical backgrounds, so it is hardly surprising that there has sometimes been conroversy and misunderstanding between them. But I advocates that APC is applied in conjuction with, rather than in competition with, SPM. So a comparision of SPM and APC is first presented and then provided how to integrate SPM and APC for total system improvement (Box et. al, 1992)

Both fields seek to reduce deviations of some characteristic from a target value. In SPM, however, this is accomplished by monitoring a process so as to detect and remove root causes of variability. On the other hand, APC seeks to counteract the effects of root causes through continual process adjustment. That is, SPM thus resembles a system 'of continuous statistical hypothesis testing, whereas, APC can be thought of as a process of statistical estimation of the current level of the disturbance that is used to apply appropriate compensatory adjustment. SPM is ordinarily appropriate when it is reasonable to expect successive process measurements to be well modeled as iid and one is concerned with detecting departures from such an idea. By contrast, APC is ineffective on an iid process. It is most effective in the context of a continually wandering process - for example, a process could be well modeled by an sautoregressive moving average time series. Thus, in usual applications, the ultimate effect of applying SPM has been to optimally adjust an existing process(Wander et. al,1992)

In reality, SPM and APC are not competitive approaches but rather complement each other very effectively. So this paper proposes techniques from both methodologies are empolyed in a synergistic way to make quality gains that would not be attainable with the use of just either one. Such dual implementation is especially natural in the continuous process industries, where quality improvement is a key to profitability and where one is apt to find measurements that are correlated over time so that the process appears to wander.

* Dept. of Industrial Engineering, University of Ulsan, Ulsan, Korea.
** Dept. of Industrial Engineering, Kyungwon University, Sungnam, Korea.
접수 : 1992. 10. 30.
확정 : 1992. 11. 9.

2. Integrated Process Control and Monitoring System

The procedure proposed in this paper is as follows:

Step 1. Multivariate integrated moving average (e.g IMA(1,1)) class of stochastic models for representing stationary and nonstationary disturbances is developed and method for identifying, fitting, and diagnostically checking such models is devised with linear difference equations to represent dynamics, these models is then used to slove problems.

Step 2. Design a control rule for the estimated model. In nonstationary multivariate IMA(1,1) model, proportional Integral (PI) adjustment of APC produces minimum mean squared error (MSE) forecast of Z_t made at time t-1 (that is, multivariate EWMA).

Step 3. Multivariate Hotelling T^2 statistic is used to monitor the complete system to detect and remove unexpected root cause of variation.

2.1 Multivariate IMA(1,1)

Multivariate IMA(1,1) model is given by

$$(1-B)\dot{X}_t = (I-\theta_B)a_t$$

or

$$\dot{X}_t = \dot{X}_{t-1} + a_t - \theta a_{t-1} \qquad (1)$$

For m = 2

$$\begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} = \begin{bmatrix} \dot{X}_{1t-1} \\ \dot{X}_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} - \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix}\begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix}$$

It is improtant to recognize that EWMA's have at least two distinctly different uses (1) as SPM chart, (Lucas et al, 1990) to detect changes from past operation (2) as forecasts, to guide in active control of a process. One must understand and distinguish cleary between these two uses of EWMA's. The former is a smoothing problem. The EWMA is a statistic with the characteristics that it gives less and less weight to data as they get older and older. A plotted on an EWMA chart can be given a long memory, thus providing a chart similar to the ordinary CUSUM chart, or it can be given a short memory and provide a chart analogous to a chart. The desire to employ historical data more resourcefully has occasionally led to the use of the moving average. An example of a weighting function for the , CUSUM, moving average and EWMA charts are displayed in Figure 1 (Hunter, 1986).

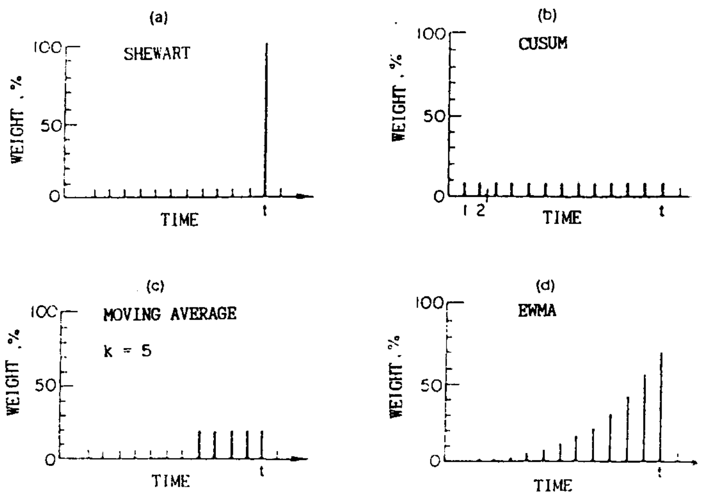

FIGURE 1. Data Weighting for the Shewart, CUSUM, Moving Average and EWMA Charts.

Lowry et. al(1992) proposed multivariate EWMA control chart to monitor a process. They assumed to be statistically dependent (multivariate) and serial uncorrelated. But this paper proposes multivariate EWMA representing serial correlation, that is, IMA(1,1) to control a process.

2.2 Minimum MSE Forecast Equation

A recursive equation for updating the minimum mean-squared-error forecast of multivariate IMA(1,1) process is

$$Z_t = (I-\theta)\dot{X}_t + \theta Z_{t-1}$$
$$= \lambda \dot{X}_t + (I-\lambda)Z_{t-1} \qquad (2)$$

For $m = 2$

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}\begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} + \begin{bmatrix} 1-\lambda_{11} & 0-\lambda_{12} \\ 0-\lambda_{21} & 1-\lambda_{22} \end{bmatrix}\begin{bmatrix} Z_{1t-1} \\ Z_{2t-1} \end{bmatrix}$$

Contrary to Lowry et. al(1992) determining λ based on ARL(Crowder,1987,1989) if the value for θ in the forecast equation (2) is a good estimate of θ in the true multivariate IMA(1,1), then $\lambda = I-\theta$.

2.3 Multivariate Hotelling T^2 statistic

Multivariate Hotelling T^2 statistic can be written as

$$T_t^2 = Z_t \Sigma_{Zt}^{-1} Z_t$$

and forecast covariance

Σ_{Zt}^{-1} can be derived from white noise covariance Σ_{at}^{-1}(Jenkins et. al, 1981)

3. Examples

In this section, we present two examples. The first is a numerical example when θ in identified multivariate IMA(1,1) is a diagonal matrix. The second example when θ in identified multivariate IMA(1,1) is a full matrix.

Example 1: A numerical example when θ in identified multivariate IMA(1,1) is a diagonal matrix is given in Table 1.

Identified multivariate IMA(1,1) process is

$$\begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} = \begin{bmatrix} \dot{X}_{1t-1} \\ \dot{X}_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} - \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}\begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix}$$

Because $\lambda = I-\theta$, multivariate EWMA for updating minimum MSE forecasts is

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}\begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} + \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}\begin{bmatrix} Z_{1t-1} \\ Z_{2t-1} \end{bmatrix}$$

Multivariate Hotelling T^2 statistic to monitor a process is

$$T_t^2 = Z_t' \Sigma_{zt}^{-1} Z_t$$

Where

$$\Sigma_{zt}^{-1} = \Sigma_{at}^{-1} \text{ (Jenkins et.al, 1981)}$$

In Table 1, observations 45, 46, and 50 seem to be out-of-control.

Tabel 1. Examplel Data

time	$\dot{x}$1(t)	$\dot{x}$2(t)	z1(t)	z2(t)	HOTELLING T^2
0	.00000	.00000	.00000	.00000	
1	1.26593	2.81122	.37978	.84337	.73617
2	.00615	1.70363	.26769	1.10145	1.45473
3	1.17286	-.01311	.53924	.76708	.59816
4	1.83320	1.55063	.92743	1.00214	1.17049
5	2.30214	1.99616	1.33984	1.30035	2.18027
6	1.48262	2.07386	1.38268	1.53240	2.68355
7	1.50299	1.27798	1.41877	1.45608	2.58447
8	2.16948	2.60176	1.64398	1.79978	3.73644
9	1.49729	1.88582	1.59998	1.82559	3.73066
10	-.77900	3.39773	.88628	2.29723	5.65558
11	.51781	1.29797	.77574	1.99745	4.26904
12	.36194	1.27040	.65160	1.77934	3.43644
13	1.06150	1.39504	.77457	1.66405	2.84735
14	.10247	.53930	.57294	1.32662	1.83765
15	2.54362	2.45283	1.16415	1.66448	2.81328
16	.69650	1.59860	1.02385	1.64472	2.70724
17	2.41996	2.70319	1.44268	1.96226	3.96046
18	.96118	.78557	1.29823	1.60925	2.76262
19	-.36996	-.10931	.79777	1.09368	1.22745
20	-.13663	.37037	.51745	.87669	.76869
21	.64700	2.29566	.55632	1.30238	1.77537
22	-.17115	1.62125	.33808	1.39804	2.34631
23	-.53337	1.54255	.07664	1.44140	3.04832
24	1.06869	2.14367	.37426	1.65208	3.32417
25	.21217	-.19027	.32563	1.09937	1.38292
26	-1.37978	1.15939	-.18599	1.11738	2.39456
27	-1.06811	.11642	-.45063	.81709	2.05086
28	-.80759	.46482	-.55772	.71141	2.02073
29	-1.40295	1.33675	-.81129	.89901	3.65880
30	-.83929	-.13261	-.81969	.58952	2.49890
31	-.23417	.85253	-.64403	.66843	2.15337
32	-1.26739	-.20779	-.83104	.40556	1.96805
33	1.64680	1.35005	-.08769	.68891	.86683
34	-1.05398	.23486	-.37758	.55269	1.09134
35	.62026	1.05714	-.07822	.70403	.88728
36	1.99673	1.72105	.54426	1.00913	1.02421
37	-.84148	-.80306	.12854	.46548	.25217
38	-.14663	.78323	.04599	.56080	.44635
39	1.35854	1.77283	.43975	.92441	.87516
40	.83370	4.09999	.55794	1.87708	4.02810
41	.92319	2.21647	.66751	1.97890	4.33826
42	1.33557	2.45173	.86793	2.12075	4.75326
43	-1.15008	-.13993	.26253	1.44254	2.64907
44	-.27895	1.43282	.10008	1.43963	2.98382
45	-.19050	4.48490	.01291	2.35321	8.59579
46	-.05928	1.33190	-.00875	2.04681	6.57972
47	-1.27262	-.01462	-.38791	1.42838	4.46197
48	-2.22852	.15934	-.94009	1.04767	4.94262
49	-.61681	1.49923	-.84311	1.18314	5.16821
50	-2.75389	2.48372	-1.41834	1.57331	11.18024

Examplel 2. A numerical example when θ in identified multivariate IMA(1,1) is full matrix is a given Table 2.

Identified multivariate IMA(1,1) process is

$$\begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} = \begin{bmatrix} \dot{X}_{1t-1} \\ \dot{X}_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} - \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.5 \end{bmatrix} \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \end{bmatrix}$$

Because $\lambda = I - \theta$, multivariate EWMA for updating minimum MSE forecasts is

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = \begin{bmatrix} 0.7 & -0.1 \\ -0.1 & 0.7 \end{bmatrix} \begin{bmatrix} \dot{X}_{1t} \\ \dot{X}_{2t} \end{bmatrix} + \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.5 \end{bmatrix} \begin{bmatrix} Z_{1t-1} \\ Z_{2t-1} \end{bmatrix}$$

Multivariate Hotelling T^2 statistic to monitor a process is

$$T_t^2 = Z_t' \, \Sigma_{zt}^{-1} Z_t$$

Where

$$\Sigma_{zt}^{-1} = \Sigma_{at}^{-1}$$ (Jenkins et.al, 1981)

In Table 2, observations 8, 29, 30, 45, 48, 49, and 50 seem to be out-of-control.

Table 2. Example 2 Data

time	$\dot{x}1(t)$	$\dot{x}2(t)$	z1(t)	z2(t)	HOTELLING T^2
0	.00000	.00000	.00000	.00000	
1	1.26593	2.82623	.60353	1.28652	1.70716
2	.79514	2.72074	.33693	1.86376	3.52786
3	1.93051	.38778	1.22728	.89903	1.81588
4	2.78241	1.72758	2.05321	.91233	4.31219
5	3.16871	1.98688	2.54414	.92742	6.50225
6	3.12387	2.75926	2.58128	1.27654	6.94013
7	3.28765	1.98866	2.74922	1.04571	7.61187
8	3.96694	3.23162	3.17389	1.46705	10.36491
9	3.70046	2.94310	3.10147	1.51764	9.99803
10	2.48920	4.37701	2.08342	2.38826	7.75710
11	2.65525	2.44622	2.00025	1.94337	5.98391
12	2.24814	2.23356	1.75608	1.66362	4.50395
13	2.71268	1.91115	2.06822	1.34051	4.84728
14	1.85476	1.20446	1.66430	.88019	2.92932
15	3.00356	1.94842	2.31892	.94752	5.44709
16	2.24792	2.02938	1.97153	1.03177	4.09998
17	3.79589	2.60688	2.88472	1.24258	8.47794
18	2.96377	1.20236	2.69556	.63762	7.29819
19	1.31972	.39954	1.62876	.11706	2.80457
20	1.18510	.65730	1.24076	.10579	1.61750
21	1.43913	2.37232	1.13181	.97107	1.71926
22	.83662	2.24863	.60321	1.41301	2.03192
23	.25166	2.36902	-.02108	1.80553	3.60792
24	1.55858	2.43995	.66013	1.96899	3.88223
25	1.21685	1.18826	.73411	1.39093	2.04498
26	-.57640	2.52882	-.57522	1.94410	5.25426
27	-.86888	1.14114	-1.08931	1.68703	5.64316
28	-1.04391	1.19472	-1.34571	1.65420	6.46476
29	-1.80923	2.39172	-2.07477	2.33845	13.93855
30	-1.46095	1.10747	-1.98969	2.07654	11.81302
31	-.93608	1.50199	-1.61001	2.08184	9.82119
32	-1.72806	.73952	-1.97478	1.74449	9.90109
33	.85821	1.16691	-.28283	1.56736	3.07975
34	-.84176	1.22633	-.95345	1.50930	4.45107
35	-.03250	1.29961	-.58967	1.50305	3.44908
36	1.89243	1.65366	.83213	1.44808	2.27074
37	-.08072	.10062	.03826	.69921	.52122
38	-.09869	1.24786	-.25231	.97958	1.28740
39	1.34573	1.87369	.58099	1.31729	1.77319
40	1.25254	3.49152	.57019	2.22105	4.94323
41	1.74827	2.79725	.89302	2.27731	5.23450
42	2.33205	2.99357	1.37325	2.31293	5.85684
43	.14225	1.05994	.17426	1.53489	2.44589
44	.02044	2.00015	-.28692	1.74805	3.77904
45	.04890	3.84484	-.61114	2.82025	10.28727
46	.22385	1.63871	-.47254	2.26821	6.60565
47	-1.13715	.29238	-1.19382	1.44126	4.98333
48	-2.79190	.38396	-2.49500	1.31118	10.88686
49	-1.99128	.97831	-2.37134	1.59337	11.46062
50	-3.06921	2.06186	-3.22537	2.37167	22.65666

4. Conclusion

This paper presents the integrated process control and monitoring system in the process industry. The integrated system consists of i) multivariate IMA(1,1) model ii) multivariate EWMA representing the minimum MSE forecasts iii) multivariate Hotelling T^2 statistic.This integrated system seeks to exploit the merits of both APC and SPM.

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