Multivariate Autoregressive Moving Average(ARMA) Process Control in Computer Integrated Manufacturing Systems(CIMS)
– CIMS에서 다변량 ARMA공정제어 –

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요 지

본 논문은 CIMS에서 적용되는 ARMA공정제어의 새로운 3단계절차를 제안한다. 첫번째 단계는 다변량 ARMA모델을 식별하여 모수를 추정하고, white noise로 진단된 잔차 series에 대하여 다변량 제어통계량(즉, 다변량 Hotelling T^2 통계량, 다변량 CUSUM, 다변량 EWMA 통계량, 다변량 MA 통계량)등을 계산한다. 마지막으로 본 논문에서 제안한 8가지 다변량 제어통계량을 상호비교하여 이상점을 발견한다.

1. Introduction

As the technology of manufacturing moves toward more automated processes, quaility engineering in general and process control in particular will assume newer and somewhat different roles than existed prior to the introduction of automated processes. In CIMS, the processes are much more complex and the cost of data collection is considerably reduced. With automation, machines (sensor) can be instrumented to collect data, computers can be programmed to analyze the data, and a message can then be sent to the machine to perform the corrective action. In this environment, the potential exists for much wider application of methods that address the complexity of the processes, i.e., multivariate and stochastic technique(Alwan et.al,1988).

In such processes, observations may exhibit statistically dependent (multivariate) and time dependent (serial autocorrelated, stochastic, time series) behavior.

This paper proposes a new procedure of multivariate stochastic process control in CIMS. The first procedure is multivariate time series model building. Multivariate time series model building is an iterative procedure. It starts with model identification and parameter estimation (filtering). After parameter estimation, we have to assess model adequacy by checking whether the model assumptions are satisfied. The basic assumption is that the $\{a_t\}$ are white noise. That is, the a_t's are uncorrelated random shocks with zero mean and constant variance. For any estimated model, the residuals $\hat{a}_t$'s are estimates of these unobserved white noise a_t's. Hence, model diagnostic checking is accomplished through a careful analysis of the residual series $\{a_t\}$. After diagnostic checking various multivariate control techniques (e.g. multivariate Hotelling T^2 control, multivariate CUSUM (Cumulative Sum) control, multivariate EWMA(Exponentially Weighted Moving Average) control, multivariate MA(Moving Average) control) are applied to the residual series. Finally out-of-control signals are detected by comparing computed multivariate control statistic.

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2. MARAMA (Multivariate ARAMA) Process Control

2.1 MARAMA Model Building (English et.al. 1991)

A useful class of parsimonious models is the MARAMA(p,q)process

$$\Phi_p(B)\dot{Z}_t = \theta_q(B)a_t \qquad (1)$$

where

$$\Phi_p(B) = \Phi_0 - \Phi_1 B - \Phi_2 B^2 - \cdots \Phi_p B^p$$

and

$$\theta_q(B) = \theta_0 - \theta_1 B - \theta_2 B^2 - \cdots \theta_q B^q$$

are the autoregressive and moving average matrix polynomials of orders p and q, respectively, and Φ_0 and θ_0 are nonsingular $m \times n$ matrices.

When p=0, the process becomes a vector MA(q) model

$$\dot{Z}_t = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}.$$

When q=0, the process becomes a vector AR(p) model

$$\dot{Z}_t = \Phi_1 \dot{Z}_{t-1} + \cdots + \Phi_p Z_{t-p} + a_t.$$

Model identification is to match patterns in the sample ACF(Autocorrelated Function) and sample PACF(Partial AutoCorrelated Function) with the theoretical patterns of the ACF and PACF. The method of moments (Yule - Walker equations), maximum likelihood method, least squares method and Bayesian estimation method may be used to estimate the parameteres Φ and θ.

To check whether the residuals are white noise, we compute the sample ACF and PACF of the residuals to see whether they do not form any pattern that are all statistically insignificant, i.e., within two standard deviations if $\alpha = 0.05$. Another useful test is the Portmanteau lack of fit test(Box et.al, 1976)

2.2 Various Multivariate Control Techniques

2.2.1 MHT(Multivariate Hotelling T^2) Control (Doganaksoy, 1991)

Suppose that $a' = [a_1, a_2, \ldots, a_m]$ is a vector of the residual series and $\bar{a}=0$.

MHT statistic is

$$MHT = (a_t - \bar{a})' \Sigma^{-1}(a_t - \bar{a}). \qquad (2)$$

2.2.2 MCUSUM(Multivariate CUSUM) Control (Croiser, 1988)

MCUSUM statistic is

$$MCUSUM = \max \{ \frac{1}{t} [0, (a_t - \bar{a})' \Sigma^{-1}(a_t - \bar{a}) - p + (t-1)C_{t-1}]\} \qquad (3)$$

2.2.3 MEWMA(Multivariate EWMA) Control (Lowry et. al. 1992)

MEWMA vector can be written as

$$Z_I = r\, a_I + (1-r)Z_{I-1} \qquad (4)$$

$i = 1, 2, \ldots, m$, where $Z_0 = 0$, $\bar{z} = 0$ and $0 < r_1 = r_2 = \ldots = r_m \leq 1$.

MEWHT statistic is

$$MEWHT = (Zt - \bar{Z})' \Sigma^{-1}_{Zi}(Zt - \bar{Z}) \qquad (5)$$

MEWHT statistic is

$$MEWHT = \max\{ \frac{1}{t}[0, (Z_t - \bar{Z})' \Sigma^{-1}_{Z_I} (Z_t - \bar{Z}) - p + (t-1)C_{t-1}]\} \qquad (6)$$

From equations (5) and (6) assuming independence, the correct time-varying expression for the covariance of the MEWMA is

$$\Sigma Z_i = \{r[1-(1-r)^{2i}]/(Z-r)\}\Sigma \quad (7)$$

This covariance converges quickly to the asymptotic value

$$\Sigma Z_i = \{r/(Z-r)\}\Sigma \quad (8)$$

2.2.4 MMA(Multivariate MA) Control

MMA vectors can be written as

$$Z_i = \frac{a_i + a_{i-1} + \dots + a_{i-N+1}}{N} \quad (9)$$

$i = 1, 2, \dots, m$, $\bar{Z}=0$ and $N \leq n$.

MMAT statistic is

$$MMAT = (Z_t - \bar{Z})' \Sigma^{-1} Z_i (Z_t - \bar{Z}) \quad (10)$$

MMAT statistic is

$$MMAT = \max\{\frac{1}{t}[0, (Z_t - \bar{Z})' \Sigma^{-1} Z_i (Z_i - \bar{Z}) - p + (t-1)C_{t-1}]\} \quad (11)$$

The covariance of the MMA is

$$\Sigma Z_i = \frac{1}{N}\Sigma \quad (12)$$

2.3 Interpretation of Out-Of-Control Signals

The interpretation of out-of-signals from multivariate control techniques can be quite difficult.

So we recommend monitoring and comparing eight multivariate control statistic proposed in this paper.

3. EXAMPLE

A sample of 54 measurements in Table 5 indicates that $\dot{Z}_{1,t}$ and $\dot{Z}_{2,t}$ can be modeled by the multivariate AR(z) process

$$\dot{Z}_{1,t} = 0.74\,\dot{Z}_{1,t-1} + 0.54\,\dot{Z}_{1,t-2} - 0.25\,\dot{Z}_{1,t-2} - 0.54\,\dot{Z}_{2,t-2} + 0.51\,\dot{Z}_{1,t-3} + a_{1,t}$$

and

$$\dot{Z}_{2,t} = 1.48\,\dot{Z}_{2,t-1} + 0.27\,\dot{Z}_{1,t-2} - 0.48\,\dot{Z}_{2,t-2} - 0.27\,\dot{Z}_{1,t-3} + a_{2,t}$$

Eight multivariate control statistic proposed in this paper are represented in Table 2,3,4.

In Table 5, observations that appear to be most out-of-control are arranged within each statistic in descending order.

Table 5. out-of-control Observation

		Observations
Exact covariance :	MHT	1, 2, 12, 40, 20
	MCUSUM	1, 2, 3, 4, 5
	MEWMAT	1, 20, 12, 17, 30
	MEWMAC	1, 2, 3, 4, 5
Approximate covariance :	MEWMAT	1, 20, 12, 17, 30
	MEWMAC	1, 2, (24, 3, 22)
	MMT	1, 24, 23, 18, 17
	MMC	1, 2, 3, 4, 5

In Table 5, observations 1, 2, 12, 20 and 30 seem to be out-of-control.

Table 1. Process Data

Time	$\dot{Z}_{1t}$	$\dot{Z}_{2t}$
1	608.0000	1016.0000
2	451.0000	921.0000
3	529.0000	934.0000
4	543.0000	976.0000
5	525.0000	930.0000
6	549.0000	1052.0000
7	525.0000	1184.0000
8	578.0000	1089.0000
9	609.0000	1087.0000
10	504.0000	1154.0000
11	752.0000	1330.0000
12	613.0000	1980.0000
13	862.0000	2223.0000
14	866.0000	2203.0000
15	1016.0000	2514.0000
16	1360.0000	2726.0000
17	1482.0000	3185.0000
18	1608.0000	3351.0000
19	1800.0000	3438.0000
20	1941.0000	2917.0000
21	1229.0000	2359.0000
22	1373.0000	2240.0000
23	1611.0000	2196.0000
24	1568.0000	2111.0000
25	983.0000	1806.0000
26	1046.0000	1644.0000
27	1453.0000	1814.0000
28	1504.0000	1770.0000
29	807.0000	1518.0000
30	339.0000	1103.0000
31	562.0000	1266.0000
32	745.0000	1473.0000
33	749.0000	1423.0000
34	862.0000	1767.0000
35	1034.0000	2161.0000
36	1054.0000	2336.0000
37	1164.0000	2602.0000
38	1102.0000	2518.0000
39	1145.0000	2637.0000
40	1012.0000	2177.0000
41	836.0000	1920.0000
42	941.0000	1910.0000
43	981.0000	1984.0000
44	974.0000	1787.0000
45	766.0000	1689.0000
46	920.0000	1866.0000
47	964.0000	1896.0000
48	811.0000	1684.0000
49	789.0000	1633.0000
50	802.0000	1657.0000
51	770.0000	1569.0000
52	639.0000	1390.0000
53	644.0000	1387.0000
54	564.0000	1289.0000

Table 2. MHT and MCUSUM Statistic

Time	a_{1t}	a_{2t}	MHT	MCUSUM
1	608.00	1016.00	22.8261	20.8261
2	-547.56	-582.68	8.2626	13.5443
3	398.56	222.76	1.6948	8.9278
4	-52.81	-6.63	.0127	6.1990
5	2.74	-45.10	.0419	4.5676
6	51.30	147.86	.4614	3.5499
7	-92.82	68.58	.1334	2.7761
8	-12.28	-151.88	.4757	2.2386
9	83.84	37.12	.0582	1.7741
10	-68.83	82.27	.1595	1.4126
11	200.33	152.21	.6471	1.1612
12	-223.11	537.17	6.1538	1.4106
13	-11.66	-2.04	.0007	1.1483
14	-133.37	-174.17	.7001	.9734
15	288.83	387.83	3.4512	1.0053
16	217.10	63.80	.2835	.8352
17	173.46	3797.74	3.3857	.8676
18	85.30	38.56	.0614	.7117
19	197.34	40.26	.1983	.5794
20	208.20	-528.74	5.9415	.7475
21	-296.08	-256.08	1.7219	.6986
22	332.11	186.91	1.1867	.6299
23	-23.42	-179.12	.6631	.5444
24	116.08	-25.00	.0699	.4413
25	-428.90	-199.97	1.6027	.4078
26	53.67	-27.21	.0274	.3162
27	212.51	89.81	.3574	.2437
28	97.15	-108.59	.2828	.1736
29	-452.41	-120.99	1.1684	.1390
30	487.13	-280.27	2.6229	.1551
31	-30.05	174.01	.6275	.1058
32	-85.72	2.40	.0312	.0410
33	53.53	-89.15	.1758	.0000
34	234.37	417.41	3.8211	.0536
35	17.66	229.96	1.0905	.0260
36	-90.41	16.39	.0402	.0000
37	108.42	228.44	1.1246	.0000
38	-166.84	-206.28	.9943	.0000
39	128.34	-189.02	.8056	.0000
40	-217.70	-533.86	6.0707	.1018
41	59.75	-24.59	.0276	.0512
42	130.19	77.45	.1953	.0070
43	-17.06	31.28	.0214	.0000
44	16.99	-204.17	.8598	.0000
45	-83.04	7.36	.0303	.0000
46	149.27	222.15	1.1108	.0000
47	-117.62	-111.12	.3129	.0000
48	-79.22	-184.82	.7301	.0000
49	75.14	62.64	.1047	.0000
50	-43.21	7.17	.0090	.0000
51	-52.80	-105.46	.2409	.0000
52	-85.17	-133.25	.3964	.0000
53	51.28	74.28	.1248	.0000
54	-143.89	-131.93	.4462	.0000

Table 3. MEWMAT and MEWMAC statistic(r = 0.2)

Time	Z_{1t}	Z_{2t}	MEWMAT*	MEWMAC*	MEWMAT**	MEWMAC**
1	121.600	203.200	16.1597	14.1597	5.81748	3.8175
2	-12.232	46.024	.9673	6.5635	0.57106	1.1943
3	69.926	81.371	1.6552	4.2607	1.22134	.5366
4	45.379	63.771	.7579	2.8850	0.63072	.0602
5	36.851	41.997	.3733	1.9827	0.33318	.0000
6	39.741	63.169	.6172	1.4217	0.57474	.0000
7	13.229	64.252	.6364	1.0238	0.60838	.0000
8	8.127	21.025	.0608	.6535	0.05908	.0000
9	23.270	24.244	.1265	.3727	0.12427	.0000
10	4.850	35.849	.2051	.1559	0.20269	.0000
11	43.946	59.121	.5647	.0113	0.56052	.0000
12	-9.465	154.731	4.8408	.2471	4.81792	.2348
13	-9.904	123.377	3.1547	.3169	3.14519	.3049
14	-34.597	63.868	1.6308	.2679	1.62761	.2565
15	30.088	128.660	2.3874	.2758	2.38446	.2650
16	67.490	115.688	1.8699	.2505	1.86765	.2402
17	88.684	172.098	4.0045	.3537	3.99971	.3437
18	88.007	145.391	2.9958	.3893	2.99218	.3797
19	109.874	124.365	2.9485	.4188	2.94510	.4095
20	129.539	-6.256	4.9198	.5438	4.91487	.5347
21	44.415	-56.221	1.7467	.5058	1.74484	.4971
22	101.954	-7.595	3.1198	.5338	3.11672	.5253
23	76.879	-41.900	2.7682	.5440	2.76540	.5357
24	84.719	-38.520	3.0841	.5665	3.08094	.5584
25	-18.004	70.804	1.3306	.5170	1.32917	.5093
26	-3.670	-62.085	.6638	.4457	0.66306	.4383
27	39.566	-31.706	.9308	.3896	0.92986	.3824
28	51.155	-47.083	1.7310	.3661	1.72925	.3591
29	-49.558	-61.864	.6568	.3072	0.65602	.3003
30	-137.072	-105.546	3.8384	.3582	3.83424	.3515
31	-115.668	-49.634	2.8246	.3733	2.82173	.3666
32	-109.678	-39.228	2.6227	.3811	2.62000	.3746
33	-77.037	-49.212	1.1977	.3452	1.19643	.3389
34	-14.755	44.112	.5802	.2933	0.57955	.2871
35	-8.272	81.282	1.4084	.2680	1.40678	.2620
36	-24.700	68.303	1.4464	.2452	1.44488	.2393
37	1.924	100.331	1.8223	.2337	1.82015	.2279
38	-31.829	39.009	.8683	.1978	0.86735	.1921
39	0.205	69.011	.8804	.1640	0.87936	.1585
40	-43.376	-51.563	.4796	.1219	0.47909	.1165
41	-22.751	-46.169	.2862	.0772	0.28590	.0718
42	7.837	-21.445	.1434	.0311	0.14322	.0259
43	2.858	-10.900	.0319	.0000	0.03186	.0000
44	5.684	-49.554	.5331	.0000	0.53250	.0000
45	-12.061	-38.171	.1996	.0000	0.19936	.0000
46	20.206	13.893	.0825	.0000	0.08238	.0000
47	-7.359	-11.109	.0183	.0000	0.01827	.0000
48	-21.732	-45.852	.2812	.0000	0.28083	.0000
49	-2.357	-24.153	.0960	.0000	0.09590	.0000
50	-10.528	-17.889	.0448	.0000	0.04480	.0000
51	-18.982	-35.403	.1708	.0000	0.17064	.0000
52	-32.220	-54.972	.4229	.0000	0.42241	.0000
53	-15.520	-29.122	.1154	.0000	0.11531	.0000
54	-41.194	-49.683	.4385	.0000	0.43803	.0000

* : Exact Covariance
** : Approximate Covariance

Table 4. MMAT and MMAC Statistic (N=9)

Time	Z_{1t}	Z_{2t}	MMAT	MMAC
1	608.000	1016.00	16.1603	14.1603
2	30.220	216.66	1.6380	6.8991
3	153.000	218.69	2.4434	4.7472
4	101.548	62.36	.9251	3.2917
5	81.786	120.87	1.2170	2.4767
6	76.705	125.37	1.4900	1.9789
7	52.487	117.26	1.4238	1.6139
8	44.391	83.61	.8442	1.2677
9	48.774	78.45	.8820	1.0026
10	-26.429	-25.30	.1530	.7177
11	56.670	56.35	.7170	.5358
12	-12.404	91.29	1.8630	.4797
13	-7.832	91.79	1.7530	.4238
14	-22.956	77.46	1.6910	.3715
15	3.437	104.12	1.9260	.3418
16	37.827	103.59	1.4410	.2855
17	58.510	164.66	3.6530	.3659
18	58.672	164.82	3.6590	.4378
19	88.247	160.15	3.5160	.4945
20	89.121	84.49	1.7270	.4561
21	81.013	-3.65	1.9170	.4305
22	119.210	17.35	3.5450	.4811
23	131.427	16.80	4.3670	.5631
24	112.232	-29.07	4.4840	.6432
25	40.454	-58.38	1.6640	.6040
26	27.144	-105.60	2.9690	.6180
27	41.279	-99.90	3.3280	.6443
28	30.187	-116.44	3.6210	.6792
29	-43.214	-71.13	.7170	.6115
30	-64.442	-73.82	1.0230	.5586
31	-104.682	-75.26	2.2180	.5476
32	-111.604	-55.09	2.5700	.5483
33	-118.554	-62.21	2.8770	.5583
34	-44.858	6.38	.6420	.5019
35	-48.859	34.95	1.3110	.4679
36	-82.517	26.80	2.5810	.4710
37	-81.304	64.24	3.8870	.5093
38	-49.575	54.77	1.9040	.4934
39	18.811	106.91	1.7310	.4738
40	-2.039	28.26	.1630	.4160
41	14.125	25.26	.0880	.3593
42	22.642	43.77	.2590	.3093
43	-5.294	.87	.0090	.2558
44	-5.369	-47.37	.3620	.2127
45	-4.550	-48.37	.3860	.1721
46	-0.011	-49.07	.4460	.1346
47	5.458	-38.50	.3340	.0963
48	-17.604	-80.04	.9330	.0721
49	14.933	-13.76	.1470	.0328
50	3.493	-10.23	.0310	.0000
51	-16.839	-30.55	.1280	.0000
52	-24.407	-48.83	.3200	.0000
53	-20.597	-17.89	.0890	.0000
54	-27.358	-33.37	.1950	.0000

5. Conclusion

A new procedure of multivariate ARMA process control in CIMS is presented. This procedure consists of i) MARAMA model building ii) various multivariate control techniques iii) interpretation of out-of-control signals. Especially in multivariate control techniques eight multivariate control statistic are proposed and examined with an example. To detect out-of-control signals we recommend comparing eight multivariate control statistic.

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