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Reliability Analysis to Contaminant Migration in Saturated Sandy Soils : System Reliability Approach

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요 지

2차원 수평흐름에서의 비반응성 오염물 이동에 대한 연속시스템(series system) 신뢰성해석을 농도와 노출시간의 두 극한상태 함수에 대해 시행하였다. 시간에 따라 변화하는 오염원에서의 오염이동모델을 시스템 신뢰해석 모델과 결합하여 오염이 예상되는 지점의 최대농도가 주어진 기간동안에 특정농도를 초과하거나 또는 오염예상 지점이 보통의 농도에 노출되어질 시간이 특정 한계를 초과할 확률을 구하였다. 본 신뢰성 해석의 결과 상기의 두조건을 초과할 확률은 각각의 조건을 초과할 확률보다 큰 것을 확인하였으며 보다 큰 초과확률을 갖는 요소에 의하여 지배되는 것을 발견하였다. 예민도 해석은 투수계수 외에도 횡단 분산계수(transverse dispersivity)도 이차원 오염이동 신뢰성 모델의 중요한 매개변수임을 보여주었다. 시스템 예민도는 두 요소의 예민도를 동시에 반영하고 있으며 큰 초과확률을 갖는 요소가 解에 보다 큰 영향을 미치는 것을 알게 되었다.

Abstract

Series system reliability analysis of non-reactive contaminant transport is performed in a two dimensional horizontal domain with two different limit state functions: (1) concentration threshold and (2) exposure time threshold. The transient source transport model is combined with the system reliability model to evaluate the probability that a specified maximum concentration at a node of interest would be exceeded or that a moderate concentration would exceed some exposure limit over a given period of time.

The results give probabilities of exceedence greater than probability of each component and they tend to be dominanted by the component with larger probability. Transverse dispersivity turns out to be an important parameter in addition to hydraulic conductivity in a two-dimensional contaminant transport model with transient source. System sensitivity is found to reflect the corresponding sensitivity of both components, with the component with larger probability having a greater influence.

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1. Introduction

In the companion paper, the first and the second order reliability has been treated in the component reliability context which deals with only one limit state function. In this paper, a problem of evaluating the system probability that the concentration at a certain point will exceed either a given maximum concentration or some other moderate lower concentration for a given period of time, which have potential in toxicity and exposure risk assessment, is considered.

System reliability methods were first developed in structural engineering to evaluate the probability of failure of structural systems. First order reliability bounds have been used to solve the series system reliability problem by Ditlevsen[1] and Madsen et al.[2] Efficient Monte Carlo simulation techniques, e.g. directional simulation and importance sampling, have been used to solve the general system problem.[3,4,5] Some applications of system reliability analysis can be found in geotechnical engineering, for example, Luckman[6] for slope stability analysis and Kim[7] for earth retaining structures. However, it was first time by Jang et al.[8] that the system reliability is used to groundwater flow and contaminant transport in saturated sandy soils.

The formulation of limit state functions and the system reliability theory are introduced briefly to the extent to understand these analyses and the system reliability analyses are performed using two-dimensional contaminant transport model with transient source. The first order reliability bound method and the directional simulation method are used to solve the general system reliability analysis of contaminant transport problems.

2. Formulation of Limit State Functions

In the case of contaminant transport with a transient source, concentration may exceed the threshold concentration only for a finite period of time. In this type of problem the "failure" event is defined as exceeding the threshold at any time during the specified interval and the limit state function, gl(X), is formulated as:

$$g_1(X) = C_t - \max_{0<t\leq t_t} C(x, y, t) \quad (1)$$

in which $\max_{0<t\leq t_t} C(x, y, t)$ denotes the maximum concentration that is reached during the interval $0<t\leq t_t$ at location(x, y) and t_t denotes the period of time of interest. The gradient of the limit state function is then obtained at the time of maximum concentration in the form:

$$\frac{\partial g_1(X)}{\partial X_i} = \frac{\partial \max_{0<t\leq t_t} C(x,y,t)}{\partial X_i} = \frac{\partial C(x, y, t_{Cmax})}{\partial X_i} \quad (2)$$

in which t_{Cmax} is the time when maximum concentration is reached at the location of interest. The maximum concentration is found using quadratic interpolation of the time history of concentrations at the point of interest and the probability is computed when the maximum concentration is reached at the location of interest. The formulation given by equation (2) assumes that there is only a single peak in the concentration profile and this concept can be extended to a plume with multiple peaks by considering the largest peak or treating each peak as a component of a system.

Another formulation of the limit state function considers the duration of exceedence of a specified concentration threshold, i.e. exposure time. The limit state function, $g_2(X)$, then has the form

$$g_2(X) = T_t - T_{C>C_e}(x,y,t) \quad (3)$$

in which $T_{C>C_e}$ is the time interval during which the concentration at the location (x,y) exceeds the threshold C_e and T_t is the allowable exposure time. The exposure interval, T, is thus a function of the location (x,y) and the total elapsed time, t as well as the basic random variables. The problem in this paper becomes a system reliability problem, since "failure" in this case is defined as a condition when either the threshold concentration or a given duration is exceeded.

3. System Reliability Theory

In general terms in a system reliability problem the failure domain is given by

$$F = \{\cup_k \cap_{i \subset C_k} g_i(X) \leq 0\} \quad (4)$$

where $g_i(\cdot)$ is the i-th limit state function and C_k denotes the k-th cut set of the system. A cut set is a set of limit states whose joint exceedence constitutes the failure of the system, for example, a set of nodes at which threshold concentration is simultaneously exceeded. All the cut sets of a system are summed up as a union in equation (4). In a special case, when each cut set has only one element for each k, such as considered herein, the system is called a series system. The probability integral is then given by

$$P_{sys} = \int_F f_x(x)dx \quad (5)$$

For a given system which has a set of random variables, X, and a set failure mode with limit state function $g_i(X)$ i = 1,···, m, a failure probability for a series system is defined by

$$P_{sys} = P[\cup_i \ g_i(X) \leq 0] \quad (6)$$

For a simple case with two modes of failure,

$$P_{sys} = P[g_1(X) \leq 0 \ \cup \ g_2(X) \leq 0] = P_1 + P_2 - P_{12} \quad (7)$$

where $P_k = P[g_k(X) \leq 0]$ k = 1, 2 are the individual probabilities of failure and $P_{12} = P[g_1(X) \leq 0 \cap g_2(X) \leq 0]$ is the joint probability of failure of mode 1 and 2. Fig. 1 illustrates a limit state surface in the two-dimensional standard normal space by coordinated u_1 and u_2. It also shows, two failure modes and the corresponding first order tangential planes at the respective design point and **u** represents the input random variables in the transformed space.

The bimodel bounds on P_{sys} are represented by the individual failure mode and the joint failure mode as suggested by Ditlevsen.[1)]

$$P_1 + \sum_{i=2}^{k} \max \{P_i - \sum_{j=1}^{i-1} P_{ij}, \ 0\} \leq P_{sys}$$

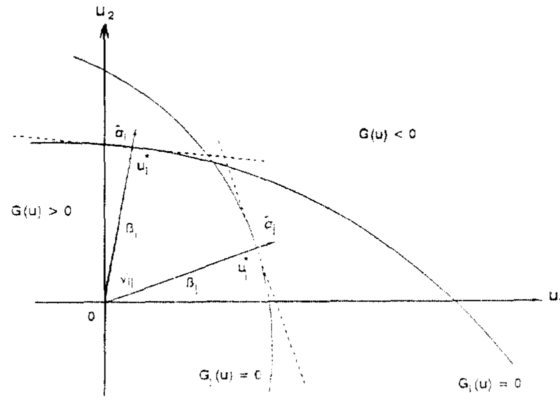

Fig. 1. Representation of limit state surface for two failure modes in standard space.

$$\leq P_1 + \sum_{i=2}^{k} \{P_i - \max_{j<i} \ P_{ij}\} \quad (8)$$

By including bimodal effects, these bounds are usually quite narrow. The bounds depend on the ordering of the failure probabilities and the best results are usually obtained if the modes are numbered in order of decreasing value of Pi.[(2)] Thus P_1 in equation (8) is the probability of the most likely mode of failure.

The first order approximation of joint probability for failure modes i and j can be represented by an intergral.[(2)]

$$P_{ij} = \Phi(-\beta_i)\Phi(-\beta_j) + \int_0^{\rho_{ij}} \phi^2(-\beta_i, \ -\beta_j, \rho)d\rho \quad (9)$$

in which $\Phi(-\beta_i)$ and $\Phi(-\beta_j)$ are individual failure modes, ρ_{ij} is the correlation coefficient between two failure modes and $\phi^2(\cdot)$ is the probability density function for a bivariate normal vector. The correlation coefficient ρ_{ij} is given by:

$$\rho_{ij} = \hat{\alpha}_i \ \hat{\alpha}_j^T \quad (10)$$

where $\hat{\alpha}_i$ and $\hat{\alpha}_j^T$ are the unit normal vectors at the design points of the two failure modes directed toward the failure region and ρ_{ij} is a scalar product of $\hat{\alpha}_i$ and $\hat{\alpha}_j$. Referring to Fig. 1, it follows that

$$\rho_{ij} = \cos \ v_{ij} \quad (11)$$

This relation has a simple geometric meaning, i.e. failure modes exhibiting zero correlation have limit state surface at right angles to one another, while, those which have perfect correlation have parallel limit state surfaces.

Finally, the generalized reliability index for the system is given by[9]

$$\beta_{sys} = \Phi^{-1}(1-P_{sys}) \qquad (12)$$

The reliability bounds are based on the first order approximation and simulation methods may be needed if a more exact estimate of the probability of exceedence is desired. The zero-one indicator based Monte Carlo method is used herien for verification of the reliability analysis[10] and the directional simulation which is a semi-analytical, conditional simulation method is used for refining the results of the first order bimodal bound method. In the directional simulation method the probability of failure is computed by conditioning on a simulated direction in the standard space[3,11] in the form:

$$P_{sys} = \int_{\mathbf{a} \subset \Omega n} P_{sys:A}(\mathbf{a}) f_A(\mathbf{a}) d\mathbf{a} \qquad (13)$$

where **A** is a unit directional vector having a uniform distribution over the n-dimensional unit sphere Ω in standard space, $P_{sys\ F:A}(\mathbf{a})$ is the probability of failure of the system given that the outcome of the variables in the standard space lies on a vector from the origin in direction **a** and $f_A(\mathbf{a})$ is the constant density of A on the unit sphere.

Assume that $r_i(i = 1, 2, \cdots, l)$ are the positive roots of the limit state function $G_j(r\mathbf{a}) = 0$ and define an indicator function associated with direction a as follows:

$$I(r\mathbf{a}) = \begin{cases} 0 & \text{if } r\mathbf{a}\ [\cup_k \cap_{j \subset Ck} G_j(r\mathbf{a}) \le 0)] \\ 1 & \text{otherwise} \end{cases} \qquad (14)$$

Here $I(r\mathbf{a})=0$ defines the system failure and $I(r\mathbf{a})=1$ denotes the system survival for outcome $\mathbf{u}=r\mathbf{a}$ in the standard normal space. The conditional probability of failure in direction **a** is represented by Bjerager[11]

$$P_{sys:A}(\mathbf{a}) = 1 - \sum_{i=0}^{l} I[0.5(r_i + r_{i+1})a] \qquad (15)$$
$$[\chi_n^2(r_{i+1}^2) - \chi_n^2(r_i^2)]$$

where $r=0$ and $r_{l+1}=\infty$, χ_n^2 is the Chi square distribution of $R^2=U^TU$ and U is a set of random variables with n degrees of freedom in the standard space. By performing N simulations of the unit vector **A**, the mean of the probability of exceedence is estimated as an average of the sample values $P_{sys:A}$.

4. Problem Formulation

An interesting and useful application of the reliability analysis is considering two possible failure modes: (1) exceedence of a threshold concentration and/or (2) exceedence of a threshold concentration over a certain period of time. Thus, the problem posed here is to estimate the probability that either the concentration exceeds 0.35 C_o during the first 150 days following the release of a contaminant or the concentration exceeds 0.2 C_o at the location of interest over a period of more than 75 days during the first 150 days following the contaminant release. Transient source with duration of 20 days is assumed. The finite element mesh is shown in Fig. 2 together with the source and the node of interest.

In this case the hydraulic conductivity is modelled as a random field, while dispersivities are treated as simple random variables. Thus, the mo-

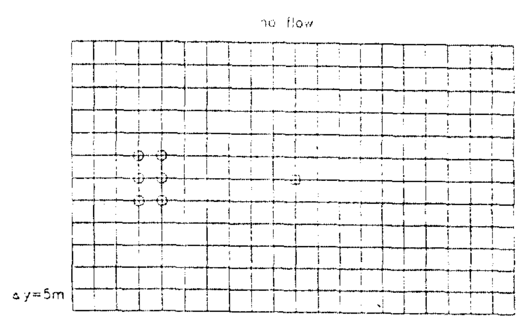

Fig. 2. Finite element mesh for system reliability application

del consist of 268 random variables which includes 240 hydraulic conductivities, two random variable dispersivities, and 26 boundary heads. The limit state functions are formulated as follows:

$$g_1(X) = 0.35\ C_o - \max_{0<t\leq 150} C(50,\ 30,\ t) \quad (16a)$$
$$g_2(X) = 75 - T_{C>0.2C_o}\ (50,\ 30,\ 150) \quad (16b)$$

where $\max_{0\leq t\leq 150} C(50, 30, t)$ is the maximum concentration that can be reached at the location (50, 30) during the time of interval 0 to 150 days and $T_{C>0.2C_o}$ represents the time during which the concentration exceeds 0.2 C_o at the location (50, 30) up to 150 days.

The distance between the source node and the node of interest is 30 m and the characteristics of the soil are given by $\mu_K = 3$ m/day, $\sigma_K = 0.3$ m/day; $\mu_{\alpha L} = 3$ m, $\sigma_{\alpha L} = 0.3$ m; and $\mu_{\alpha T} = 1.5$ m, $\sigma_{\alpha T} = 0.15$ m. The mean values of boundary heads are 4.6 and 0.6 m at the upgradient and the downgradient boundaries, respectively, with negligible standard deviation, giving a mean flow velocity of 0.4 m/day. Distribution of K is assumed lognormal and distributions of the other variables are normal. The correlation length used in this analysis is 20 m and the porosity is a constant value of 0.3.

5. Component Analysis Results

The component analysis is an essential step performing a system reliability analysis if FORM /SORM analyses are used. The contours of design point values of parameters and sensitivities are obtained from the first-order reliability analysis.

The concentration contours at the design point for the two limit state functions are compared with the mean concentration contours in Fig. 3. In the concentration threshold case the two profiles overlap around the node of interest and the design point concentration profile has significantly higher peak concentration than that of the mean concentration (Fig. 3a). The deterministic estimate of the maximum concentration reached during the assigned time interval of 150 days is 0.305 C_o at t=84.49 days, while the maximum concentration at the design point is reached at t=79.40 days which is about 5 days earlier.

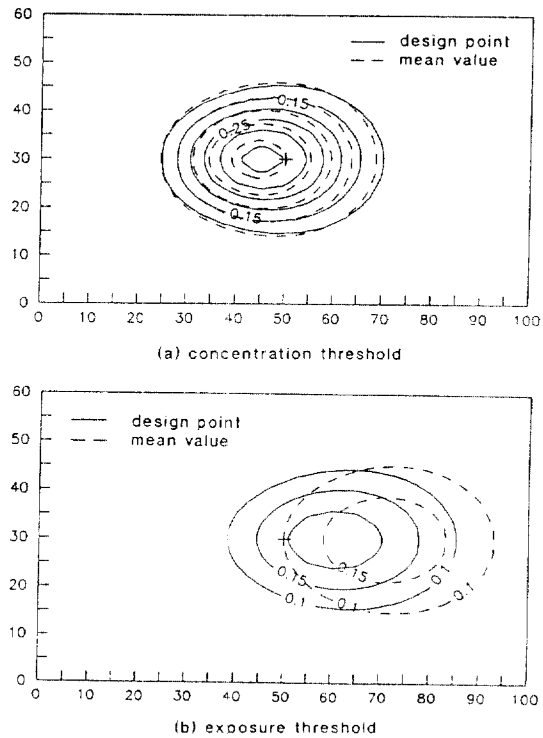

Fig. 3. Comparison of design point concentration profile with deterministic mean concentration profile

The design point concentration profiles for the exposure time analysis are shown in Fig. 3b. The mean time interval during which the concentration reached or exceeded 0.2 C_o begins at t=57.03 days after the start of the contaminant leakage and lasts 65.62 days. At the design point the time interval starts at t=59.05 days and lasts over 75 days. The dashed line indicates the deterministic mean value concentration profile at the last time step and the solid line is the design point concentration profile at t=135 days at which time the concentration becomes less than 0.2 C_o. As can be seen, the design point concentration plume moves less than the mean profile and it is less dispersed in the transverse direction.

Fig. 4a shows contours of K at the design point for the concentration threshold analysis. Design

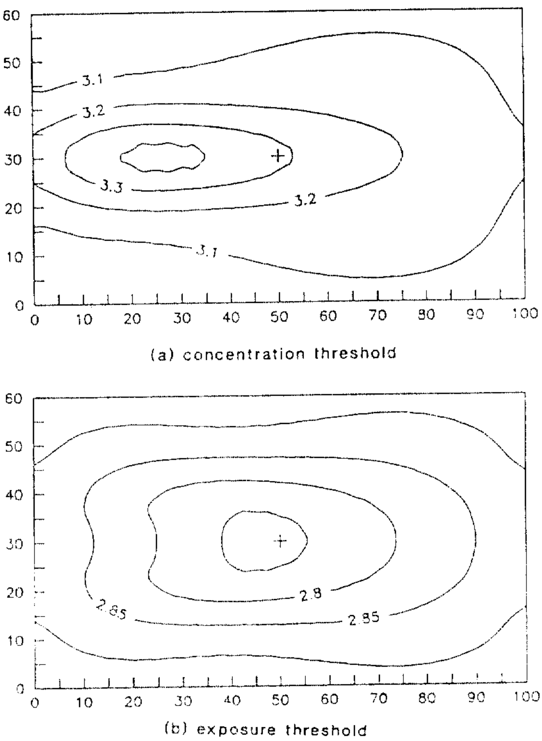

Fig. 4. Contours of K at the design point

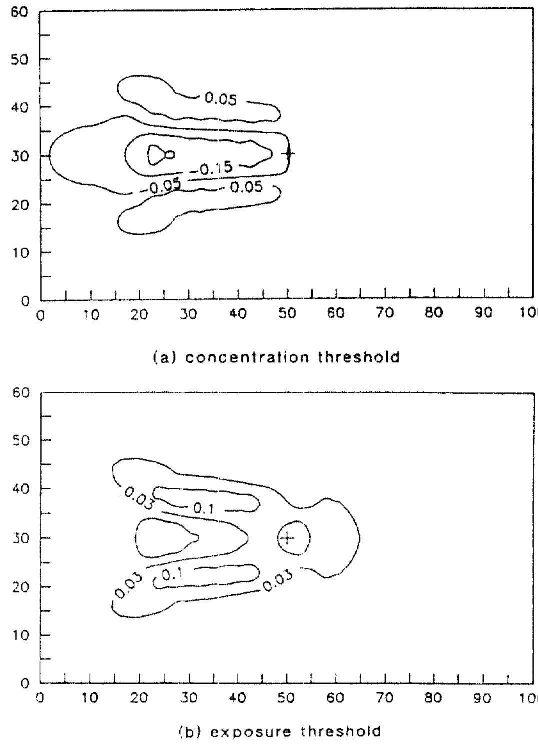

Fig. 5. Local sensitivity of β to μ_K

point values of K are generally higher than the mean of hydraulic conductivity, μ_K, in the entire flow domain and higher K values are located along the contaminant transport path. In contrast, the contours of K at the design point for exposure time threshold analysis (Fig. 4b) show smaller values of K than μ_K in the entire flow domain.

The local sensitivities of β to μ_K and σ_K are shown in Fig. 5 and 6. In the concentration threshold example, local sensitivity of β to K is high along the contaminant travel path (Fig. 5a). High sensitivity shown adjacent to the axis of the contaminant travel path indicates importance of transverse dispersion. Fig. 5b shows the local sensitivity of β to μ_K in the exposure time threshold analysis. Positive sensitivity obtained in the whole domain except in the area adjacent to the source shows that when K values increase, β increases. Highest negative sensitivity of K is distributed adjacent to the center of the contaminant travel path, because transverse dispersion coefficient has

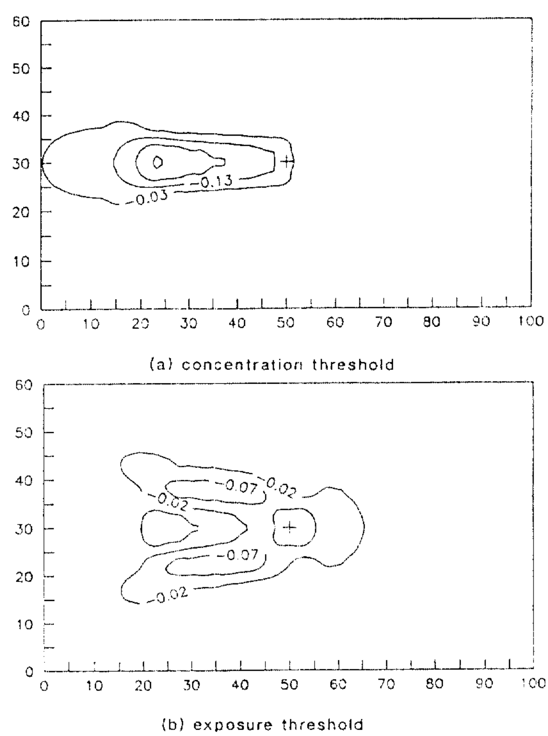

Fig. 6. Local sensitivity of β to σ_K

a significant influence on the probability of exceedence and K smaller than μ_K leads to lesser transverse dispersion.

The local sensitivity of β to σ_K is high along the contaminant travel path for the concentration threshold case(Fig. 6a). Negative sensitivity indicates that an increase in σ_K results in a decrease in β, which means that reducing the uncertainly in the hydraulic conductivity will lead to a reduction in the probability of exceedence, as would be expected. In the exposure time analysis negative sensitivity of β to σ_K is high near the node of interest and adjacent to the axis of the contaminant travel path, indicating that transverse dispersivity and hydraulic conductivity are important parameter in this analysis.

Table 1. Results of Component Reliability Analysis

Case	Model	β	P
Concentration target=0.35 C_0	FORM	3.1576 (tol=0.001)	0.000795
	SORM	3.1848	0.000724
	Monte Carlo	3.2038	0.000678* (c.o.v.=0.5)
Duration time target=75 days	FORM	2.6355 (tol=0.001)	0.004200
	SORM	2.7501	0.003414
	Monte Carlo		N.A.

SORM = Point fit. SORM with Tvedt's exact integral; tol=tolerance in FORM analysis; *6000 simulations

Table 2. Results of System Reliability Analysis

First order bimodal bounds	bounds of β	$2.58<\beta<2.58$
	bounds of P_{sys}	$0.00494<P_{sys}<0.00494$
	ρ_{12}	0.3813
Directional simulation		
first order	β	2.5996
	P_{sys}	0.00466
	c.o.v.	0.0900
second order	β	2.6659
	P_{sys}	0.00384
	c.o.v.	0.1026
Monte Carlo Simulation	β	2.6267
	P_{sys}	0.00431
	c.o.v.	0.1995

ρ_{12}: correlation coeffcient of two failure modes; c.o.v.=coeffcient of variation of P_{sys} in Monte Carlo analysis.

The results of the component reliability analysis are shown in Table 1. The FORM analysis for concentration threshold case converged to P=7.95 (10^{-4}) after 7 iteration with tolerance 0.001. The solution required 1883 g-function calls and CPU time was 22.3 min on Cray X-MP. The number of g-function calls for the SORM analysis was about the same as for the FORM analysis and the CPU time was 23.8 min. The solution of Monte Carlo simulation did not converge to a reliable value after 6,000 simulations. In the exposure time threshold case, the FORM solution converged to tolerance 0.001 after 5 iterations and 1345 g-function calls, and 17.9 min CPU time were needed. The CPU time for SORM analysis was 31.3 min.

6. System Analysis Results

Determining the probability of the union of the two failure events, known as series system analysis, is the next step in this analysis. First- order bimodal reliability bounds and directional simulation with first- and second-order limit state surfaces are performed and the results were compared with those of Monte Carlo simulation (Table 2). All three methods give P_{sys} of the system greater than the P of the individual components. According to the results of the first-order bimodal reliability bounds, the upper and lower bounds are practically identical, which is often the case if P_{sys} is small. The correlation coefficient between the two failure modes is 0.3813.

P_{sys} obtained from the directional simulation with the first order limit state surface is slightly higher than that obtained with the second order limit state surface. The results of the Monte Carlo simulation are closer to those of the directional simulation, although P_{sys} of Monte Carlo simulation has not converged to a value with sufficiently

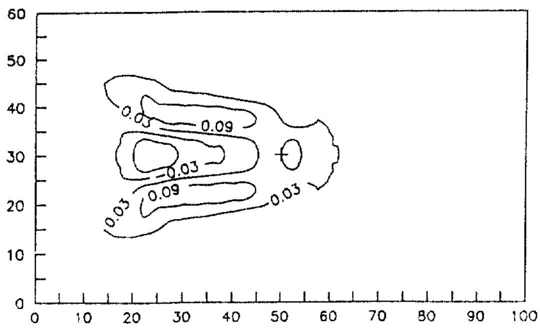

Fig. 7. Local system sensitivity of β to μ_K

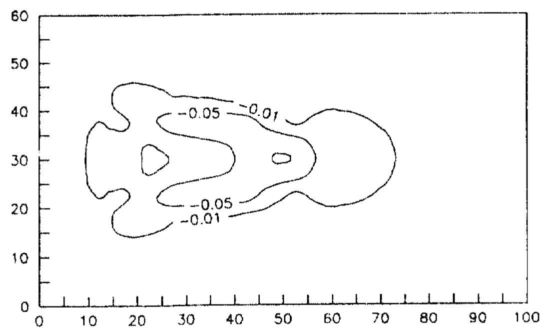

Fig. 8. Local System Sensitivity of β to σ_K

small c.o.v. The results of Monte Carlo simulation were obtained after 6,000 simulation of both limit state functions and the total CPU time spent was 138.6 min on a Cray X-MP which is greater than total CPU time spent for the FORM and SORM analysis of the two limit state functions(96.4 min). The CPU time spent for the directional simulation methods with the first and the second-order limit state surface is negligible(less than or equal to 1 min) compared to that of FORM and SORM, because analytical solutions for finding the roots of the limit state surface are available.

The contours of local sensitivity of β_{sys} to distribution parameters μ_K and σ_K can be obtained using the first order bimodal bounds (Fig. 7 and 8). Contours of sensitivity of β_{sys} to both distribution parameters obtained at the upper bound of the bimodal bounds reflect the corresponding sensitivities of both components. However, the influence of the component which has a larger probability of exceeding the threshold, i.e. the exposure time limit state in this analysis, is more significant.

7. Conclusions

Series system reliability methodology is presented and an example reliability analysis is performed using a transient source transport model in a horizontal two dimensional domain. Two limit state functions, (1) concentration threshold and (2) duration time threshold, are used in this analysis. For concentration threshold analysis, transverse dispersivity turns out to be an important parameter in addition to hydraulic conductivity. This is because dispersion, especially in the transverse direction apparently plays a major role when threshold concentration is greater than the maximum concentration obtained from a mean value solution. This trend is similar for duration time threshold case. Sensitivities of hydraulic conductivities located adjacent to the axis of the plume and near the node of interest are high because of the significance of transverse dispersion. The results of the series system reliability analysis give probabilities of exceedence greater than those of each component and they tend to be dominated by the component with larger probability. System sensitivity is found to reflect the corresponding sensitivity of both components, with the component with larger probability having a greater influence.

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