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Optimal Sampling Plans of Reliability Using the Complex Number Function in the Complex System.

Chung Hwan Oh *
Jong Chul Lee **
Nam Ho Cho ***

Abstract

This paper represents the new techniques for optimal sampling plans of reliability applying the mathematical complex number(real and imaginary number) in the complex system of reliability.

The research formulations represent a mathemtical model which preserves all essential aspects of the main and auxiliary factors of the reserch objetctives. It is important to formule the problem in good agreement with the objective of the research considering the main and auxilary factors which affect the system performance. This model was repeatedly tested to determine the required statistical chatacteristics which in themselves determine the actual and standard distributions. The evaluation programs and techniques are developed for establishing criteria for sampling plans of reliability effectiveness, and the evaluation of system performance was based on the complex stochastic process(derived by the Runge-Kutta method, by kolmogorv's criterion and the transform of a solution to a Sturon-Liouville equation.)

The special structure of this mathmatical model is exploited to develop the optimal sampling plans of reliability in the complex system.

* Samil Coopers & Lybrand
* * Dept. Shoe-design Eng., Osan Junior College
* * * Dept. of I.E., col. of Eng., Kon-Kuk Univ.

1. Introductiom

The formulation of a statistical model is the basic condition of obtaining objective results. The statistical computerized simulation with the aid of finite difference and computer analysis guided a fine solution of the research problem. The distribution of the data corresponds rather accurately to the given curves, which are the inverses of the functions of the theoretical dirstribution based on the formalized approach.

The variability and effectiveness of product varies with the degree of defective level within the inspected product and the probability of detection of these measures is not same. If these levels are broken into groups by statistical and logical considerations, a method can be established for determining the product variability and effectiveness within these groups, evaluation programs and techniques may be developed for establishing criteria for sampling plans of reliability effectiveness.

Thus the more realistic sampling plans, the product variability and the effectiveness of fair decision may be developed.

These mentioned above are the issue of main research.

For the new techniques, computerized statistical treatment of formulation by the Runge-Kutta method, the transform of a solution to a Sturon-Liouville equation and Laplace transform could ve derived.

Notation

R : system reliability

$$R = \frac{Y_n}{Y'_n} = \frac{Y_n - Y_{n-1}}{Y'_n - Y_{n-1}}$$

Y_N : performance of system

Y_n : performance of system subgroups at component n

Y'_n : the condition to optimize subgroups $Y'_n = mX_g + b$. (m, b is constant)

X_g : performance of system subsystems at group g

l : fraction of components between specified limits for specified number of components

Y_o : performance of groups

X_s : performance of subsystems

Y_s : performance of subgroups

N : number of components

Φ : specified s-confidence limits

f : a funtion of D/t

D : fraction of defective unit

t : life time

h : a function of W/t

W : a function of sample size and standard error

λ a function of 1 and Φ

α a function of W and Φ

β a function of D and Φ

2. Derivation of Formulated Model in Complex System Reliability.

In complex system components into subgruops, subgroups into groups of subsystem, and the subsystems into a system whose performance depends on the characteristics of its subgrouping all the way down to the lowest component. Variations in these components are inavoidable causes to the tolerences of different components in various conditions.

Refering to a complex system in service, input reliability is the initial reliability of the system, output reliability is the deteriorated value of the initial reliability at the end of servise time. That is, input of conserved quantity into a system minus outputs of conserved quantity from the system equals accumulation of conserved quantity in the system. We can rewrite

$$\begin{bmatrix} \text{input} = 1Y_{n-1} + \Phi X_{g-1}, \\ \text{output} = fY_n + hX_g = f(D/t)Y_n + h(W/t)X_g \\ \text{input} = \text{output} \end{bmatrix}$$

These can be analyzed using finite difference and the method of undetermined coefficients.

$$1(Y_{n-1} + Y_n) + \Phi(X_{g+1} - X_g) = W\frac{\partial X_g}{\partial t} + \Phi\frac{\partial Y_n}{\partial t} \quad \cdots\cdots (1)$$

The boundery conditions are :

$$\begin{aligned} &X_g = X_s, && t=0 \\ &Y_n = mX_s + b = Y_s, && t=0 \\ &X_{g+1} = X_s, && t>0 \quad \cdots\cdots (2) \\ &Y_{n=0} = Y_o && t>0 \end{aligned}$$

for the condition to optimize subgroups with subsystems

$Y'_n = mX_g + b$, substituting into (1) we get :

$$1(Y_{n-1} - Y_n) + \frac{\Phi}{m}(Y'_{n-1} - Y'_n) = \frac{\partial}{\partial t}\left(\frac{W}{m}Y'_n + DY_n\right) \cdots\cdots (3)$$

Combine $R=\frac{Y_n}{Y'_n}=\frac{Y_n-Y_{n-1}}{Y'_t-Y_{n-1}}$ and(3) :

$$1(Y_{n-1}-Y_n)+\frac{\Phi}{mR}(Y_{n+1}-Y_n)+(R-1)(Y_n-Y_{n-1})$$
$$=\frac{\partial}{\partial}\left[\frac{W}{mR}(Y_n+(R-1)\quad Y_{n-1})+DY_n\right]\cdots\cdots(4)$$

Considering the system failure rate λ, producer risks α and consumer risks β, and also taking into account the boundary condition of(2) and rearrangement the terms by using the Laplas transformation with respect to time of(4)is :

$$Y_{n+1}+(\beta+\alpha R)Y_s+[\lambda+1-R+\alpha(R-1)P]Y_{n-1}$$
$$-[\lambda+2-R+(\alpha'+\beta')P]Y_n=0\cdots\cdots(5)$$

Here P is based on the binomial plans and the solution is developed by using the complex number : that is,

$$\overline{Y_n}=A[1/2(2+\lambda-R+(\alpha+\beta)P)+i\sqrt{4(1+\lambda-R-\alpha(R-1)P)-(2+\lambda-R+(\alpha+\beta)P)^2}\;]^n$$
$$+B[1/2(2+\lambda-R+(\alpha+\beta)P)+i\sqrt{4(1+\lambda-R-\alpha(R-1)P)-(2+\lambda-R+(\alpha+\beta)P)^2}\;]^n$$
$$+Y_s/P\cdots\cdots(6)$$

Here A and are constants. If n=1, g=1, then(1) becomes

$$1(Y_0-Y_1)+\Phi(X_2-X_1)=\frac{\partial}{\partial t}(WX_1+\Phi Y_1)\cdots\cdots(7)$$

Replace the X term in (7) by means of R and $Y'_n=mX_n+b$, taking into account that Y_0 is constant. Then (7) becomes

$$(1+\lambda-R)Y_0-(2+\lambda-R)Y_1+Y_2=(\alpha+\beta)\frac{\partial Y_1}{\partial t}\cdots\cdots(8)$$

The Laplas transform of (8) is :

$$\overline{Y_2}-[\lambda+2-R+(\alpha+\beta)P]\overline{Y_1}+\frac{1+\lambda-R}{p}Y_0+(\alpha+\beta)Y_s=0\cdots\cdots(9)$$

By use of complex numbers, let $Z_1=1/2(c+i\sqrt{4d-c^2})$, $Z_2=1/2(c-i\sqrt{4d-c^2})$, $C=\lambda+2-R+(\alpha'+\beta')P$
$d=\lambda+1-R-\alpha'(R-1)P$, and hence become complex conjugates.

$Z_1 = 1/2(c + \sqrt{4d-c^2}) \equiv r(\cos\theta + i\ \sin\ \theta)$

$Z_2 = 1/2(c - \sqrt{4d-c^2}) \equiv r(\cos\theta - i\ \sin\ \theta)$

$r = \sqrt{d}$

$C = 2\sqrt{d}\ \cos\ \theta \equiv 2r\ \cos\theta.$

Then Equation(6) becomes

$$\overline{Y_n} = d^{n/2}[(A+B)\cos\ n\theta + i(A-B)\sin\ n\theta] + \frac{Y_s}{P} \quad \cdots\cdots (10)$$

$$A+B = \frac{(1+\lambda-R)(Y_0-Y_1)}{dP} \quad \cdots\cdots (11)$$

$$A-B = i(A+B)\frac{\sqrt{d}\ \cos(N+1)\theta + (R-1)\ \cos N\theta}{\sqrt{d}\ \sin(N+1)\theta + (R-1)\ \sin N\theta} \quad \cdots\cdots (12)$$

Then Eq. (10) is

$$\overline{Y_n} = ([(1+\lambda-R)(Y_0-Y_1)d^{(n-2)/2}] + \{(\cos\ n\theta)\ [d^{1/2}\sin(n+1)\theta + (R-1)\sin\ N\theta] - (\sin\ n\theta)\ [d^{1/2}\cos(n-1)\theta + (R-1)\cos\ N\theta]\} + Y_s[d^{1/2}\sin(n+1)\theta + (R-1)\sin\ N\theta])/P[d^{1/2}\sin(N+1)\theta + (R-1)\sin\ N\theta] \quad \cdots\cdots (13)$$

If n=N, then(13) becomes

$$\overline{Y_n} = ([(1+\lambda-R)(Y_0-Y_s)d^{(n-1)/2}\sin\theta] + Y_s[d^{1/2}\sin(N+1)\theta + (R-1)\sin\ N\theta] / P[d^{1/2}\sin(N+1)\theta + (R-1)\sin\ N\theta] \quad \cdots\cdots (14)$$

Assuming order K(p) is at least one greater than that of j(p), then for k(p)=0. p=0, $d^{1/2}\sin(N+1)\theta + (R-1)\sin\ N\theta = 0$.

The transform of a solution to a Sturm-Liouville equation is analytic for all finite P except for those which correspond to the eigenvalues of the system. The residue of the analytic function at P=O is $P_o(t) = j(o)/k(o)$, and

$\mu'(0) = [d^{1/2}\sin(N+1)\theta + (R-1)\sin\ N\theta]_{p=0}$

Hence

$$\frac{j(0)}{\mu'(0)} = \frac{(Y_0-Y_s)d^N\ (1-d)}{(1-d^{N-1}) + (R-1)(1-d^N)} + Y_s$$

$$= \frac{(Y_0-Y_s)(R-\lambda)(1+\lambda-R)^N}{R-\lambda(1+\lambda-R)^N)} + \lambda_s$$

So, Eq. (14) becomes

$$\frac{Y_N-Y_s}{Y_0-Y_s} = \frac{(R-\lambda)(1+\lambda-R)^N}{R-\lambda(1+\lambda-R)^N} \quad \cdots\cdots (15)$$

If R=1, then

$$\frac{Y_N-Y_s}{Y_0-Y_s}=\frac{(1-\lambda)(\lambda)^N}{1-\lambda^{N+1}}$$

$$Y_N=\frac{[(Y_0-Y_s)d^{(N-2)/2}\sin\theta+Y_s\sin(N+1)\theta]}{P\ \sin\ (N+1)\theta}$$

for p=0 and $\sin(N+1)\theta=0$,

$$d=\lambda$$

$$C=1+\lambda+(\alpha+\beta)P=2\sqrt{\lambda}\ \cos\theta \quad\cdots\cdots(16)$$

The condition $\sin(N+1)\theta=0$ requires

$$\theta=K\pi/(N+1) \quad\cdots\cdots(17)$$

$$K=1,\ 2,\ 3,\ 4,\ \cdots\cdots,\ N+2$$

Denote the values of θ Which satisfy (17) by θ_k and the corresponding values of P by P_k. Then

$$P_k=\frac{2\sqrt{\lambda}\ \cos\theta_k-(1+\lambda)}{\alpha\ +\ \beta} \quad\cdots\cdots(18)$$

In order to keep the complex system on the some function, P_k is simple and θ_k is restricted to the range $\pi > \theta_k=0$ which gives following distribution.

$$\frac{d}{dP}[P\ \sin(N+1)\theta]P_k=[P(N+1)\theta\ \frac{d\theta}{dP}]p_k$$

$$=\frac{-[2\lambda^{1/2}\cos\theta_k-(1+\lambda)]\ (N+1)\ \cos\ (N+1)\theta_k}{2\lambda^{1/2}\sin\theta_k} \quad\cdots\cdots(19)$$

Hence,

$$\Sigma P_k=\sum_{K=0}^{K=N+1}\left\{\frac{(Y_0-Y_s)2\ \lambda^{(N+1)/2}\sin^2\ K\pi/(N+1)}{(-1)^k(N+1)\ [1+\lambda-2\lambda^{1/2}\cos K\pi/(N+1)]}\right\}$$

$$\times\ \{\exp(-[\ \frac{1+\lambda-2\lambda^{1/2}\cos\ K/(N+1)}{\alpha+\beta}\ ]t)\} \quad\cdots\cdots(20)$$

And when R=1, the formulation is, then

$$\frac{Y_N-Y_s}{Y_0-Y_s}=\frac{(1-\lambda)(\lambda)^N}{1-\lambda^{N+1}}$$

$$+\sum_{K=1}^{K=N}\left\{\frac{(-1)^k 2\lambda^{(N+1)/2}\sin^2 K\pi/(N+1)}{(N+1)[1+\lambda-2\lambda^{1/2}\cos K\pi/(N+1)]}\right\}$$

$$\times\left\{\exp(-[1+\lambda-2\lambda^{1/2}\cos K\pi/(N+1)])\frac{t}{\alpha+\beta}\right\} \quad \cdots\cdots (21)$$

The desired solution of $\overline{Y_N}$(Eq. 14) requires only positive values of θ which makes positive. The values of θ are determined by trial and error. From the differntiation of the denominator of Eq.(14), the only term which contributes to the residue is

$$P\frac{d[d^{1/2}\sin(N+1)\theta+(R-1)\ \sin N\theta]}{dP}=P\frac{[(1-R\alpha\sin\theta\ \sin(N+1)\theta]}{2\sin\ d^{1/2}}$$

$$+\frac{(1-R)\ \alpha\ \cos\theta}{\beta d^{1/2}}[(N+1)d^{1/2}\cos(N+1)\theta+(R-1)N\ \cos\ N\theta]$$

$$-(\alpha-\beta)\ [(N+1)d^{1/2}\ \cos(N+1)\theta+(R-1)N\ \cos N\theta]$$

$$=\frac{P_i\ K_i}{2\sin\ d^{1/2}} \quad \cdots\cdots (22)$$

Therefore,

$$\frac{Y_N-Y_s}{Y_0-Y_s}=\frac{(R-\lambda)\ (1+\lambda-R)^N}{R-\lambda\ (1+\lambda-R)^N}=\frac{(R-\lambda)\ (1+\lambda-R)^N}{R-(1+\lambda-R)^N}+$$

$$2(1+\lambda-R)\ \Sigma\ \frac{[\exp(p_i t)]\ (\sqrt{di})^N\ \sin^2\theta_i}{P_i\ K_i} \quad \cdots\cdots (23)$$

These above models which describe a mathematical formulations was repeatedly tested to determine the required statistical characteristics which determine the practical distributions.

The statistical simulation allowed us to standardize to solution of our research for the requred objectives, with the aid of finite differences and computer systems and to represent a mathematically formulated model.

3. Analytic Approach and Application

By use of complex numbers, the solution of equation(5) was equation(6).
If $Z_1=1/2(C+i\sqrt{4d-c^2})$, $Z_2=1/2\ (C-i\sqrt{4d-c^2})$, $C=2+\lambda-R+(\alpha+\beta)P$, $d=1+\lambda-R-\alpha(R-1)P$,

We may make Eq.(6) be the $\overline{Y_n}=A\ Z_1^n+B\ Z_2^n+\frac{Y_s}{P}$

and also $Z = r(\cos\theta \pm i \sin\theta)$ $r = \sqrt{d}$, $C = 2r\cos\theta$.

These complex numbers was used and derived to formule the model of eqation(21) and (23), Which was repeatedly tested to determine the required statistical and standard distribution. It was possible to estimate the required number of observations based on a distribution which allowed more accuracy than required when the suggested values of samples were chosen and used.

Computerized statistical trearments of equation(21) and (23) by the Runge-Kutta method were made. The results from the formalized approach was first tested by Kolmogorv's criterion to evaluate their S-significance. This enabled us to ensure a good fit of all variables and to avoid biased ones as well as to verify the truth of our hypothesis that the samples were randomly drawn from a continuous distribution.

The computation for the distributions and for the inverse line, by the method of correlation and regression, were treated as shown in following example. The example show the relation between sample sizes, the confidence limits, the number of failed units during the sample test, and reliability.

The following table is an example, showing several plans for a sample size 10, 15, 20, 25 failed units μ and confidence limit 95%, 90%

SAMPLE SIZE	CONFIDENCE LIMIT 95%					CONFIDENCE LIMIT 90%				
	$\mu=1$	$\mu=2$	$\mu=3$	$\mu=4$	$\mu=5$	$\mu=1$	$\mu=2$	$\mu=3$	$\mu=4$	$\mu=5$
10	0.74	0.61	0.49	0.39	0.30	0.79	0.66	0.55	0.45	0.36
15	0.83	0.71	0.62	0.57	0.50	0.88	0.75	0.67	0.61	0.56
20	0.90	0.76	0.71	0.65	0.59	0.92	0.81	0.75	0.69	0.65
25	0.92	0.82	0.77	0.71	0.67	0.94	0.86	0.81	0.75	0.71

4. Conclusions

The qualitative and quantitative techniques for reliability sampling plans by use of complex numbers with the statistical simulation in the complex system were described.

As a consequence of logical and statistical considerations together with simulations and basic physical laws, We conclude under the basic assumption that : Input of conserved quantity into a system minus outputs of conserved quantity from the system equals accumulation of conserved in the system. In our research, we identified the reliability of the system as the conserved quantity in the system. An example showed to apply the techniques to detect service life and the effectiveness of reliability. Table showed the relation between sample size(10, 15, 20, 25 units), failed Units($\mu=1$, $\mu=2$, $\mu=3$, $\mu=4$, $\mu=5$,), confidence limits 95% and 90%, and the fraction of reliability. The evaluation of system performance was based on the complex stochastic process. This result was then used to derive the main and auxiliary factors by means of the finite element method.

It is expected that more realistic sampling plans and control chart for the required reliability, the life times, the quality control, the tolerance, the product variability and the effectiveness of fair decision could be developed in the future.

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