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A NOTE ON THE CLASS $\mathbf{A}_{1,\aleph_0}$

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1. Introduction

Let $\mathcal{H}$ denote a separable, infinite dimensional, complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators on $\mathcal{H}$. A *dual algebra* is a *weak** closed unital subalgebra of $\mathcal{L}(\mathcal{H})$. Recall that if $\mathcal{A}$ is a dual algebra and m and n are cardinal numbers, where $1 \le m, n \le \aleph_0$, then $\mathcal{A}$ is said to have property $(\mathbf{A}_{m,n})$ if each system of simultaneous equations

$$(1) \qquad [L_{i,j}] = [x_i \otimes y_j], \quad 0 \le i < m, \quad 0 \le j < n$$

in the predual $Q_{\mathcal{A}}$ of $\mathcal{A}$ has a solution $\{x_i : 0 \le i < m\}$, $\{y_j : 0 \le j < n\}$, where x_i and y_j are vectors from $\mathcal{H}$.

Here $[x \otimes y]$ denotes the class in $Q_{\mathcal{A}}$ of the rank-one operator defined by $(x \otimes y)(z) = (z, y)x$, $z \in \mathcal{H}$.

If $\rho > 0$ then $\mathcal{A}$ has property $(\mathbf{A}_{m,n}(\rho))$ if for each $s > \rho$, vectors x_i and y_j can be chosen to satisfy (1), and also the inequalies

$$\|x_i\| < (s \sum_{0 \le j < n} \|[L_{i,j}]\|)^{1/2}, \quad 0 \le i < m$$

and

$$(2) \qquad \|y_j\| < (s \sum_{0 \le i < m} \|[L_{i,j}]\|)^{1/2}, \quad 0 \le j < n$$

It is clear that if m and n are finite cardinals and $\mathcal{A}$ has property $(\mathbf{A}_{m,n}(\rho))$ for some $\rho > 0$, then $\mathcal{A}$ also has property $(\mathbf{A}_{m,n})$. In this note, we are concerned with several classes of contractions appearing in the theory of dual algebras and we continue the study of a geometric

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criterion for membership in the class $\mathbf{A}_{1,\aleph_0}$ (or more precisely, one of the classes $\mathbf{A}_{1,\aleph_0}(\rho)$). The results of this note and [2] are same with different methods. And the following theorem is generalization of [6].

2. Notations and preliminaries

The notation and terminology herein agree with that in [2], [4]. Let $\mathbf{N}$ be the set of positive integers, and let $\mathbf{D}$ be the open unit disc in $\mathbf{C}$. set $\Lambda \subset \mathbf{D}$ is said to be *dominating* for $\mathbf{T} = \partial\mathbf{D}$ if almost every point of $\mathbf{T}$ is a nontangential limit of a sequence of points from Λ. The spaces $L^p = L^p(\mathbf{T})$ and $H^p = H^p(\mathbf{T})$, $1 \le p \le \infty$, are the usual Lebesque and Hardy function spaces relative to normalized Lebesque measure on $\mathbf{T}$.

If $T \in \mathcal{L}(\mathcal{H})$ then $\mathcal{A}_T$ denotes the dual algebra generated by T in $\mathcal{L}(\mathcal{H})$ and Q_T denotes the predual $Q_{\mathcal{A}_T}$ of $\mathcal{A}_T$. If T is also absolutely continuous (i.e., if the maximal unitary direct summand of T is either absolutely continuous or acts on the space(0)),then one knows (cf. 1. Thm 4.1]) that the Sz- Nagy - Foias functional calculus Φ_T is a weak*- continuous, norm-decreasing, algebra homomorphism of H^∞ onto a weak* dense subalgebra of $\mathcal{A}_T$ and $\mathbf{A} = \mathbf{A}(\mathcal{H})$ denotes the class of all absolutely continuous contractions for which the Sz-Nagy-Foias functional calculus Φ_T is an isometry. If $T \in \mathbf{A}$, then it follows easily from general principals that there exists an isometry ϕ_T from Q_T onto L^1/H_0^1 (the predual of H^∞) such that $\phi_T^* = \Phi_T$. If m and n are cardinal numbers, $1 \le m, n \le \aleph_0$, then we define the class $\mathbf{A}_{m,n}$ to be the set of those $T \in \mathbf{A}$ such that the dual algebra $\mathcal{A}_T$ has property $(\mathbf{A}_{m,n})$ and the class $\mathbf{A}_{m,n}(\rho)$ similarly. We recall from [4] that if $\mathcal{M}$ is a *weak**-closed subspaces of $\mathcal{L}(\mathcal{H})$ and $0 \le \theta < 1$, then $\mathcal{E}_\theta^r(\mathcal{M})$ denotes the set of all $[L]$ in $Q_\mathcal{M}$ for which there exist sequences $\{x_n\}$ and $\{y_n\}$ in the closed unit ball of $\mathcal{H}$ satisfying

(a) $\overline{\lim}\|[L] - [x_n \otimes y_n]\| \le \theta$
and

(b^r) $\|[x_n \otimes z]\| \to 0 \quad \forall z \in \mathcal{H}$

(c^r) $\{y_n\}$ converges weakly to zero.

The corresponding subset $\mathcal{E}_\theta^l(\mathcal{M})$ of $Q_\mathcal{M}$ is obtained by replacing conditions (b^r) and (c^r) by

(b^l) $\|[z \otimes y_n]\| \to 0 \quad \forall z \in \mathcal{H}$

(c^l) $\{x_n\}$ converges weakly to zero.

We next recall from [4] that a *weak**-closed subspace $\mathcal{M}$ of $\mathcal{L}(\mathcal{H})$ is said to have property $E^r_{\theta,\gamma}$ (for some $0 \leq \theta < \gamma \leq 1$) if the closed absolutely convex hull of the set $\mathcal{E}^r_\theta(\mathcal{M})$ (notation : $\overline{aco}\{\mathcal{E}^r_\theta(\mathcal{M})\}$) contains the closed ball in $Q_\mathcal{M}$ centered at 0 with radius γ ; property $E^l_{\theta,\gamma}$ is defined similarly.

It is well-known fact that every contraction $T \in \mathbf{A}(\mathcal{H})$ has a minimal co-isometric extension $B = B_T \in \mathcal{L}(\mathcal{K})$ that is unique up to untary equivalence. We have under consideration an absolutely continuous contraction T in $\mathcal{L}(\mathcal{H})$ whose minimal coisometric extension B has a Wold Decomposition $B = S^* \oplus R$, where $S \in \mathcal{L}(\mathcal{S})$ is a unilateral shift of some multiplicity and $R \in \mathcal{L}(\mathcal{R})$ is an absolutely continuous unitary operator.

The projection of $\mathcal{K}$ onto $\mathcal{S}$ is denoted by Q, the projection of $\mathcal{K}$ onto $\mathcal{R}$ by A, and the projection of $\mathcal{K}$ onto $\mathcal{H}$ by P.

Thus every vector $x \in \mathcal{K}$ has a unique decomposion $x = Qx + Ax = Qx \oplus Ax$.

PROPOSITION 1 [2. PROPOSITION.2.1]. *Suppose $T \in \mathbf{A}(\mathcal{H})$ and its minimal co-isometric extension $B = S^* \oplus R$ in $\mathcal{L}(\mathcal{K})$.*

Then $B \in \mathbf{A}(\mathcal{K})$, $\Phi_T \circ \Phi_B^{-1}$ is an isometric algebra isomorphism and a weak-homeomorphism from $\mathcal{A}_B$ onto $\mathcal{A}_T$, and $J = \varphi_B^{-1} \circ \varphi_T$ is a linear isometry of Q_T onto Q_B satisfying*

$$J([x \otimes y]_T) = [x \otimes y]_B, \quad x, y \in \mathcal{H},$$

and

$$[x \otimes z]_B = [x \otimes Pz]_B, \quad z \in \mathcal{K}.$$

PROPOSITION 2 [2. PROPOSITION.2.2]. *Suppose that T is an absolutely continuous contraction in $\mathcal{L}(\mathcal{H})$, and $B = S^* \oplus R$ is its minimal co-isometric extension in $\mathcal{L}(\mathcal{S} \oplus \mathcal{R})$ with $\mathcal{R} \neq (0)$.*

Then there exists a Borel set $\sigma \subset \mathbf{T}$ such that $m|\sigma$ is a scalar spectral measure for R. Moreover, $\mathcal{R}$ contains a reducing subspace $\mathcal{R}_0$ for R such that:

(a) $R_0 = R|\mathcal{R}_0$ is unitarily equivalent to multiplication by the position function on $L^2(\sigma)$

(b) if we denote by $\mathcal{R}_0^+$ the subspace of $\mathcal{R}_0$ corresponding to $H^2(\sigma)$ under the unitary equivalence in (a), then $\mathcal{R}_0^+ \subset \overline{A\mathcal{H}}$.

In the case where $\mathcal{M}$ is the dual algebra generated by an absolutely continuous contraction, we consider now the weak property $F^r_{\theta,\gamma}$ and $F^l_{\theta,\gamma}$.

DEFINITION 3 [2. Definition.3.2] *Let T be an absolutely continuous contraction in $\mathcal{L}(\mathcal{H})$ with minimal coisometric extension $B = S^* \oplus R$ in $\mathcal{L}(\mathcal{K})$ and let $\sigma \subset \mathbf{T}$ be as in Proposition 2 (if $\mathcal{R} = (0)$, then $\sigma = \phi$).*

We say that the dual algebra $\mathcal{A}_T$ has property $F^r_{\theta,\gamma}$ (for some $0 \leq \theta < \gamma \leq 1$) if

$$\overline{aco}\{\mathcal{E}^r_\theta(\mathcal{A}_T) \cup \varphi_T^{-1}\{[f] : f \in L^1(\sigma), \|f\| \leq 1\}\}$$

contains the closed ball in Q_T of radius γ centered at the origin. Moreover, we say that $\mathcal{A}_T$ has property $F^l_{\theta,\gamma}$ if $\mathcal{A}_{T^*}$ has property $F^r_{\theta,\gamma}$.

Obviously, we say that if $\mathcal{A}_T$ has property $E^r_{\theta,\gamma}$, then it has property $F^r_{\theta,\gamma}$.

Let A_0 denote the orthogonal projection of $\mathcal{K}$ onto $\mathcal{R}_0$ and let $z \mapsto \{z\}$ denote the isomorphism from $\mathcal{R}_0$ onto $L^2(\sigma(R))$.

The following Lemma is proved in [2].

LEMMA 4. *[2. Proposition.3.4] If $T \in \mathbf{A}(\mathcal{H})$ with minimal coisometric extension $B \in \mathcal{L}(\mathcal{S} \oplus \mathcal{R})$ and $\mathcal{A}_T$ has property $F^r_{\theta,\gamma}$ (for some $0 < \theta < \gamma \leq 1$) .*

Suppose that we are given $0 < \rho < 1$, $N \in \mathbf{N}$, $\{[V_j]_B\}_{j=1}^N \subset Q_B$, $a \in \mathcal{H}$, $\{w_j\}_{j=1}^N \subset \mathcal{S}$, $\{b_j\}_{j=1}^N \subset \mathcal{R}_0$ and positive scalars $\{\mu_j\}_{j=1}^N$, $\{d_s\}_{j=1}^t \subset \mathcal{K}$, $\{z_l\}_{l=1}^r \subset \mathcal{S}$ satisfying

$$\|[V_j]_B - [a \otimes (w_j + b_j)]_B\| < \mu_j, \quad 1 \leq j \leq N.$$

Then there exist $a' \in \mathcal{H}$, $u \in \mathcal{H}$, $\{w_j'\}_{j=1}^n \subset \mathcal{S}$, $\{b_j'\}_{j=1}^N \subset \mathcal{R}_0$ such that

$$\|[V_j]_B - [a' \otimes (w_j' + b_j')]_B\| < (\frac{\theta}{\gamma})\mu_j, \quad 1 \leq j \leq N,$$

$$\|a' - a\| < \frac{3}{\gamma^{1/2}}(\sum_{j=1}^{N} \mu_j)^{1/2},$$

$$\|w_j' - w_j\| < (\mu_j/\gamma)^{1/2}, \quad 1 \le j \le N,$$

$$\|b_j'\| < \frac{1}{\rho}\{\|b_j\| + (\mu/\gamma)^{1/2}\}, \quad 1 \le j \le N,$$

$$|\{A_0 a'\}(e^{it})| \ge \rho |\{A_0(a+u)(e^{it})|, \quad e^{it} \in \mathbf{T},$$

$$\|[u \otimes d_s]\| < \epsilon, \quad 1 \le s \le t,$$

$$\|[(a' - a) \otimes z_l]\| < \epsilon, \quad 1 \le l \le r.$$

3. Main Results

We are now prepared to prove the main result . It's proof follows the main ideas from [5. Lemma 5] and [4. Theorem 4.7].

THEOREM 5.. *Suppose $T \in \mathbf{A}(\mathcal{H})$ with minimal co isometric extension $B \in \mathcal{L}(\mathcal{S} \oplus \mathcal{R})$ and suppose that $\mathcal{A}_T$ has property $F^r_{\theta,\gamma}$ (for some $0 < \theta < \gamma \le 1$) .*

Then for each sequence of element $\{[L_j]_T : j \ge 1\}$ from Q_T such that $\sum \|[L_j]_T\|^{1/2} < \infty$, there exist $\hat{a} \in \mathcal{H}$ and $\{w_j + b_j\}_{j=1}^{\infty} \subset \mathcal{S} \oplus \mathcal{R}$ such that

$$[L_j] = [\hat{a} \otimes P(w_j + b_j)] \quad , \quad j \ge 1,$$

$$\|\hat{a}\| \le \frac{3}{1 - (\theta/\gamma^{1/2})} \cdot \sum_{j \ge 1} \mu_j^{1/2},$$

$$\|w_j\| \le \frac{1}{1 - (\theta/\gamma^{1/2})} \cdot \mu_j^{1/2} \quad , \quad j \ge 1,$$

$$\|b_j\| \le \frac{2}{1 - (\theta/\gamma^{1/2})} \cdot \mu_j^{1/2} \quad , \quad j \ge 1.$$

In particular, $\mathcal{A}_T$ has property $(\mathbf{A}_{1,\aleph_0})$.

proof. Let $\{[L_j]_T\}_{j=1}^{\infty} \subset Q_T$ and let $[V_j]_B = \varphi_B^{-1} \circ \varphi_T([L_j]_T)$ for each positive integer j. Let $\mu_j > 0$ such that $\sum \mu_j^{1/2} < \infty$.

Assume that $\|[V_j]_B\| < \mu_j$, for each j.

Let us denote $\epsilon_{j,k} = \mu_j(\frac{\theta}{\gamma})^k$, for all $j \geq 1, \quad k \geq 0$.

We select a strictly decreasing sequence $\{s_n\}_{n=1}^{\infty}$ of positive numbers such that $s_1 = 1$ and $\lim_{n\to\infty} s_n = \frac{1}{2}$ and let $\rho_n = \frac{s_{n+1}}{s_n}, \quad n \geq 1$.

Let $B : \mathbf{N} \times \mathbf{N} \longrightarrow \mathbf{N}$ be a bijection such that $j \leq j'$ and $k \leq k'$ implies $B(j,k) \leq B(j',k')$.

Let $w_{j,0} = 0$ in $\mathcal{S}$, $b^n_{j,0} = 0$ in $\mathcal{R}_0$ $\forall j \geq 1, n \geq 1$.

We shall construct, by the induction (on the range of B) sequence $\{a_n\} \subset \mathcal{H}$, $\{w_{j,k}\}_{j,k\geq 1} \subset \mathcal{S}$ for $n \geq 1$, finite sequence $\{b^n_{j,k}\}_{B(j,k)\leq n} \subset \mathcal{R}_0$ such that

$$\|[V_j]_B - [a_n \otimes (w_{j,k} + b^n_{j,k})]_B\| < \epsilon_{j,k}, \qquad B(j,k) \leq n, \tag{3}$$

$$\|a_n - a_{n-1}\| < 3\epsilon^{1/2}_{j,k-1}, \qquad \text{for } n = B(j,k) \tag{4}$$

$$\|w_{j,k} - w_{j,k-1}\| < \epsilon^{1/2}_{j,k-1}, \qquad \forall j,k \geq 1 \tag{5}$$

$$\|b^k_{j,k}\| < \frac{1}{\rho_n}\|b^{n-1}_{j,k}\| \qquad \text{if } n > B(j,k) \tag{6}$$

$$\|b^n_{j,k}\| < \frac{1}{\rho_n}\{\|b^{n-1}_{j,k-1}\| + \epsilon^{1/2}_{j,k-1}\} \quad \text{if } n = B(j,k) \tag{7}$$

For $n = 1 = B(1,1)$.

Apply Lemma 4, with $N = 1, a = 0, w_{1,0} = 0 \in \mathcal{S}, 0 = b^1_{1,0} \in \mathcal{R}_0$, there exist $a_1 \in \mathcal{H}, w_{1,1} \in \mathcal{S}, b_{1,1} \in \mathcal{R}_0$ such that

$$\|[V_j]_B - [a \otimes (w_{1,1} + b^1_{1,1})]\| < \epsilon_{1,1}$$

$$\|a_1\| < 3(\mu_1/\gamma)^{1/2}$$

$$\|w_{1,1}\| < \mu_1/\gamma)^{1/2}$$

$$\|b^1_{1,1}\| < \frac{1}{\rho_1}(\mu_1/\gamma)^{1/2}$$

Suppose now that vectors $\{a_1, \cdots, a_n\} \subset \mathcal{H}$, $\{w_{j,k}\}_{B(j,k)\leq n} \subset \mathcal{S}$, and $\{b^n_{j,k}\}_{B(j,k)\leq n} \subset \mathcal{R}_0$ have been chosen so that (3) - (5) are satisfied ;

Let $n+1 = B(p,q)$.

Apply Lemme 4, with $[V_p]$,$a = a_n$, $w = w_{p,q-1}$, $b = b^n_{p,q-1}$, $\rho = \rho_{n+1}, \mu = \epsilon_{p,q-1}$, $\{d_s\} = \{b^n_{j,k}\}_{B(j,k)\leq n}$, $\{z_l\} = \{w_{j,k}\}_{B(j,k)\leq n}$ and $\epsilon > 0$ sufficiently small to obtain $a_{n+1} \in \mathcal{H}$, $w_{p,q} \in \mathcal{S}$, $b^{n+1}_{p,q} \in \mathcal{R}_0$, $u_{n+1} \in \mathcal{H}$ such that

$$\|[V_p]_B - [a_{n+1} \otimes (w_{p,q} + b^{n+1}_{p,q})]_B\| < \epsilon_{p,q},$$

$$\|a_{n+1} - a_n\| < 3\epsilon^{1/2}_{p,q-1},$$

$$\|w_{p,q} - w_{p,q-1}\| < \epsilon^{1/2}_{p,q-1},$$

$$\|b^{n+1}_{p,q}\| < \frac{1}{\rho_{n+1}}\{\|b^n_{p,q-1}\| + \epsilon^{1/2}_{p,q-1}\},$$

$$|\{A_0 a_{n+1}\}(e^{it})| > \rho_{n+1}|\{A_0(a_n + u_{n+1})\}(e^{it})|, \quad e^{it} \in \mathbf{T},$$

$$\|[(a_{n+1} - a_n) \otimes w_{j,k}]\| < \epsilon, \quad for \quad B(j,k) \leq n,$$

$$\|[u_{n+1} \otimes b^n_{j,k}]\| < \epsilon, \quad for \quad B(j,k) \leq n,$$

Let us define for each (j,k) with $B(j,k) \leq n$,

$$\overline{\{b_{j,k}^{n+1}\}}(e^{it}) = \begin{cases} \dfrac{\{A_0(a_n + u_{n+1})\}(e^{it})}{\{A_0(a_{n+1})\}(e^{it})} \cdot b_{j,k}^n(e^{it}) \\ \qquad if \{A_0(a_{n+1})\}(e^{it}) \neq 0 \\ \\ 0 \\ \\ \qquad if \{A_0(a_{n+1})\}(e^{it}) = 0. \end{cases}$$

then $b_{j,k}^{n+1} \in \mathcal{R}_0$, $\|b_{j,k}^{n+1}\| < \frac{1}{\rho_{n+1}}\|b_{j,k}^n\|$

and $[a_{n+1} \otimes b_{j,k}^{n+1}]_B = [(a_n + u_{n+1}) \otimes b_{j,k}^n]_B$ for all (j,k) such that $B(j,k) \leq n$.

For $\epsilon > 0$ sufficiently small,

$$\|[V_j]_B - [a_{n+1} \otimes (w_{j,k} + b_{j,k}^{n+1})]_B\| < \epsilon_{j,k}, \quad if \quad B(j,k) \leq n+1.$$

Therefore (3) -(7) are fulfilled for $n+1$. and from (2),(3), we do obtain Cauchy sequences $\{a_n\}_{n=1}^{\infty}$ and for each $j \geq 1$, $\{w_{j,k}\}_{k=1}^{\infty}$, whose limits $\hat{a}$ and w_j satisfy

$$\begin{aligned} \|\hat{a}\| = \|\sum_{n=1}^{\infty}(a_n - a_{n+1})\| &\leq \sum_{n=1}^{\infty} \|(a_n - a_{n+1})\| \\ &= \sum_{j,k}^{\infty} 3\epsilon_{j,k}^{1/2} = 3\sum_{j=1}^{\infty}\sum_{k=0}^{\infty} \mu_j^{1/2}(\frac{\theta}{\gamma})^{1/2\cdot k} \\ &= \frac{3}{1-(\frac{\theta}{\gamma})^{1/2}} \cdot \sum_{j=1}^{\infty} \mu_j{}^{1/2} \end{aligned}$$

Similarly,

$$\|w_j\| \leq \frac{1}{1-(\frac{\theta}{\gamma})^{1/2}} \cdot \mu_j{}^{1/2} \quad \text{for all } j \geq 1.$$

And from (6), (7), $\{b_{j,k}\}_{k=1}^{\infty}$ is bounded sequence for all $j \geq 1$, where $b_{j,k} = b_{j,k}^{B(j,k)}$ for all $j,k \geq 1$.

Hence we may suppose that $\{b_{j,k}\}_{k=1}^{\infty}$ converges weakly to some $b_j \in \mathcal{R}_0$. By Proposition 1,

$$[L_j]_T = [\hat{a} \otimes P(w_j + b_j)]_T \quad \text{for all } j \geq 1.$$

From (6), (7),

$$\begin{aligned} s_{n+1}\|b_{j,k}\| &\leq s_{B(j,k-1)+1}\|b_{j,k-1}\| + \epsilon_{j,k}{}^{1/2} \\ &\leq s_{B(j,1)+1}\|b_{j,1}\| + \sum_{l=1}^{k-1} \epsilon_{j,l}{}^{1/2} \leq \sum_{l=0}^{k-1} \epsilon_{j,l}{}^{1/2} \\ &= \sum_{l=0}^{k-1} \mu_j{}^{1/2}(\frac{\theta}{\gamma})^{l/2} \leq \mu_j{}^{1/2}\frac{1}{1-(\frac{\theta}{\gamma})} \end{aligned}$$

Letting $n \to \infty$, $s_{n+1} \to \frac{1}{2}$,

$$\|b_{j,k}\| < \frac{2}{1-(\frac{\theta}{\gamma})^{1/2}}\mu_j{}^{1/2}, \quad \textit{for all} \quad j \geq 1 \quad \textit{and} \quad k \geq 0.$$

Hence

$$\|b_j\| \leq \frac{2\mu_j{}^{1/2}}{1-(\frac{\theta}{\gamma})^{1/2}}, \quad \textit{for all} \quad j \geq 1.$$

From the above relations,
$T \in \mathbf{A}_{1,\aleph_0}(\rho)$, where $\rho \leq \frac{3}{1-(\frac{\theta}{\gamma})^{1/2}}$

The following corollary is immediate from [4. Proposition.4.3] and Theorem 5.

COROLLARY. *Suppose* $T \in \mathbf{A}(\mathcal{H})$, $0 \leq \theta < 1$, *and* $\Lambda \subset \mathbf{D}$ *is dominating for* $\mathbf{T}$. *If for each* $\lambda \in \Lambda$, *there exists a sequence* $\{x_n^\lambda\}$ *in the closed unit ball of* $\mathcal{H}$ *such that*

$$\overline{\lim}_n \|[C_\lambda]_T - [x_n^\lambda \otimes x_n^\lambda]_T\| \leq \theta$$

and

$$\|[x_n^\lambda \otimes z]_T\| \longrightarrow 0, \qquad y \in \mathcal{H},$$

then $T \in \mathbf{A}_{1,\aleph_0}(\rho)$, *where* $\rho \leq \frac{3}{1-\theta^{1/2}}$.

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