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FUZZY WEAKLY IRRESOLUTE MAPPINGS

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The concept of a fuzzy set, which was introduced in [9], provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. The idea of fuzzy topological spaces was introduced by Chang [3]. The idea is more or less a generalization of oridinary topological spaces.

In [2], Chae, Dube and Panwar have studied weakly irresolute mappings in topological spaces. In this paper, we generalize the concept of weakly irresolute mappings in fuzzy setting.

Let X and Y be two sets of points. A fuzzy set in X is a mapping from X into the closed unit interval $I = [0,1]$ on the real line. For X, I^X denotes the collection of all mappings from X into I. The union $\cup\lambda_\alpha$ (the intersection $\cap\lambda_\alpha$) of a family $\{\lambda_\alpha\}$ of fuzzy sets in X is defined to be the mapping $\sup\lambda_\alpha(\inf\lambda_\alpha)$. For any two members λ and μ of I^X; $\lambda \leq \mu$ if and only if $\lambda(x) \leq \mu(x)$ for each $x \in X$, and in this case λ is said to be contained in μ, or μ is said to contain λ. 0 and 1 denote constant mappings taking whole of X to 0 and 1, respectively. The complement λ' of a fuzzy set λ in X is $1-\lambda$, defined by $(1-\lambda)(x) = 1-\lambda(x)$ for each $x \in X$.

A fuzzy point p in X is a fuzzy set in X defined by

$$p(x) = \begin{cases} k \in (0,1) & \text{for } x = x_p, \\ 0 & \text{otherwise }, \end{cases}$$

for each $x \in X$, where x_p and k are the support (written x_p=supp p) and the value of p, respectively. A fuzzy point p is said to belong to a fuzzy set λ in X, written $p < \lambda$, iff $p(x_p) < \lambda(x_p)$.

Let $f : X \to Y$ be a mapping. If λ is a fuzzy set in X, then $f(\lambda)$ is a fuzzy set in Y defined by

$$f(\lambda)(y) = \begin{cases} \sup_{z\in f^{-1}(y)} \lambda(z) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise} \end{cases}$$

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for each $y \in Y$. If p is a fuzzy point in X with the support x_p and $p(x_p) = k$, then $f(p)$ is also a fuzzy point in Y defined by

$$f(p)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} p(z) = k & \text{if } y = f(x_p) \\ 0 & \text{otherwise} \end{cases}$$

for each $y \in Y$. If μ is a fuzzy set in Y, then $f^{-1}(\mu)$ is a fuzzy set in X defined by $f^{-1}(\mu)(x) = \mu(f(x))$ for each $x \in X$.

A subfamily τX of I^X is called a fuzzy topology on X ([1, 3]) if (i) 0 and 1 belong to τX, (ii) any union of members of τX is in τX, (iii) a finite intersection of members of τX is in τX.

Members of τX are called fuzzy open sets in X and their complements fuzzy closed sets. A set X with fuzzy topology τX is called a fuzzy topological space, written $(X, \tau X)$ (or shortly, X).

In what follows, $(X, \tau X)$ and $(Y, \tau Y)$ (or shortly X and Y) would mean fuzzy topological spaces unless otherwise specified.

DEFINITION 1 ([1]). Let λ be a fuzzy set in X. The closure $\mathrm{Cl}\lambda$ and the interior $\mathrm{Int}\lambda$ of λ are defined by

$$\mathrm{Cl}\lambda = \inf\{\nu : \nu \geq \lambda,\ \nu' \in \tau X\},$$

and

$$\mathrm{Int}\lambda = \sup\{\nu : \nu \leq \lambda,\ \nu \in \tau X\}.$$

DEFINITION 2 ([1]). Let λ be a fuzzy set in X.

(a) λ is called a fuzzy semi-open set in X if there exists $\nu \in \tau X$ such that $\nu \leq \lambda \leq \mathrm{Cl}\nu$.

(b) λ is called a fuzzy semi-closed set in X if there exists $\nu' \in \tau X$ such that $\mathrm{Int}\nu \leq \lambda \leq \nu$.

LEMMA 1 ([1]). *Let λ be a fuzzy set in X. Then the following are equivalent:*

(a) λ is a fuzzy semi-closed set.

(b) λ' is a fuzzy semi-open set.

(c) $\mathrm{IntCl}\lambda \leq \lambda$.

(d) $\mathrm{ClInt}\lambda' \geq \lambda'$.

DEFINITION 3 ([8]). Let λ be a fuzzy set in X. The semi-closure sClλ and the semi-interior sIntλ of λ are defined by

$$\mathrm{sCl}\lambda = \inf\{\mu : \lambda \leq \mu,\ \mu \text{ is fuzzy semi-closed}\}$$

and

$$\mathrm{sInt}\lambda = \sup\{\mu : \mu \leq \lambda,\ \mu \text{ is fuzzy semi-open}\}.$$

LEMMA 2 ([7]). *Let $f : X \to Y$ be a mapping. Then;*
(a) If $\lambda \leq \mu$ for any $\lambda, \mu \in I^X$, then $f(\lambda) \leq f(\mu)$.
(b) If $\lambda \leq \mu$ for any $\lambda, \mu \in I^Y$, then $f^{-1}(\lambda) \leq f^{-1}(\mu)$.
(c) If $\lambda \in I^X$, then $\lambda \leq f^{-1}(f(\lambda))$.
(d) If $\lambda \in I^Y$, then $f(f^{-1}(\lambda)) \leq \lambda$ and $f^{-1}(\lambda') = (f^{-1}(\lambda))'$.
(e) If $\lambda_i \in I^X$ for each $i \in T$, $f(\cup_{i \in T}\lambda_i) = \cup_{i \in T} f(\lambda_i)$.
(f) If $\lambda_i \in I^Y$ for each $i \in T$, $f^{-1}(\cup_{i \in T}\lambda_i) = \cup_{i \in T} f^{-1}(\lambda_i)$.
(g) If $\lambda_i \in I^Y$ for each $i \in T$, $f^{-1}(\cap_{i \in T}\lambda_i) = \cap_{i \in T} f^{-1}(\lambda_i)$
(h) If f is one to one and $\lambda \in I^X$, then $f^{-1}(f(\lambda)) = \lambda$.
(i) If f is onto and $\lambda \in I^Y$, then $f(f^{-1}(\lambda)) = \lambda$.
(j) Let g be a function from Y to Z. If $\lambda \in I^Z$ and $\mu \in I^X$, then $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$ and $(g \circ f)(\mu) = g(f(\mu))$.
(k) If f is bijective and $\lambda \in I^X$, then $f(\lambda)' = f(\lambda')$.

LEMMA 3 ([8]). *Let λ and μ be fuzzy sets in X satisfying $\lambda \leq \mu$. Then;*
(a) $sCl\lambda \leq sCl\mu$,
(b) $sInt\lambda \leq sInt\mu$,
(c) $\lambda \leq sCl\lambda \leq Cl\lambda$,
(d) $\lambda \geq sInt\lambda \geq Int\lambda$.

PROPOSITION 1. *Let λ be a fuzzy set in X. Then;*

$$1 - sInt\lambda = sCl(1 - \lambda) \text{ and } 1 - sCl\lambda = sInt(1 - \lambda).$$

Proof.

$$\begin{aligned}
&\mathrm{sCl}(1-\lambda)\\
&= \inf\{\mu : 1 - \lambda \leq \mu,\ \mu \text{ is a fuzzy semi-closed set in } X\}\\
&= \inf\{1 - (1 - \mu) : \lambda \geq 1 - \mu,\ 1 - \mu \text{ is a fuzzy semi-open set in } X\}\\
&= 1 - \sup\{1 - \mu : \lambda \geq 1 - \mu,\ 1 - \mu \text{ is a fuzzy semi-open set in } X\}\\
&= 1 - \mathrm{sInt}\lambda
\end{aligned}$$

and

$$\begin{aligned}
&\mathrm{sInt}(1-\lambda)\\
&=\sup\{\mu : 1-\lambda \le \mu,\ \mu \text{ is a fuzzy semi-open set in } X\}\\
&=\sup\{1-(1-\mu) : \lambda \le 1-\mu,\ 1-\mu \text{ is a fuzzy semi-closed set in } X\}\\
&=1-\inf\{1-\mu : \lambda \le 1-\mu,\ 1-\mu \text{ is a fuzzy semi-closed set in } X\}\\
&=1-\mathrm{sCl}\lambda.
\end{aligned}$$

PROPOSITION 2. *If λ is a fuzzy semi-open set in X, then $sCl\lambda$ is also a fuzzy semi-open set in X.*

Proof. Since λ is a fuzzy semi-open set in X, there exists a fuzzy open set μ in X such that $\mu \le \lambda \le \mathrm{Cl}\mu$. Then by Lemma 3, we have $\mu \le \mathrm{sCl}\mu \le \mathrm{sCl}\lambda \le \mathrm{Cl}\mu$. So, $\mathrm{sCl}\lambda$ is a fuzzy semi-open set in X.

In [2], Chae, Dube and Panwar defined the notion of a weakly irresolute mapping in topological spaces as follows;

DEFINITION 4. A mapping $f : X \to Y$ is called a weakly irresolute mapping if for each $x \in X$ and each semi-nbd $V \subset Y$ of $f(x)$, there exists a semi-nbd U of x such that $f(U) \subset \mathrm{sCl}(V)$.

We generalize the above definition in fuzzy setting.

DEFINITION 5. A mapping $f : X \to Y$ is called a fuzzy weakly irresolute mapping if for each fuzzy point p in X and each fuzzy semi-open set λ in Y satisfying $f(p) < \lambda$, there exists a fuzzy semi-open set μ in X such that $p < \mu$ and $f(\mu) \le \mathrm{sCl}\lambda$

THEOREM 1. *Let $f : X \to Y$ be a mapping. Then the following are equivalent:*

(a) f is fuzzy weakly irresolute.
(b) For any fuzzy semi-open set λ in Y, $f^{-1}(\lambda) \le sInt(f^{-1}(sCl\lambda))$.
(c) For any fuzzy semi-closed set λ in Y, $sCl(f^{-1}(sInt\lambda)) \le f^{-1}(\lambda)$.
(d) For any fuzzy semi-open set λ in Y, $sCl(f^{-1}(\lambda)) \le f^{-1}(sCl\lambda)$.

Proof. (a)$\Longrightarrow$(b) Let λ be any fuzzy semi-open set in Y and $p < f^{-1}(\lambda)$. Then by Lemma 2, $f(p) < f(f^{-1}(\lambda)) \le \lambda$. Also, since f is a fuzzy weakly irresolute mapping, there exists a fuzzy semi-open set λ in X such that $p < \mu$ and $f(\mu) \le \mathrm{sCl}\lambda$. This implies $p < \mu \le f^{-1}(\mathrm{sCl}\lambda)$

and so by Lemma 3, $p < \mu = \mathrm{sInt}\mu \leq \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda))$. Thus, we have $f^{-1}(\lambda) \leq \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda))$.

(b)$\Longrightarrow$(a) Let p be a fuzzy point in X and λ a fuzzy semi-open set in Y satisfying $f(p) < \lambda$. Then by (a), we have

$$p < f^{-1}(\lambda) \leq \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda)).$$

Putting $\mu = \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda))$, then μ is a fuzzy semi-open set in X satisfying $p < \mu$. Thus, by Lemma 2,

$$f(\mu) \leq f(f^{-1}(\mathrm{sCl}\lambda)) \leq \mathrm{sCl}\lambda.$$

(b)$\Longrightarrow$(c) Let λ be a fuzzy semi-closed set in Y. Then λ' is a fuzzy semi-open set in Y and by (b), we have $f^{-1}(\lambda') \leq \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda'))$. Now, by Lemma 2 and Proposition 1, we also have

$$\begin{aligned}\{f^{-1}(\lambda)\}' = f^{-1}(\lambda') &\leq \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda')) \\ &= \mathrm{sInt}(f^{-1}((\mathrm{sInt}\lambda)')) = \mathrm{sInt}(f^{-1}(\mathrm{sInt}\lambda))' \\ &= \{\mathrm{sCl}(f^{-1}(\mathrm{sInt}\lambda))\}',\end{aligned}$$

so that $\mathrm{sCl}f^{-1}(\mathrm{sInt}\lambda) \leq f^{-1}(\lambda)$.

By the same method, we conclude that (c)$\Longrightarrow$(b).

(c)$\Longrightarrow$(d) Let λ be a fuzzy semi-open set in Y. Then $\mathrm{sCl}\lambda$ is a fuzzy semi-closed set and by Lemma 3, $\lambda \leq \mathrm{sInt}(\mathrm{sCl}\lambda)$. Also, by (c), Proposition 2 and Lemma 2, we have

$$f^{-1}(\mathrm{sCl}\lambda) \geq \mathrm{sCl}(f^{-1}(\mathrm{sInt}(\mathrm{sCl}\lambda)))$$

$$= \mathrm{sCl}f^{-1}(\mathrm{sCl}\lambda)) \geq \mathrm{sCl}(f^{-1}(\lambda))$$

(d)$\Longrightarrow$(c) Let λ be a fuzzy semi-closed set in Y. Then $\mathrm{sInt}\lambda$ is a fuzzy semi-open set in Y and by Lemma 3, $\mathrm{sCl}(\mathrm{sInt}(\lambda)) \leq \lambda$. So, by Lemma 2 and (d), we have $\mathrm{sCl}(f^{-1}(\mathrm{sInt}\lambda)) \leq f^{-1}(\mathrm{sCl}(\mathrm{sInt}\lambda)) \leq f^{-1}(\lambda)$.

DEFINITION 6 ([1]). A fuzzy set λ in X is called (i) a fuzzy regular open set in X if $\mathrm{IntCl}\lambda = \lambda$, and (ii) a fuzzy regular closed set in X if $\mathrm{ClInt}\lambda = \lambda$.

LEMMA 4 ([1]).

(a) The closure of a fuzzy open set is a fuzzy regular closed set.

(b) The interior of a fuzzy closed set is a fuzzy regular open set.

PROPOSITION 3. *Let λ be a fuzzy set in X. Then;*

$$sCl\lambda = \lambda \cup IntCl\lambda.$$

Proof. Let λ be a fuzzy set in X. Then $\mathrm{sCl}\lambda$ is a fuzzy semi-closed set. By Lemmas 1 and 3,

$$\mathrm{IntCl}\lambda \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{sCl}\lambda)) \leq \mathrm{sCl}\lambda.$$

This implies $\lambda \cup \mathrm{IntCl}\lambda \leq \mathrm{sCl}\lambda$.

Conversely, it is clear that $\mathrm{IntCl}\lambda \leq \lambda \cup \mathrm{IntCl}\lambda \leq \mathrm{Cl}\lambda$. Hence, by Definition 2, we know that $\lambda \cup \mathrm{IntCl}\lambda$ is a fuzzy semi-closed set and so $\mathrm{sCl}\lambda \leq \lambda \cup \mathrm{IntCl}\lambda$. Thus we have $\mathrm{sCl}\lambda = \lambda \cup \mathrm{IntCl}\lambda$.

DEFINITION 7. Let λ be a fuzzy set in X. Then λ is called a fuzzy semi-regular open set in X if it is both fuzzy semi-open and fuzzy semi-closed.

REMARK. Following Proposition 2, we know that $\mathrm{sCl}\lambda$ is a fuzzy semi-regular open set for a fuzzy semi-open set λ.

PROPOSITION 4. *Let λ be a fuzzy set in X. Then the following are equivalent:*

(a) λ is fuzzy semi-regular open.

(b) $\lambda = sInt(sCl\lambda)$.

(c) there exists a fuzzy regular open set μ in X such that $\mu \leq \lambda \leq Cl\mu$.

Proof. (a)$\Longrightarrow$(b) If λ is fuzzy semi-regular open, then

$$\mathrm{sInt}(\mathrm{sCl}\lambda) = \mathrm{sInt}\lambda = \lambda.$$

(b)$\Longrightarrow$(c) Suppose $\lambda = \mathrm{sInt}(\mathrm{sCl}\lambda)$. By Proposition 3, $\mathrm{IntCl}\lambda \leq \mathrm{sCl}\lambda$ and so by Lemma 3, $\mathrm{IntCl}\lambda \leq \mathrm{sInt}(\mathrm{sCl}\lambda) = \lambda$. Also, since λ is fuzzy semi-open, $\lambda \leq \mathrm{ClInt}\lambda$. By Lemmas 1 and 3, we have $\mathrm{IntCl}\lambda \leq \lambda \leq \mathrm{ClInt}\lambda \leq \mathrm{ClIntCl}\lambda$. Taking $\mu = \mathrm{IntCl}\lambda$, we have that μ is fuzzy regular open.

(c)$\Longrightarrow$(a) Clearly, λ is a fuzzy semi-open set. By (c), $\mathrm{Cl}\lambda = \mathrm{Cl}\mu$ and so $\mathrm{IntCl}\lambda = \mathrm{IntCl}\mu = \mu \leq \lambda$. By Proposition 3, $\mathrm{sCl}\lambda = \lambda \cup \mathrm{IntCl}\lambda = \lambda$. and hence λ is fuzzy semi-closed.

THEOREM 2. *Let $f : X \to Y$ be a mapping. Then the following are equivalent:*

(a) f is fuzzy weakly irresolute.

(b) If λ is a fuzzy semi-regular open set in Y, then $f^{-1}(\lambda)$ is a fuzzy semi-regular open set in X.

Proof. Let λ be a fuzzy semi-regular open set in Y. Since f is fuzzy weakly irresolute,

$$\mathrm{sCl}(f^{-1}(\lambda)) \le f^{-1}(\mathrm{sCl}\lambda)$$

and

$$f^{-1}(\lambda) \le \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda)).$$

By Proposition 2 and Lemma 3,

$$\begin{aligned}\mathrm{sInt}(\mathrm{sCl}(f^{-1}(\lambda))) &\le \mathrm{sCl}(f^{-1}(\lambda)) \le f^{-1}(\mathrm{sCl}\lambda) = f^{-1}(\lambda)\\ &\le \mathrm{sInt}(f^{-1}(\mathrm{sCl}\lambda)) \le \mathrm{sInt}(\mathrm{sCl}(f^{-1}(\mathrm{sCl}\lambda)))\\ &= \mathrm{sInt}(\mathrm{sCl}(f^{-1}(\lambda))).\end{aligned}$$

Thus we have $f^{-1}(\lambda) = \mathrm{sInt}(\mathrm{sCl}(f^{-1}(\lambda)))$ and so $f^{-1}(\lambda)$ is a fuzzy semi-regular open set.

Conversely, let p be a fuzzy point in X and λ a fuzzy semi-open set in Y satisfying $f(p) < \lambda$. Then $\mathrm{sCl}\lambda$ is a fuzzy semi-open set and a fuzzy semi-closed set and so $\mathrm{sCl}\lambda$ is a fuzzy semi-regular open set in Y. By hypothesis, $f^{-1}(\mathrm{sCl}\lambda)$ is a fuzzy semi-open in X satisfying $p < f^{-1}(\mathrm{sCl}\lambda)$. Putting $\mu = f^{-1}(\mathrm{sCl}\lambda)$, by Lemma 2, $f(\mu) = f(f^{-1}(\mathrm{sCl}\lambda)) \le \mathrm{sCl}\lambda$.

DEFINITION 8 ([8]). A mapping $f : X \to Y$ is a fuzzy irresolute mapping if for any fuzzy semi-open set λ in Y, $f^{-1}(\lambda)$ is a fuzzy semi-open set in X

REMARK 1. Clearly, every fuzzy irresolute mapping is a fuzzy weakly irresolute mapping. But a fuzzy weakly irresolute mapping need not be fuzzy irresolute as shown in the following example.

EXAMPLE 1. Let λ and μ be fuzzy sets in the unit closed interval I defined by $\lambda(x) = \frac{1}{3}(x)$ and $\mu(x) = \frac{2}{3}(x)$, respectively. Consider fuzzy topologies $\tau_1 I = \{0, 1, \lambda\}$, and $\tau_2 I = \{0, 1, \mu\}$ on I, and take $f : (I, \tau_1 I) \to (I, \tau_2 I)$ defined by $f(x) = x$ for each $x \in I$. We will show that f is fuzzy weakly irresolute, but is not fuzzy irresolute. Let η be a fuzzy semi-open set in $(I, \tau_2 I)$. Then η =0 or $\eta \geq \mu$. So, we have

$$f^{-1}(\mathrm{sCl}\eta) = \begin{cases} 0 & \text{if } \eta=0 \\ 1 & \text{if } \eta \geq \mu. \end{cases}$$

Also, in the case $\eta = 0, sClf^{-1}(\eta) = 0$. Thus, $\mathrm{sCl}f^{-1}(\eta) \leq f^{-1}(\mathrm{sCl}\eta)$ and f is fuzzy weakly irresolute. While, let ν be a fuzzy set defined by $\nu(x) = \frac{2}{3}x + \frac{1}{3}$. Then ν is a fuzzy semi-open set in $(I, \tau_2 I)$. But, $f^{-1}(\nu) = \nu \circ f = \nu$ is not a fuzzy semi-open set in $(I, \tau_1 I)$ and hence f is not fuzzy irresolute.

LEMMA 5 ([8]). *Let $f : X \to Y$ be a mapping. Then the following are equivalent:*

(a) f is fuzzy irresolute,

(b) For each fuzzy point p in X and each fuzzy semi-open set λ in Y satisfying $f(p) \lessdot \lambda$, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu \leq f^{-1}(\lambda)$,

(c) For each fuzzy point p in X and each fuzzy semi-open set λ in Y satisfying $f(p) \lessdot \lambda$, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \lambda$.

DEFINITION 9. A fuzzy space X is said to be fuzzy strongly semi-regular if for each fuzzy point p in X and each fuzzy semi-open set ν satisfying $p \lessdot \nu$, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu \leq \mathrm{sCl}\mu \leq \nu$.

THEOREM 3. *Let $f : X \to Y$ be a fuzzy weakly irresolute mapping. If Y is fuzzy strongly semi-regular, then f is fuzzy irresolute.*

Proof. Let p be a fuzzy point in X and let λ be a fuzzy semi-open set in Y satisfying $f(p) \lessdot \lambda$. Since Y is fuzzy strongly semi-regular, there exists a fuzzy semi-open set μ in Y such that $f(p) \lessdot \mu \leq \mathrm{sCl}\mu \leq \lambda$. Also since f is fuzzy weakly irresolute, there exists a fuzzy semi-open set ν in X such that $p \lessdot \nu \leq f^{-1}(\mathrm{sCl}\mu) \leq f^{-1}(\lambda)$. Hence by Lemma 5, f is fuzzy irresolute.

THEOREM 4. *Let $f : X \to Y$ and $g : Y \to Z$ be mappings. If f is fuzzy weakly irresolute and g is fuzzy weakly irresolute, then $g \circ f$ is fuzzy weakly irresolute.*

Proof. Let p be a fuzzy point in X and λ a fuzzy semi-open set in Z satisfying $(g \circ f)(p) = g(f(p)) \lessdot \lambda$. Since g is fuzzy irresolute, there exists a fuzzy semi-open set ν in Y such that $f(p) \lessdot \nu$ and $g(\nu) \leq \mathrm{sCl}\lambda$. Also, since f is fuzzy weakly irresolute, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \mathrm{sCl}\nu$. So, by Theorem 1, we have

$$\begin{aligned}\mu &\leq f^{-1}(\mathrm{sCl}\nu) \leq f^{-1}(\mathrm{sCl}(g^{-1}(\mathrm{sCl}\lambda))) \\ &\leq f^{-1}(g^{-1}(\mathrm{sClsCl}\lambda)) = f^{-1}(g^{-1}(\mathrm{sCl}\lambda)) \\ &= (g \circ f)^{-1}(\mathrm{sCl}\lambda).\end{aligned}$$

Thus $g \circ f$ is fuzzy weakly irresolute.

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