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The Impact of Reliability Growth on Spares Provisioning

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Abstract

Reliability growth modeling can be a requirement when bidding on large military hardware systems. Under current reliability warranty legislature, the reliability growth model can be later translated into necessary reliability performance which must be demonstrated over an extended period of usage. In this paper the modeling situation is concerned with determining the number of spares needed to support a projected reliability growth both at the flightline and in a depot inventory. The model differs from existing models for logistics planning in that we allow for the phenomena of reliability growth. The model can also be used to determine central depot staffing requirements based upon a specified system utilization.

1. Introduction

Under current reliability warranty legislature, military contractors are required to provide warranties on parts and materiels specifically against product failure. Coupling the reliability warranty requirement for military hardware, written guarantees should be provided in connection with the procurement of spare parts. Therefore, the reliability group must be well aware of what the equipment will actually demonstrate over an extended period of time including growth (or improvement) in systems reliability. The concept of reliability growth recognizes that increased usage will identify product deficiencies through failures and as the deficiencies are corrected, the product slowly improves resulting in reliability growth. In expensive military materiels with complex hardware subsystems and with even moderate volumes of production, the failure to perform from a reliability standpoint could be an extremely expensive proposition. This requires considerable diligence in the application of reliability techniques.

This paper presents a methodology for spare provisioning of military flight hardware under re-

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liability growth. In this work, two basic problems were addressed : (1) determining the number of spares needed at the flightlines to replace failed items : and (2) determining the number of spares needed at the depot to supply replaceable items to the flightlines. Outputs from this model include recommended parts stocking levels at both the flightlines and central depot which are driven by specifications on risk of stockout at each of these locations. This model can also be used to determine staffing requirements. The throughput of a depot depends on the size of the physical facility and on the staffing. Once the physical size of the depot is determined it is exceedingly difficult to expand the facility and increased throughput can only be accomplished by adding repair personnel or overtime operation.

We consider a two-echelon inventory system where a repairable item may be demanded at several flightlines and the flightlines are supported by a centrally located repair depot. When an item fails at the flightline, it is replaced with a spare item from the flightline's inventory. The failed item is sent to the depot for repair. The depot provides a replacement item taken from the depot's spare stock if it is available, otherwise the replacement request is backordered at the depot until a spare is available. When the failed item arrives at the repair depot, it enters the repair process. Upon completion of repair, the item is put into the depot buffer inventory or fills a backorder if any exists.

Several authors [1, 3, 4, 5, 8, 10, 12, 14] have worked on the control of repairable items with multi-echelon spares provisioning systems. The multi-echelon structure introduces a complexity due to having a stock level requirement at each echelon on the system, which in turn, has an effect on service further down the system, in that stockouts at the specific echelon (for example, a central depot) will increase the effective lead time for the next lower echelon (flightline). Most of the multi-echelon spares provisioning models have been developed for large scale, military provisioning systems. None of the models were found to accommodate nonstationary failure processes due to reliability growth. The complexity of this problem lies in ascertaining how to control inventories of a depot-flightline system with time varying demand and service rates, and how to allocate spare items to maximize the availability of serviceable items.

2. Characterzing the Failure Process under Reliability Growth

By system improvement, we mean a tendency for a system's interarrival times to become larger. In other words, the expected number of ailures in any initial interval is no less than the expected number of failures in any interval of the same length occurring later. One popular stochastic process to represent this situation is the nonhomogeneous Poisson process (NHPP). The properties of NHPP satisfy all the conditions for a Poisson process except that the mean rate varies with time. The NHPP has been used widely as a model for a system subject to improvement. Within the class of NHPP models, the Duane [2] model is most commonly discussed in the literature. See the survey on reliability growth models in [7]. We assume that demand from flightlines is described by the Duane reliability growth curve.

Duane observed that a plot of cumulative failure rate versus cumulative operating hours on log-log paper followed a straight line. System failure times of the Duane model follow an NHPP process.

We define

$r_c(t)$ = cumulative failure rate at time t

$r_i(t)$ = instantaneous failure rate at time t

$M_c(t)$ = cumulative MTBF at time t

$M_i(t)$ = instantaneous MTBF at time t

$N(t)$ = cumulative expected number of failures by time t

K = constant, K depends upon the system complexity, design objective and margin

t = total operational time items

m = growth rate

Then the cumulative failure rate is

$$\log r_c(t) = \log K - m\log t \tag{1}$$

$$r_c(t) = Kt^{-m}. \tag{2}$$

Using N(t), the cumulative failure rate can be observed as

$$r_c(t) = \frac{N(t)}{t}. \tag{3}$$

Substituting Equation (2) into Equation (3), we have

$$N(t) = Kt^{1-m}. \tag{4}$$

The instantaneous rate of change of $N(t)$ is the instantaneous failure rate of the system at time t and is

$$\frac{dN(t)}{dt} = K(1-m)t^{-m} = r_i(t). \tag{5}$$

By substituting Equation (5) into Equation (2), the relationship between the cumulative failure rate and the instantaneous failure rate can then be seen as

$$r_i(t) = (1-m)r_c(t). \tag{6}$$

The expected number of failures over a small interval of time dt would be

$$r_i(t)dt. \tag{7}$$

and we would expect the total number of failures for a time interval $[t_1, t_2]$ to be

$$N(t_2)-N(t_1) = \int_{t_1}^{t_2} r_i(t)dt, \quad 0 \le t_1 \le t_2 . \tag{8}$$

The time dependent value, $N(t_2)-N(t_1)$, is regarded as the mean value function for an NHPP. For example, the probability of x failures in a time interval of specified duration $[t_1, t_2]$ follows a Poisson distribution according to

$$Pr\{X_{12}=x\} = \frac{[N(t_2)-N(t_1)]^x e^{-[N(t_2)-N(t_1)]}}{x!} \tag{9}$$

where X_{12} = number of failures over the time interval $[t_1, t_2]$.

The original Duane model was applied to a cumulative failure rate which decreases as time (and experience) is gained with the product. Most recent applications have used the MTBF which increases. Obviously, both are related. The reciprocal of Equation (5) gives the instantaneous MTBF as

$$M_i(t) = \frac{t^m}{K(1-m)} . \tag{10}$$

By applying Equation (6)

$$M_c(t) = (1-m)M_i(t) \tag{11}$$

and taking a logarithm from Equation (10) we have

$$\log M_i(t) = m\log t + \log\left(\frac{1}{K(1-m)}\right). \tag{12}$$

This equation will plot as a straight line on log-log paper with m being the slope of the line measured on a linear scale. If we have information on the MTBF at two points in time, say t_1, t_2 where $[t_2 > t_1]$, then we can estimate the growth rate (m) and constant (K). The slope (m) could be found by

$$m = \frac{\log M_i(t_2) - \log M_i(t_1)}{\log t_2 - \log t_1}. \tag{13}$$

Now we can substitute Equation (13) into Equation (10) to determine K and we have

$$\log K = m\log t_1 - \log M_i(t_1) - \log(1-m). \tag{14}$$

The Equations 1-14 are sufficient to provide answers to the usual questions arising in a reliability growth situation particularly for the purpose of predicting reliability growth early in a program.

2.1 Limiting Values for Reliability Growth

The Duane model predicts that reliability improves forever, but the design improvements will not go on forever. We modify the Duane model to yield finite MTBFs for the operational period. This means that we would specify a mature MTBF value taken at what we believe to be the maturity time in total cumulative flight hours. The mature MTBF is predictable from previous experience prior to the start of the reliability planning. The starting MTBF is usually recognizable in the beginning of the program. We use these MTBFs as instantaneous values and develop the cumulative growth curve from them. The actual cumulative growth curve that we would expect the hardware to follow approaches the limiting MTBF value from the bottom in a smooth fashion as shown in Figure 1.

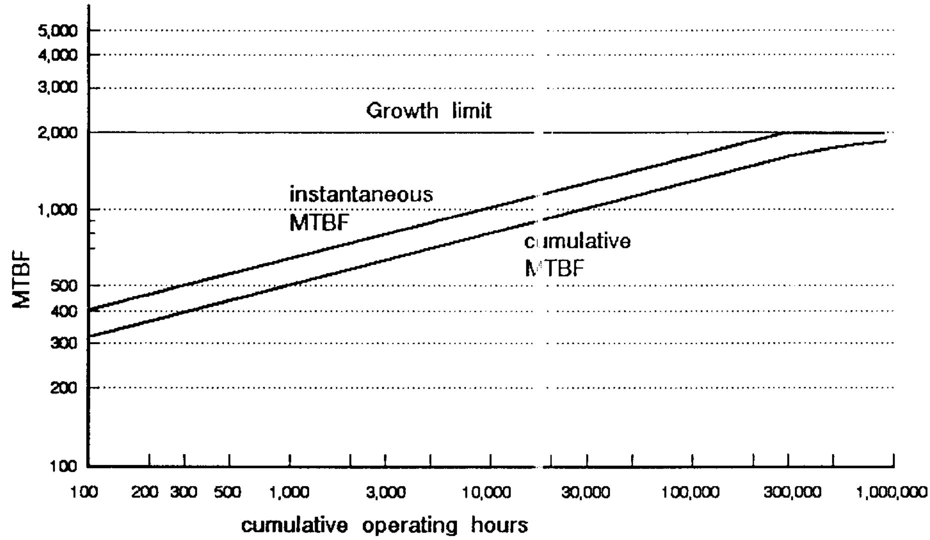

Figure 1. Reliability growth curve with limiting values

2.2 An Application for Estimating the Expected Number of Failures at the Flightlines

Let us now show how the modified Duane model might be used in a typical situation. Suppose we believe that our hardware system will ultimately achieve an instantaneous MTBF of 2, 000 hr by 300,000 hr of flight time. This means that if we stopped the design activity at this point and fixed the known design defects (applying the same diligence as in the past) the resultant fielded MTBF would be 2,000 hr. We will have a threshold MTBF of 400 hr; that is, we would expect our hardware to demonstrate an instantaneous MTBF of 400 hr which we will assign at 100 hr of flight time. Then, we find the growth curve constant to be $K=0.00625$ and a growth rate of $m=0.20$ from Equations (13) and (14). Using Equation (11), we also find the cumulative MTBF at maturity as 1,600 hr and cumulative MTBF of 320 hr at the beginning of the program. Figure 1 shows the growth curve.

The mean value function is found by substitution into Equation (4) and is

$$N(t)=0.00625t^{0.8},\ \text{for}\ t\leq 300,000\ \text{hr}.$$

At growth maturity time we have

$$N(300,000\ hr)=150.51.$$

Table 1. Flight Statistics

Flihgtline	Year	Flight Hours	Operating Hours
1	1	30,000	42,000
1	2	32,000	44,800
1	3	36,000	50,400
2	1	22,000	30,800
2	2	22,000	30,800
2	3	24,000	33,600
3	1	40,000	56,000
3	2	42,000	58,800
3	3	42,000	58,800

Service To Flight Ratio=1.40

Percentage Erroneous Faults=30%

Once MTBF has reached maturity, it remains constant thereafter. This implies that beyond 300,000 hr we have

$$N(t) = 150.51+\frac{1}{2,000}(t\text{-}300,000), \text{ for } t>300,000 \text{ hr.}$$

As an example, let the flight hours over the first three years of the program be as given in the Table 1. The difference between the flight hours and the operating hours recognizes the fact that different aircraft systems are operational for more time than the actual flight hours and can still fail during this operational usage time. For the specific flight hardware under consideration, one must know the service-to-flight ratio (R_{SF}). In this instance, we have used a R_{SF} of 1.4. In other words,

$$\text{Operating Hours} = \text{Flight Hours} \times R_{SF}$$

For the first year, at flightline 1, the operating hours per month would be

$$42,000 \text{ hr}/12 = 3,500 \text{ hr.}$$

And at the end of the first month the cumulative number of failures would be

$$N\ (3,500 \text{ hr}) = 0.00625\ (3,500)^{0.8} = 4.28$$

while at the end of the second interval we would find

N(7,000 hr) = 7.45.

This means that the number of failures in the second interval would be

N(7,000) − N(3,500) = 3.17.

The expected number of failures by time t, $\Lambda(t)$, must be adjusted for false failures to include such factors as misdiagnosis, no problem found, etc. One must maintain historical maintenance data to determine the correct factor for a particular hardware system. In this case, we are using a rate of 30% and the average number of true hardware failures must be inflated by this amount. Therefore, the adjusted mean value for the first month is

$1.3 \times 4.28 = 5.56$

and 4.11 for the second month.

Table 2 shows the calculations for the expected number of failures at the flightlines by month. As the reliability improves, the number of failures goes down in the first year, but is impacted by the increased flight hours in subsequent years. We also observe that the decreasing rate of the first year is more rapid than subsequent years. This phenomenon is due to the nature of the reliability growth model, and the failure process is a combination of reliability growth and usage.

Table 2. Expected Number of Failures at the Flightlines

Month →		1	2	3	4	5	6	7	8	9	10	11	12
Flightline 1	Year 1	5.56	4.11	3.70	3.46	3.29	3.16	3.05	2.96	2.89	2.83	2.77	2.72
	Year 2	2.85	2.81	2.76	2.73	2.69	2.66	2.63	2.60	2.57	2.55	2.52	2.50
	Year 3	2.79	2.77	2.74	2.73	2.73	2.73	2.73	2.73	2.73	2.73	2.73	2.73
Flightline 2	Year 1	4.08	3.02	2.71	2.54	2.41	2.31	2.24	2.17	2.11	2.07	2.03	1.99
	Year 2	1.96	1.93	1.90	1.87	1.85	1.83	1.81	1.79	1.77	1.75	1.74	1.72
	Year 3	1.86	1.84	1.83	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82
Flightline 3	Year 1	7.42	5.49	4.94	4.61	1.38	4.21	4.07	3.95	3.85	3.77	3.69	3.63
	Year 2	3.74	3.68	3.63	3.58	3.53	3.49	3.45	3.41	3.38	3.34	3.31	3.28
	Year 3	3.26	3.23	3.20	3.19	3.18	3.18	3.18	3.18	3.18	3.18	3.18	3.18

3. Spares Provisioning Analysis under Reliability Growth

Delay in the depot-flightline system is due to the time in the three processes : (i) transit time from flightline to the depot, (ii) delay time at the depot, and (iii) depot to flightline shipment time. Assuming unlimited space capacity at the repair depot, the repair and the shipment processes in the system may be viewed as a network series of queues, each with infinite capacity. With a NHPP failure process which is generated by a reliability growth curve, and general shipment times, the transit process for the flightline-to-depot can be modeled as a nonstationary $M/G/\infty$ system. Mirasol [9] showed that the output of a nonstationary $M/G/\infty$ is a Poisson, regardless of the distribution of service times. Therefore, using a common scenario the repair process at the depot can be modeled as an $M/M/s$ system that is independent of the number of items in-transit. In this case, the output of the $M/M/s$ system becomes the input of the depot to flightline shipment system.

We will use the following notation.

J	= number of flightlines
$D_0(t)$	= aggregated outstanding orders at time t
$D_j(t)$	= outstanding orders at flightline j at time t
$L_1(t)$	= aggregate expected number of failed items in-transit to the depot at time t
$L_2(t)$	= aggregate expected number of failed items either in repair or in the repair queue at the depot at time t
$B_0(t\|s_0)$	= aggregated backorders at time t at the depot given the depot stocks s_0
$B_j(t\|s_0)$	= disaggregated backorders at time t at flightline j given the depot stocks s_0
$s_0(t)$	= planned stock level at the depot at time t
$s_j(t)$	= planned stock level at flightline j at time t
τ_j	= deterministic shipment time for the depot-to-flightline j
$N_j(t, t+\tau_j)$	= expected number of failures at flightline j during τ_j
$\lambda_j(t)$	= expected number of arrivals for repair at the depot from flightline j at time t
$1-\alpha$	= service level of the depot spare stock
$1-\beta$	= service level of the spare stock for flightline j

3.1 Depot Spares Requirement

Estimating the expected number of total system backorders is one of the key elements in modeling the inventory systems with probabilistic demand items [6, 13]. This modeling approach requires the probabilistic distribution of the depot backorders at time t given the depot spare stocks. The backorder distribution at time t can be obtained by subtracting from the aggregated outstanding orders at time t in the system, the planned stock level at the depot. Hence, at certain times, negative backorders denote serviceable spares on hand. The distribution of outstanding orders is the convolution of the following two distributions :

(i) the number of failed items in-transit from the flightlines to the repair depot.
(ii) the number of failed items either in repair at the depot or in the repair queue.

For (i), the number of failed items in-transit is the occupancy level in an $M/G/\infty$ system, in which sufficient service capacity exists to prevent virtually any failed item waiting time. We assume that the number of failed items in-transit is independent of the number of items in the repair process. The number of failed items in (ii) is the occupancy level of an $M/M/s$ system that is independent of the numbers in (i). The convolution of the two distributions can be solved by computer using :

$$P_r\{D_0(t)=n\}=\sum_{i=0}^{n} Pr\{L_1(t)=i\}\times Pr\{L_2(t)=n-i\}. \tag{15}$$

The orders occurring when the depot is out-of-stock are backordered. For a given depot stock level, the depot backorder is given by

$$B_0(t\,|\,s_0(t))=\max\{0, D_0(t)-s_0(t)\}. \tag{16}$$

Hence, the net inventory level at the depot is $s_0(t)-D_0(t)$. The spares requirement at the depot must be planned corresponding to this level. A positive safety stock above the average outstanding order provides a buffer against larger-than-average outstanding orders. Therefore, we can choose the safety stock level with a designated service level 100 $(1-\alpha)\%$. that is

$$Pr\{D_0(t)\geq s_0(t)\}\leq\alpha \tag{17}$$

where $Pr\{\cdot\}$ is the probability of stockout given the depot stock level $s_0(t)$. For convenience, we will henceforth omit the argument t in $s_0(t)$ except when it is required to avoid ambiguity.

3.2 Flightline Spares Requirement

The outstanding orders at flightline j are either backordered at the depot or in-transit to the flightline. It is clear that no orders from flightline j at time t can resupply stock to the flightline by time $t+\tau_j$, since the shipping time for the depot-to-flightline j is τ_j. Thus, outstanding orders for flightline j at time $t+\tau_j$ are

$$D_j(t+\tau_j)=B_j(t|s_0)+N_j(t,\ t+\tau_j) \tag{18}$$

where $N_j(t,\ t+\tau_j)$ is the failures over the shipment time of depot-to-flightline j. The method for finding outstanding orders using Equation (18) was proposed by Graves [4], and conceptually similar to that of finding the net inventory in a continuous-review, constant lead time inventory control system [13]. The logic of Equation (18) is that at time t the outstanding orders at the flightlines either are backordered at the depot or are in-transit to the flightlines. Since the shipping time from depot to the flightline j is τ_j, none of the depot backorders at time t can arrive at the flightline j by time $t+\tau_j$. For this reason, any failure in the time interval $(t,\ t+\tau_j)$ and the depot backorders from time t must be outstanding. Note that $B_j(t|s_0(t))$ and $N_j(t,\ \tau_j)$ are independent random variables.

For $D_j(t)$, we need to find the backorder distribution for flightline j from Equation (16). If we assume that the depot backorders are filled on a first-come, first-served basis, then the disaggregation of the depot backorders are directly proportional to the demand rate of the flightline. This implies that the conditional distribution is a binomial distribution, and the distribution of backorders for flightline j is

$$\begin{aligned} Pr[B_j(t|s_0)=i] &= \sum_{n=i}^{J} Pr[B_0(t|s_0)=\mathrm{n}]\ Pr[B_j(t|s_0)=i\,|\,B_0(t|s_0)=n] \\ &= \sum_{n=i}^{J} Pr[B_0(t|s_0)=\mathrm{n}]\ \binom{n}{i}\, q_j^i(1-q_j)^{n-i} \end{aligned} \tag{19}$$

where $q_j=\lambda_j(t)\,/\,\lambda(t)$ and $\lambda(t)=\sum_{j=1}^{J}\lambda_j(t)$.

From Equations (18) and (19), the distribution of the outstanding orders at flightline j is found as

$$\begin{aligned} Pr[D_j(t)=k] &= \sum_{i=0}^{k}[Pr\{B_j(t|s_0)=i\}\times Pr\{N_j(t,\ t+\tau_j)=k\text{-}i\}] \\ &= \sum_{i=0}^{k}[\{\sum_{n=i}^{J} Pr\{B_0(t|s_0)=n\}\binom{n}{i}q_j^i(1-q_j)^{n-i}\}\times Pr\{N_j(t,\ t+\tau_j)=k\text{-}i\}]. \end{aligned} \tag{20}$$

Once the outstanding orders for flightlines are obtained, a service rate constraint is given by

$$Pr[D_j(t) \geq s_j(t)] \leq \beta \qquad (21)$$

where $Pr\{\cdot\}$ is the probability of no stockout given the flightline stock level $s_j(t)$. A risk level constraint of β is imposed on the stock-out condition. We desire a capability such that at least $100(1-\beta)\%$ of the requests for spare items at the flightlines are immediately filled from on-hand spare inventory.

3.3 An Application for Spares Provisioning Analysis

In order to illustrate an application of the model, we will rely upon the growth curve information (Figure 1) and associated data for the example discussed in the section of the Application for Estimating the Expected Number of Failures. We will use the following data for this application.

1. Transit service times for the flightlines-to-depot are assumed to be exponential with parameters $\delta_1=10$ days, $\delta_2=15$ days, and $\delta_3=$ 0 days for flightlines 1, 2, and 3 respectively.
2. The required number of servers is estimated in the case of $0.6<\rho<0.8$ where ρ is the utilization factor for the M/M/s system.
3. The mean service time is identically 10 days for each server.
4. Spare stock levels for the depot and flightlines are planned for a service level of 90% availability.
5. The deterministic shipping times for the depot-to-flightlines are such that $\tau_1=7$ days, $\tau_2=10$ days, and $\tau_3=7$ days for flightlines 1, 2, and 3 respectively.

Table 3 presents the mean number of failed items in the system and the required number of servers in repair depot. Due to transient effects at time t=0 −the system is empty and idle at t=0− we do see an initial increase in the expected number of items arriving at the depot from month 1 to month 2 from 11.18 to 13.00 even though the field failures are decreased from 17.06 to 12.62. Much of this phenomena is due to delay involved in shipping the items from the flightline to the depot. It appears that the transient effects die out quickly. Later on, we see a higher expected demand rate at the beginning of the second year, from 8.41 at the end of year 1, month 12, to 8.61 at the end of year 2, month 1. This is due to the discontinuity in the specification of monthly operating hours as the transition is made from year 1 to year 2. Note that the flightline's operating hours specification is made on a yearly basis, and we approximate month-by-month flight hours as the aggregate twelve month specification divided by 12. The same phenomena is observed at the beginning o year 3.

Table 3. Mean Number of Failed Items in the Syste n and Required Number of Servers in Repair Depot

Year	Month	Aggregate Failures in the Feld	Aggregate Arrivals at Depot	Required Servers	Utilization Factor	No. of Failed Item in Depot
1	1	17.06	11.18	5	0.745	5.06
1	2	12.62	13.00	6	0.722	5.30
1	3	11.35	11.75	5	0.783	5.80
1	4	10.60	10.88	5	0.726	4.74
1	5	10.08	10.29	5	0.686	4.20
1	6	9.68	9.84	5	0.656	3.87
1	7	9.36	9.49	5	0.633	3.64
1	8	9.09	9.20	4	0.767	4.84
1	9	8.86	8.96	4	0.747	1.47
1	10	8.67	8.75	4	0.729	4.20
1	11	8.49	8.57	4	0.714	3.98
1	12	8.34	8.41	4	0.701	3.81
2	1	8.56	8.61	4	0.717	4.03
2	2	8.42	8.47	4	0.706	3.87
2	3	8.29	8.31	4	0.695	3.74
2	4	8.18	8.22	4	0.685	3.62
2	5	8.07	8.11	4	0.676	3.52
2	6	7.98	8.01	4	0.668	3.44
2	7	7.89	7.92	4	0.660	3.36
2	8	7.80	7.83	4	0.653	3.28
2	9	7.72	7.75	4	0.646	3.22
2	10	7.65	7.67	4	0.640	3.16
2	11	7.57	7.60	4	0.634	3.11
2	12	7.51	7.53	4	0.628	3.06
3	1	7.91	7.92	4	0.660	3.36
3	2	7.84	7.86	4	0.665	3.30
3	3	7.77	7.79	4	0.649	3.25
3	4	7.74	7.75	4	0.646	3.22
3	5	7.73	7.75	4	0.646	3.22
3	6	7.73	7.75	4	0.646	3.22
3	7	7.73	7.75	4	0.646	3.22
3	8	7.73	7.75	4	0.646	3.22
3	9	7.73	7.75	4	0.646	3.22
3	10	7.73	7.75	4	0.646	3.22
3	11	7.73	7.75	4	0.645	3.22
3	12	7.73	7.75	4	0.645	3.21

It is evident from Table 3 that the changing rate of the demand affects the choice of the number of required servers. Hence, the utilization factor and the number of failed items in the depot are also affected. For example, decreasing the demand (arrival) rate through year 1 allows for the removal of one service personnel at month 8 for the choice of $\rho>0.6$. This increases the utilization factor and therefore increases the expected number of failed items in the depot. The variation in the utilization factor depends on the demand rate and the number of servers. In such a system with a small number of servers, the decrement of one server affects the utilization factor more than a system with a larger number of servers.

Figure 2 depicts the expected number of $L_1(t)$ and $L_2(t)$. By using the two distributions, the aggregated outstanding orders can be calculated from Equation (15). The safety stock at the depot must be planned corresponding to this level. To provide an appropriate safety stock, we can choose a service level which may preclude the possibility of a stockout. In this application, for illustrative purposes, the depot stock level $s_0(t)$ is planned for a service level of 90% availability. In this case, the depot spare stock is constrained to be empty not more than 10% of the time. Table 4 depicts the depot stock level by month. Although the demands are decreasing through the year, the required depot stock level based on the outstanding orders is increasing in month 8 of year 1. These changes are due to the variation of the number of repair channels. Clearly, a reduced repair channel will contribute to the higher safety stock. The increased stock levels in the beginning of year 2 and year 3 are due to the increment of aggregate flight hours. Table 5 shows the spare stock level for flightline j by month. The estimates of the required spare stocks with 10% risk level is rationally identical to that of the method for the depot spare stock. Here we can also observe a jump of required stock level in month 8 of year 1 that are effects of the system performance due to the variation of the number of repair channels. In interpreting the inventory requirements from Tables 4 and 5, sufficient spare items must be bid into the contract to cover this total demand. As long as one has bid sufficient items to meet the maximum flow at a desired risk level, then one is protected from an out-of-stock situation at that risk level.

The time varying arrival of failed parts to the repair depot over successive periods is due to changes in flight hours specification, weighted by the time-varying failure intensity due to the reliability growth phenomena of the hardware. For the class of problems which we model, we do not expect to see reliability growth rates exceed 0.2. See [11] for the growth rate of the Duane curve. The example was run for an extreme case where the growth rate is 0.2. To check the accuracy of the approximation, experiments were conducted by means of comparison with the transient model. With the same data as used in this example, our experiments indicate that the average error for approximation is less than 3.6% of the overall time average for the expected number in the depot repair system.

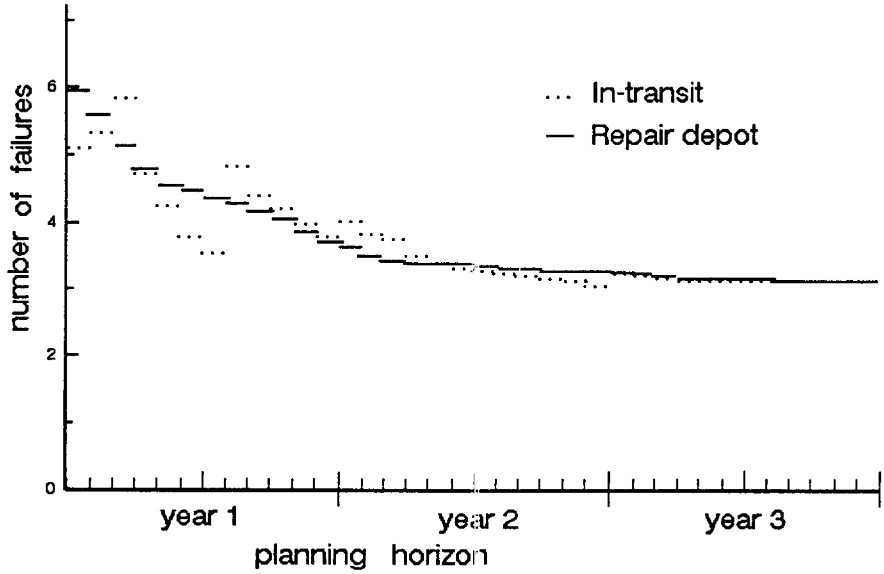

Figure 2. Aggregate occupancy level of the failed items

Table 4. Depot Stock Level Analysis by Month

Month →	1	2	3	4	5	6	7	8	9	10	11	12
Year 1	15	15	16	14	12	12	11	14	13	12	12	11
Year 2	12	11	11	11	10	10	10	10	10	10	9	9
Year 3	10	10	10	10	9	9	9	9	9	9	9	9

Table 5. Flightline Stock Level Analysis by Month

Month →		1	2	3	4	5	6	7	8	9	10	11	12
Flightline 1	Year 1	7	6	6	6	6	5	5	6	6	6	5	5
	Year 2	5	5	5	5	5	5	5	5	5	5	5	5
	Year 3	5	5	5	5	5	5	5	5	5	5	5	5
Flightline 2	Year 1	6	6	6	5	5	5	5	5	5	5	5	5
	Year 2	5	5	5	5	5	5	5	5	5	5	5	5
	Year 3	5	5	5	5	5	5	5	5	5	5	5	5
Flightline 3	Year 1	8	7	8	7	7	6	6	7	7	7	6	6
	Year 2	6	6	6	6	6	6	6	6	6	6	6	6
	Year 3	6	6	6	6	6	6	6	6	6	6	6	6

4. Concluding Remarks

Spares provisioning models have tended to focus largely on stationary conditions. Under reliability growth, the failure process is nonstationary since the failure rate of the hardware follows a time dependent reliability growth curve. We extended the basic results for cases where failures were generated by nonhomogeneous Poisson processes. Such a model has not been treated in the literature to date.

The Duane growth model implies that reliability improves forever, but since design improvements do not go on forever in most practical situations, we proposed a simple way to limit the Duane growth model. We also proposed an estimation method for the two parameters of the Duane model that can be easily applied for a military contract.

Any actual repair depot has important options available for temporary expansion of capacity via over time, additional shifts, or subcontracting. Without making any additional assumptions about the repair discipline, such asdeterministic repair time or ample repair capacity, we obtained the required number of personnel with the varying demand at the depot. Our models incorporate the mechanism which determines when to increase shop capacity and this information is extremely useful for selecting appropriate scheduling rules and spares stocking policies.

The methodology developed in this research will provide a basis for better cost estimation early in the bid stage of a project, and for improved logistics planning. We believe that this effort would be of great benefit to many Air Force contractors and would eventually lead to a basis for a new approach to logistics planning.

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