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A Probabilistic Order Level System When Delay in Payment Is Permissible

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Abstract

The probabilistic order level inventory model is developed when a supplier allows some credit period T^* for settling the accounts for purchase quantity. The credit period T^* is known constant. Mathematical models are derived for both the cases i) $T^* \leq T$ and ii) $T^* > T$. Expressions are derived for average expected total cost of the system, the optimum cycle time and for obtaining optimum order level $S=S_0$ in each case. The examples are given to illustrate the mdel.

1. Introduction

In the conventional inventory models, it is implicitly assumed that the payment of an order quantity is made as soon as the goods are received by the system. In practice, however, the supplier announces some credit period in settling the account, so that no interest charges are payable from the outstanding amount if the account is settled within the allowable delay period. The supplier will obviously charge higher interest if the account is not settled by the end of the permissible delay period. This brings some economic advantage to the system, as it would try to earn some interest from the revenue realized during the period of permissible delay. Davis and Gaither [1] have studied an EOQ model when the supplier offers one time opportunity to delay the payment of order in case an order for additional units is placed. Goyal [2] has also studied an EOQ model under this situation. Shah, Patel and Shah [5] have studied the same model by allowing shortages. Mandal and Phaujdar [3] have studied the same situation by including interest earned from the sales revenue on the stock remaining beyond the settlement period.

All the inventory models studied above are deterministic. In this paper, we have considered a

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stochastic inventory model when delay in payments is permissible. In particular, we consider the probabilistic order-level system in which the supplier allows a fixed credit period of T^* - time unit. Appropriate cost function is developed for the total average expected cost of the system, and formula for obtaining optimum cycle time $T=T_0$ and optimum value of order level $S=S_0$ have been obtained.

2. Assumptions

The model under consideration is formulated with the following assumptions :

1) On hand inventory of the system is reviewed regularly at a prescribed time period of length T. At the end of each scheduling period T, units are ordered so as to bring the on hand inventory to an order level S. S is the decision variable.

2) Lead-time is zero.

3) Shortages are not allowed.

4) The demand x−during any scheduling period T follows a probability density function (p.d.f.) $f(x|T)$, $a(T) \le x \le b(T)$ with $R=\mu(T)/T$ as the average demand rate where

$$\mu(T)=E(x|T)=\int_{a(T)}^{b(T)} f(x|T)dx \tag{1}$$

as the mean demand during T. We assume that R is known constant and that demand of x units occur with a uniform pattern over the scheduling period T.

5) During the fixed credit period T^* the unit cost of generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit price is retained by the system to meet the day to day expenses of the system. At the end of the credit period the account is settled and interest charges are payable on the account in stock. The supplier gives a credit period of length T^*. Thus, the system has not to pay interest charges on the purchase units for a fixed period of length T^* where

$$\text{i) } T^* \le T \qquad \text{or} \qquad \text{ii) } T^* > T.$$

6) The unit cost C per unit and inventory holding cost excluding interest charges C_1 per unit per time unit are known and constant during the period under consideration.

For the period during which accounts of purchase quantity is not settled, the generated sales revenue is deposited in an interest bearing account at the rate of I_e per \$ per time unit. After the account is settled the system starts paying interest charges on the outstanding amount in inventory at the rate of I_c per \$ per time unit. It is assumed that only the unit cost from the generated revenue is deposited in the interest bearing account. The ddifference between the selling price and unit cost is retained by the system for meeting day to day expenses.

3. Notations

Throughout the formulation of the model we will use the following notations :

C=unit purchase cost in \$

C_1=inventroy holding cost per unit per time unit excluding interest charges in \$

C_3=ordering cost per cycle in \$

I_e=interest that can be earned per \$ per time unit

I_c=interest charges payable per \$ per unit per time unit

(Note : We generally have $I_c > I_e$.)

T=cycle time

T^*=permissible delay period for settling accounts in time units.

T_1^o, T_2^o and S_1^o, S_2^o are the optimum values of cycle time T and optimum order level S.

4. Mathematical Model

Note that we may have either $T^* \leq T$ or $T^* > T$ (T^* is known constant) and the total cost function in both the cases will be different. In what follows we derive mathematical models for both the cases. At the beginning of each cycle on an average μ-units are scheduled for replenishment. Under the assumption that total demand x-units occur uniformly over the cycle time, average demand per time unit is x / T.

Let Q(t / x) denote on hand inventory at time t of a cycle, then

$$Q(t / x) = S - \frac{x}{T} t, \qquad 0 \leq t \leq T \tag{2}$$

and average expected inventory in the system per time unit is

$$I_1(T)=b(T)-\mu(T)/2 \tag{3}$$

where as shown by Naddor [4] we take S=b(T) the maximum possible demand during any scheduling period T. Note that all the components of total average expected cost functions are average expected.

Here, components of total average expected cost functions are as follows :

a) Inventory holding cost per time unit $=C_1\{b(T)-\mu(T)/2\}$. (4)

b) Interest earned per time unit

$$=\begin{cases} \frac{CI_e}{T}\int_0^{T^*}\frac{x}{T}\,tdt, & T^*\le T \\ \frac{CI_e}{T}\int_0^{T}\frac{x}{T}\,tdt+\frac{CI_e x}{T}(T^*-T), & T^*>T \end{cases}$$

$$=\begin{cases} \frac{CI_e x T^{*2}}{2T^2}, & T^*\le T \\ CI_e x\,[T^*/T-1/2], & T^*>T. \end{cases}$$

Hence average expected interest earned per time unit is

$$=\begin{cases} \frac{CI_e\mu(T)T^{*2}}{2T^2}, & T^*\le T \\ CI_e\mu(T)\,[T^*/T-1/2], & T^*>T. \end{cases} \tag{5}$$

c) Interest charged per time unit

$$=\begin{cases} \frac{CI_c}{T}\int_{T^*}^{T}Q(t/x)dt, & T^*\le T \\ 0, & T^*>T \end{cases}$$

$$=\begin{cases} CI_c\,[1-T^*/T]\,\{S-\frac{x}{2T}(T+T^*)\}, & T^*\le T \\ 0, & T^*>T. \end{cases}$$

Thus, average expected interest charged per time unit is

$$=\begin{cases} CI_c\{b(T)-\mu(T)/2\}-CI_c\frac{T^*}{T}\{b(T)-\frac{T^*}{2T}\mu(T)\}, & T^*\le T \\ 0, & T^*>T. \end{cases} \tag{6}$$

d) Replenishment cost per time unit $=C_3/T$. (7)

Let $K(T)$ denote the average expected total cost of the system incurred per time unit when the cycle time is T, then

$$K(T) = \begin{cases} K_1(T), & T^* \le T \\ K_2(T), & T^* > T \end{cases} \tag{8}$$

where

$$K_1(T)=(C_1+CI_c)\{b(T)-\mu(T)/2\}-CI_cT^*b(T)/T+\frac{C(I_c-I_e)\,T^{*2}\mu(T)}{2T^2}+C_3/T, \quad T^* \le T. \tag{8.1}$$

and

$$K_2(T)=(C_1+CI_c)\{b(T)-\mu(T)/2\}-CI_e\mu(T)\{T^*/T-1/2]+C_3/T, \qquad T^*>T. \tag{8.2}$$

Thus, the optimum value of $T=T_1^0$ is the solution of $dK_1(T)/dT=0$, i.e. it is the solution of

$$\begin{aligned}&2(C_1+CI_c)\{b'(T)-\mu'(T)/2\}T^3-2CI_cT^*b'(T)T^2\\&\quad+\{2CI_cT^*b(T)+C(I_c-I_e)T^{*2}\mu'(T)-2C_3\}T-2C(I_c-I_e)T^{*2}\mu(T)=0, \quad T^* \le T\end{aligned} \tag{9}$$

and the optimum value of cycle time $T=T_2^0$ is the solution of $dK_2(T)/dT=0$, i.e. it is the solution of

$$[C_1\{2b'(T)-\mu'(T)\}+CI_e\mu'(T)]\,T^2-2CI_e\mu'(T)T^*T-2C_3+2CI_e\mu(T)T^*=0, \quad T^*>T. \tag{10}$$

The condition for optimality is

$$\begin{aligned}\frac{d^2K_1(T)}{dT^2} &= (C_1+CI_c)\{b''(T)-\mu''(T)/2\}\\&\quad+\frac{C(I_c-I_e)T^{*2}}{2T^2}\{\mu''(T)-4\mu'(T)/T+6\mu(T)/T^2\}\\&\quad-\frac{CI_cT^*}{T}\{b''(T)-2b'(T)/T+2b(T)/T^2\}+\frac{2C_3}{T^3}>0\end{aligned} \tag{11}$$

when

$$T=T_1^0 \ge T^*$$

and

$$\frac{d^2K_2(T)}{dT^2} = C_1\{b''(T)-\mu''(T)/2\} + CI_e\mu''(T)/2 - \frac{CI_eT^*}{T}\{\mu''(T)-2\mu'(T)/T+2\mu(T)/T^2\} + \frac{2C_3}{T^3} > 0, \quad (12)$$

when

$$T=T_2^0 \leq T^*.$$

Hence, optimum order level $S=S_0$ is given by

$$S_0 = \begin{cases} b(T_1^0), & T_1^0 \geq T^* \\ b(T_2^0), & T_2^0 > T^*. \end{cases} \quad (13)$$

As seen above, the total average expected costs $K_1(T)$, $K_2(T)$ and optimum order levels in different situations depends on the particular forms of the function $b(T)$ – the maximum possible demand during any scheduling period T.

Consider,

$$\left.\begin{array}{l} \mu(T)=RT \\ b(T)=P\mu(T) \end{array}\right\} \quad (14)$$

where P is a positive constant $(P \geq 1)$.

Here, total average expectedd cost K(T) is

$$K(T) = \begin{cases} (C_1+CI_e)RT\,(P-1/2)-CI_ePRT^* + \dfrac{C(I_c-I_e)RT^{*2}}{2T} + C_3/T, & T^* \leq T \\ C_1RT\,(P-1/2) - CI_eR\,(T^*-T/2) + C_3/T, & T^* > T. \end{cases} \quad (15)$$

Then,

$$T_1^0 = \sqrt{\frac{2C_3+C(I_c-I_e)RT^{*2}}{2(C_1+CI_e)R(P-1/2)}}, \quad T^* \leq T_1^0 \quad (16)$$

and

$$T_2^0 = \sqrt{\frac{2C_3}{R\{2C_1(P-1/2)+CI_e\}}}. \quad (17)$$

Clearly,

$$T_1^{0\,2} - T_2^{0\,2} = \frac{1}{2(C_1+CI_c)R(P-1/2)}\{C(I_c-I_e)RT^{*2} + \frac{2CC_3I_c}{2C_1(P-1/2)+CI_c}\} > 0$$

$$(\text{since } I_c > I_e). \tag{18}$$

Also,

$$\frac{d^2K(T)}{dT^2} = \begin{cases} \frac{1}{T^3}\{C(I_c-I_e)RT^{*2} + 2C_3\} & T^* \le T \\ \frac{2C_3}{T^3}, & T^* > T. \end{cases} \tag{19}$$

Since $I_c > I_e$, in either cases $d^2k(T)/dT^2 > 0$, which shows that K(T) is a convex function and hence it has a unique minimum.

5. Computational Procedure

In order to obtain optimum total average expected cost, optimum cycle time interval between two consecutive orders we follow these steps :

STEP 1. Compute T_1^0 from eq. (9). If $T_1^0 \ge T^*$, then T_1^0 is the optimum value of T and obtain optimum cost from eq. (8.1).

STEP 2. If $T_1^0 < T^*$, then compute T_2^0 from eq. (10). Again if $T_2^0 < T^*$, then T_2^0 is the optimum value of T and optimum cost function can be computed from eq. (8.2).

6. Numerical Illustrations

Problem 1. Consider an inventory system with the following data :

C = \$ 50 per unit

i = 0.1 per annum

C_1 = \$ 5 per unit per year

R = 1000 per annum

C_3 = \$ 250 per unit yearly

I_c = 12% per annum

I_e = 20% yearly

T^* = 1/12 years

P = 1.5

then

$T_1^0 = 0.187$ yr ($T_1^0 > T^*$)

$K_1(T_1^0)$ = \$ 1568.08 is total cost and optimum order level S_1^0 is 280.5.

Problem 2. Consider data as

C_3 = \$200 per unit per year

R = 10000 per annum

keeping remaining data same as above, then

$T_1^0 = 0.067$ yrs ($T_1^0 < T^*$)

$T_2^0 = 0.0603$ yrs ($T_2^0 < T^*$)

$K_2(T_2^0)$ = \$ 1653.25 and optimum order level is $S_2^0 = 904.5$.

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