Korean Management
Science Review
Vol.10, No.2, October 1993.


# AN INTERACTIVE FACE SEARCH PROCEDURE FOR MULTIPLE OBJECTIVE LINEAR PROGRAMMING

Dong Yeup Lee*

## ABSTRACT

This papper presents a new interactive procedure for multiple objective linear programming ploblem (MOLP). In practical multiple objective linear programming applications, there is usually no need for the decision maker to consider solutions which are not efficient. Therefore, the interactive procedure presented here searches only among efficient solutions and terminates with a solution that is guaranteed to be efficient. It also can converge to nonextreme efficient final solutions rather than being restricted to only extreme efficient points of the feasible set. The procedure does not require sophisticated judgements or inputs from the decision maker. One of the most attractive features of the procedure however, is that the method allows the DM to examine the efficient faces it finds. As iteration goes, the DM can explore a wide variety of efficient faces rather than efficient faces confined to only certain subregion of the feasible set of problem MOLP since the efficient faces that the procedure finds need not be adjacent. This helps the DM explore the nature of the efficient set of problem MOLP and also helps the DM have confidence with a final solution. For these reasons, I feel that the procedure offer significant promise in solving multiple objective linear programs rapidly and in a satisfying manner to the DM.

## 1. INTRIDUCTION

Multiple criteria decision making (MCDM) problems have been of increasing interest to management scientists, due in part to the realization that many problems, particularly those of a strategic nature, and more particularly those in the public sector, must explicitly consider multiple criteria if they are to be resolved with truely good decisions. One of the more popular and practical models has been used to help make decisions involving multiple criteria is the multiple objective linear

* Myung Ji University

programming problem (MOLP) model. This model can be written

(MOLP) "max" Cx, subject to $x \in X$,

where $X = \{x \in R^n : Ax \leq b\}$ and X is a nonempty, compact polyhedron, C and A are p x n and m x n matrices, respectively, and $b \in R^m$.

In the multiple criteria decision making (MCDM) problem, 'maximization' is not well defined since the objective functions may be conflicting with each other, and usually some compromise solution is required. Numerous techniques to find the most preferred compromise solution have been proposed in the literature, where 'most preferred' depends upon the preferences of the decision maker (DM). Usually the most preferred compromise solution is required to be an efficient (nondominated, Pareto) solution.

**Definition 1.1**

A point $x^0 \in X$ is an efficient solution of problem (MOLP) if and only if there exists no $x \in X$ such that $Cx \geq Cx^0$ and $Cx \neq Cx^0$.

Numerous real and potential applications of multiple objective decision making have been reported in the literature in the past two decades. These applications have been in areas such as academic planning [9], scheduling [10], inventory planning [11], location planning [16], energy planning [12], [23], and project management [21].

Many of the approaches for analyzing problem (MOLP) involves generating efficient solutions [7], [18], [23]. Two of these, interactive approach and vector maximization approach, are quite commonly used.

The interactive approach has become one of the more popular approaches in recent years. In this approach, the decision maker (DM) dynamically interacts with a computerized algorithm, and thereby explores the feasible decisions until he finds one that he prefers the most. Through practical experience, researchers and decision makers have learned that the interactive approaches do not have many of the serious flaws which noninteractive approaches possess. It also has certain advantages that the noninteractive approaches do not have. Chief among these is that the DM is actively and dynamically involved in the decision making process. He can thereby learn about his preferences and come to a decision in which he can have confidence. Furthermore, interactive procedures can be flexibly designed to yield a variety of attractive characteristics. In contrast, the vector maximization approach has a different set of drawbacks. Among them are that it requires a great deal of computation and that the number of candidate solutions that it generates for the DM to choose among can be overwhemingly large.

Because of the advantages of the interactive approach, many knowledgeable individuals in the field of MCDM would agree with Steuer's (1986, p.361) statement that "the future of multiple objective programming is in its interactive application."

Many of the interactive procedures have been developed for the problem (MOLP) [1], [7], [8], [14], [17], [18], [20], [24]. Among these solution procedures, different magnitudes of requirements are placed on the DM in terms of both the quality and quantity of information required of him. They differ in the kind of preference information required from the DM. They may require assessments of weights [5], specifications of relaxation quantities [1], evaluations of tradeoffs [9], or pairwise preference comparisions [13].

Although numerous interactive approaches have been suggested, none has emerged as a clearly preferred approach. It is unlikely that any single procedure will emerge as universally preferred because different procedures may be better suited for different types of DMs and decision making situations. Thus in individual decision situations, analysts could select from among solution procedures which are most compatible with DM's decision processes.

This paper presents an interactive face search procedure for problem (MOLP). The procedure has four major benefits. First, it searches only among efficient solutions, and terminates with a solution that is guaranteed to be efficient. Second, it also can converge to nonextreme efficient final solutions rather than being restricted to only extreme efficient points of the feasible set. Third, it does not require sophisticated judgements or inputs from the decision makers. Finally, the procedure allows the DM to examine the efficient faces that it finds. At each iteration, if the DM wants to find an improved solution, the subset of the efficient face containing the current solution is determined by the DM's responses. Then the procedure finds an efficient solution which is preferred to the current solution by examing the subset determined by the DM's responses. Since the subset of the efficient face containing the current solution includes only the efficient solutions interesting to the DM, the procedure examining the subset of the efficient face gives the DM a better chance to find a satisfying solution. This feature would enable the DM to reach a satisfying solution in relatively few iterations. As iteration goes, the procedure allows the DM to explore a variety of efficient faces rather than efficient faces confined to only subregion of X since the efficient faces that the procedure finds need not be adjacent. Furthermore, the procedure makes the DM possible to examine the region around the current solution.

The organization of the paper is as follows. In section 2 the theory necessary for understanding the procedure is explained. Section 3 gives the steps of the procedure. In section 4 the procedure is illustrated with a small example. The final section contains concluding remarks.

## 2. THEORETICAL BACKGROUND

Let $x^o \in X$.

Consider the following linear program $P\lambda$.

$$(P\lambda) \quad \max \ <\lambda^T C, x> \quad \text{subject to} \quad Cx \geqq Cx^o \quad (1), \quad x \in X$$

The interactive face search procedure is based on the following two results.

The first result follows easily from [2]. The second result is derived from the first result by using duality theory [4].

Let $X_E$ denote the set of all efficient solutions of problem MOLP.

**THEOREM 1**

Let $x^0 \in X$. Then $x^o \in X_E$ if and only if for any $\lambda > 0$, $x^o$ is an optimal solution of the linear program $(P\lambda)$.

**THEOREM 2**

Assume that $\lambda^o > 0$ and $x^o \in X_E$.

Let $(u^{oT}, w^{oT})$ be any optimal solution to the linear programming dual $D\lambda^o$ of problem $P\lambda^o$, where $u^o$ represents the dual variables corresponding to the constraints (1). Then $x^o$ belongs to the efficient face $X_{\hat{\lambda}^o}$ of X, where $\hat{\lambda}^o = u^o + \lambda^o$ and $X_{\hat{\lambda}^o}$ denotes the optimal solution set of the weighted sum problem $(W\lambda)$ with $\lambda = \hat{\lambda}^o$ : $\max <\lambda^T C, x>$ subject to $x \in X$.

**PROOF**

To prove the desired result, we need to show that $x^o$ is an optimal solution to problem $W\lambda$ with $\lambda = \hat{\lambda}^o$. The dual linear program to problem $P\lambda^o$ is given by

$$D\lambda^o : \quad \min \ -<x^{oT} C^T, u> + <b, w>, \quad \text{subject to} \quad -C^T u + A^T w = C^T \lambda^o, \quad u, w \geqq 0.$$

By Theorem 1, since $x^o \in X_E$, $x^o$ is an optimal solution for problem $P\lambda^o$.

By duality theory of linear programming [15], $\lambda^{oT} Cx^o = -\langle x^{oT} C^T, u^o\rangle + \langle b, w^o\rangle$.

Rearranging this equation, we obtain $(u^o+\lambda^o)^T Cx^o = \langle b, w^o\rangle$. (2)

Now consider the dual linear program to problem $W\lambda$ with $\lambda = \hat{\lambda}^o$. This dual program is given by

$$\begin{aligned} D: \quad & \min \langle b, w\rangle \\ & \text{subject to} \\ & A^T w = C^T(u^o + \lambda^o)^T \\ & w \geqq 0. \end{aligned}$$

Since $(u^{oT}, w^{oT})$ is an optimal solution to problem $D\lambda^o$, $w^o$ is a feasible solution for problem D.

Let w be any feasible solution for problem D. Then it is easily seen that $(u^{oT}, w^T)$ is a feasible solution for problem $D\lambda^o$. Since $(u^{oT}, w^{oT})$ is an optimal solution for problem $D\lambda^o$, this implies that

$-\langle x^{oT} C^T, u^o\rangle + \langle b, w\rangle \geqq -\langle x^{oT} C^T, u^o\rangle + \langle b, w^o\rangle$,

or, equivalently, $\langle b, w\rangle \geqq \langle b, w^o\rangle$. It follows that $w^o$ is an optimal solution for problem D.

Notice also that since $x^o \in X_E$, $x^o$ is a feasible solution for problem $W\lambda$ with $\lambda = \lambda^o$.

To summarize, we have shown that with $\lambda = \lambda^o$, $x^o$ is a feasible solution to the linear program $W\lambda$, $w^o$ is an optimal solution to the dual linear program D of problem $W\lambda$, and by (2), that the objective function value of $x^o$ in problem $W\lambda$ equals the objective function value of $w^o$ in problem D. From linear programming duality theory [15], this implies that $x^o$ is an optimal solution to problem $W\lambda$ with $\lambda = \hat{\lambda}^o$, and the proof is complete.

The following corollary of Theorem 2 is immediate.

**Corollary 1**

Assume that $\lambda^o > 0$ and $x^o \in X_E$.

Let $(u^{oT}, w^{oT})$ be any optimal solution to the linear programming dual $D\lambda^o$ of problem $P\lambda^o$, where $u^o$ represents the dual variables corresponding to the constraints (1). Let $\hat{\lambda}^o = u^o + \lambda^o$, and let $v_o = (\hat{\lambda}^o)^T Cx^o$. Then the efficient face $X_{\hat{\lambda}o}$ of X can be represented as $X_{\hat{\lambda}o} = \{x \in X \mid (\hat{\lambda}^o)^T Cx = v_o\}$.

From corollary 1, it can be easily seen that the following linear programming P finds an efficient solution which minimizes $k$th objective function over the subset of the efficient face $X_{\hat{\lambda}o}$ which contains $x^o$

(P) $\min \langle c_k, x \rangle$

subject to

$(\lambda^o)^T Cx = (\lambda^o)^T Cx^o$

$c_j x^o - \triangle_j \leqq c_j x \leqq c_j x^o + \triangle_j \qquad 1 \leqq j \leqq p$

$x \in X$

, where $\triangle_j > 0$.

## 3. THE INTERACTIVE FACE SEARCH METHOD

The steps of the interactive face search method are as follows :

**Step 1.**

Solve the p single objective linear programming problems

$\max \{c_i x = z_i\} \qquad i = 1, \cdots\cdots, p$

subject to

$x \in X$

to obtain p efficient points, $x^i$, $i=1,\cdots\cdots,p$, to the orginal problem, and their associated images, the p criterion vectors $z^i$, $i=1,\cdots\cdots,p$, respectively. Let the DM review these p criterion vector. If the DM wishes to stop with his most preferred one among them, the procedure terminates. If the DM wishes to try to find an improved solution, let his most preferred one among them be $z^h$ and its inverse image be $x^h$. Continue with Step 2.

**Step 2.**

Construct a payoff table to obtain the ideal criterion vector $z^* \in R^p$.

Payoff Table

| | $z_1$ | $z_2$ | | $z_p$ |
|---|---|---|---|---|
| $z^1$ | $z_1^*$ | $z_{12}$ | | $z_{1p}$ |
| $z^2$ | $z_{21}$ | $z_2^*$ | · · · | $z_{2p}$ |
| $z^p$ | $z_{p1}$ | $z_{p2}$ | | $z_p^*$ |

where the rows are the criterion vectors resulting from inividually optimizing each of the objectives. The $z_i^*$ entries along the main diagonal form the $z^*$ ideal criterion vector.

**Step 3.**

Let iteration counter h=0. Let $m_i$ be the minimum value in the ith colum of the payoff table. Calculate $\pi_i$ values where

$$\pi_i = \begin{cases} \dfrac{z_i^* - m_i}{z_i^*} \left[ \sum_{j=1}^{n} (c_{ij})^2 \right]^{-\frac{1}{2}} & \text{when } z_i^* > 0 \\ \dfrac{m_i - z_i^*}{m_i} \left[ \sum_{j=1}^{n} (c_{ij})^2 \right]^{-\frac{1}{2}} & \text{when } z_i^* \leqq 0 \end{cases}$$

Term 1 Term 2

**Step 4.**

With $(x^h, z^h)$, specify the index set $I^*$ of criterion values to be relaxed and specify the amounts $(\triangle_i, i \in I^*)$ by which they are to be relaxed.

**Step 5.**

Let h=h+1. Calculate $\lambda_i$ weights where

$$\lambda_i^h = \begin{cases} \varepsilon & i \in I^* \\ \dfrac{\pi_i}{\sum_{j=1}^{p} \pi_j} & i \notin I^* \end{cases}$$

(note : $\varepsilon$ is an arbitrary small, positive constant)

**Step 6.**

Solve the linear program $(P\lambda^h)$.

$$\text{Max } \langle (\lambda^h)^T C, x \rangle$$

subject to

$$\left. \begin{array}{ll} c_i x \geqq c_i x^h & i \notin I^* \\ c_i x \geqq c_i x^h - \triangle_i & i \in I^* \end{array} \right\} \quad (1)$$

$$x \in X$$

for decision space solution $x^{h1}$.

**Step 7.**

Let $z^{h1} = z(x^{h1})$. Compare $z^{h1}$ with $z^*$.

**Step 8.**

If all components of $z^{h1}$ are satisfactory, stop with $(z^{h1}, x^{h1})$ as the final solution. Otherwise, go to Step 9.

**Step 9.**

Specify the index set $J^*$ of criterion values to be relaxed and specify the amounts $(\triangle_j, j \in J^*)$ by which they are to be relaxed. Also specify the maximum amounts $(\triangle_j, j \notin J^*)$ by which they are to be improved.

**Step 10.**

Choose the index $k (k \in J^*)$, which indicates the index of criterion values to be sacrificed the most.

**Step 11.**

Solve the dual linear program $D\lambda^h$ to problem $P\lambda^h$. Let $u^h$ be the vector of optimal dual variables corresponding to constraints (1) in problem $P\lambda^h$.

**Step 12.**

Let $\hat{\lambda}^h = (u^h + \lambda^h)$.

**Step 13.**

Solve the linear program $(P_h)$

$$\min \langle c_k, x \rangle$$

subject to

$$(\hat{\lambda}^h)^T Cx = (\hat{\lambda}^h)^T Cx^{h1}$$

$$c_j x^{h1} - \triangle_j \leqq c_j x \leqq c_j x^{h1} \qquad j \in J^*$$

$$c_j x^{h1} \leqq c_j x \leqq c_j x^{h1} + \triangle_j \qquad j \notin J^*$$

$$x \in X$$

for decision space solution $x^{h2}$.

**Step 14.**

Let $z^{h2} = z(x^{h2})$. Compare $z^{h2}$ with $z^*$.

**Step 15.**

If all components of $z^{h2}$ are satisfactory, stop with $(z^{h2}, x^{h2})$ as the final solution. Otherwise, go to Step 4 by letting $x^{h2} = x^h$ and $z^{h2} = z^h$.

Some of these steps will now be discussed in detail.

In Step 1, if the possibility exists that some objectives have a alternative optima, a way to assure that only efficient solutions are generated is to lexicographically maximize the objective [19].

In Step 2, to be guaranteed that the procedure produces the solution in fewer iteration than the number p of objective functions, the index $i(i \in I^*)$ chosen at this step will no longer be eligible for further relaxation. Thus the procedure will terminate itself, if the DM does not terminate it first. This is not really an important practical consideration, however, since the DM would be expected to terminate the procedure himself in most problem situation after relatively few iterations.

In Step 3, payoff table column minimums are used in place of the minium criterion values over the efficient set because these minimum criterion values are difficult to obtain [3]. Notice the purpose of Term 1 is to place the most weight on the objectives with the greatest relative ranges. Term 2 normalizes the gradients of the objective functions according to the $L_2$-norm.

In Step 5, to ensure that only efficient solutions are generated at step 6, for $i \in I^*$, $\lambda_i$ shoud be assigned the value of $\varepsilon$, which is an arbitrary small and positive constant.

In Step 6, it is solved for the efficient solution which may not correspond to an extreme point of the original MOLP problem. This may be desirable in most decision situations since the exploration of the interior of efficient facets is possible.

In Step 8 and Step 15, as long as some criterion vector components are more satisfactory than others, iteration should be kept because current situation can be improved by making tradeoffs.

In Step 13, the purpose is to find an efficient solution which is the most preferred point to the DM among the points in the efficient face containing the solution $x^{h1}$ found at Step 6. At this step, the DM is required to give the specification of the maximum improvement quantities of the criterion values to be improved as well as the specification of relaxation quantities of the criterion values to be relaxed. It gives the maximum value $M_{k,t}$ (the minimum value $n_{k,t}$) the $k$th objective can take at iteration t, depending on $k \notin I^*$ $(k \in J^*)$. With reasonable DM's responses, $M_{k,t}(n_{k,t})$ would be expected to be nonincreasing (nondecreasing) as iteration goes. Thus keeping $M_{k,t} \geqq M_{k,t+1}$ and $n_{k,t} \leqq n_{k,t+1}$ would keep the DM from making possible errors in judgment that the DM may inadvertently commit in giving some of his responses. Based on the DM's responses, the feasible set of the linear programming problem ($P_h$), which is the subset of the efficient face containing the solution $x^{h1}$, is determined. An efficient solution found at this step, if it is not the same as the solution $x^{h1}$ found at Step 6, is guaranteed to be more preferred solution to the solution $x^{h1}$. This is because the feasible set of the linear programming problem ($P_h$), which is the subset of the efficient face containing the solution $x^{h1}$, includes only the efficient solutions interesting to the DM. Solving the linear programming problem ($P_h$) also makes the DM possible to examine the region around the solution $x^{h1}$ by narrowing down the range each criterion can be changed.

## 4. AN ILLUSTRATION WITH A SMALL EXAMPLE

To illustrate the suggested implementation of the interactive face search procedure, consider the following MOLP problem.

$$
\begin{aligned}
&\text{Max} \quad -x_1 + 2x_2 \\
&\text{Max} \quad 2x_1 - x_2 + x_3 \\
&\text{Max} \quad x_1 + x_2 - 2x_3 \\
&\text{subject to} \\
&\quad 2x_1 + x_2 \leqq 16 \\
&\quad 8x_1 + 5x_2 \leqq 66 \\
&\quad 2x_1 + 3x_2 \leqq 27 \\
&\quad x_1 \geqq 0 \\
&\quad 0 \leqq x_2 \leqq 7 \\
&\quad 0 \leqq x_3 \leqq 2
\end{aligned}
$$

The sets X and $X_E$ are shown in Figure 1, where $X_E$ consists of the four two-dimensional faces of X which are shaded. Table 1 lists the extreme points of X.

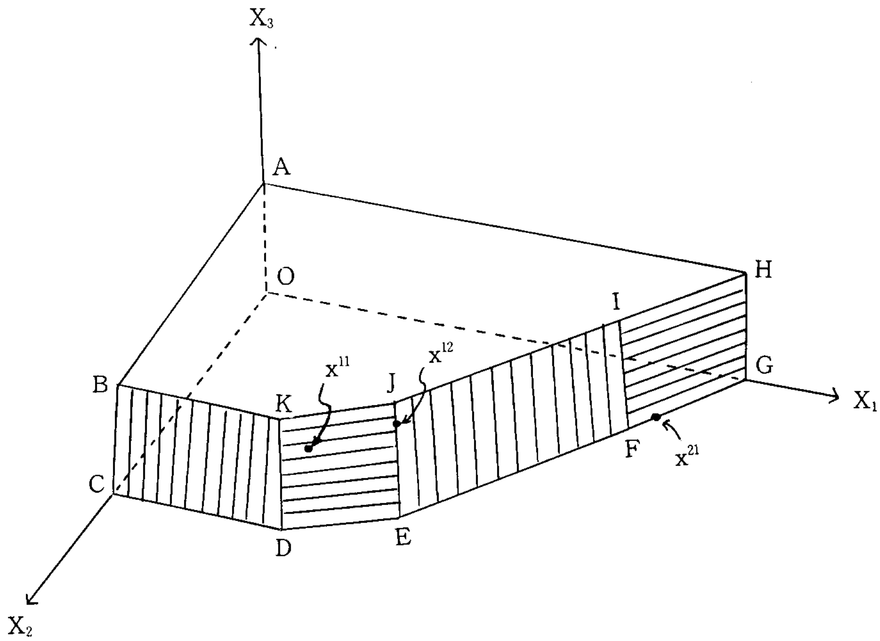


Figure 1. The Sets X and $X_E$

Table 1. Extreme Points of X

| Point | Coordinates |
|---|---|
| A | (0, 0, 2) |
| B | (0, 7, 2) |
| C | (0, 7, 0) |
| D | (3, 7, 0) |
| E | (4.5, 6, 0) |
| F | (7, 2, 0) |
| G | (8, 0, 0) |
| H | (8, 0, 2) |
| I | (7, 2, 2) |
| J | (4.5, 6, 2) |
| K | (3, 7, 2) |
| O | (0, 0, 0) |

**Step 1.**

By individually optimizing each objective function, the following three efficient points and their associated criterion vectors are found.

$x^1 = (0, 7, 0)$ $z^1 = (14, -7, 7)$

$x^2 = (8, 0, 2)$ $z^2 = (-8, 18, 4)$

$x^3 = (4.5, 6, 0)$ $z^3 = (7.5, 3, 10.5)$

Let the DM review these criterion vectors.

Suppose the DM wishes to try to find an improved soluion.

Let $x^3 = x^h$ and $z^3 = z^h$.

**Step 2.**

By constructing payoff table, $z^* = (14, 18, 10.5)$.

**Step 3.**

$h = 0$.

And $\pi_1 = 0.703$, $\pi_2 = 0.567$, $\pi_3 = 0.2916$.

**Iteration 1**

**Step 4.**

Suppose the DM choose $I^* = \{3\}$ and $\triangle_3 = 3.5$.

**Step 5.**

$h = 1$.

And $\lambda_1 = 0.45$, $\lambda_2 = 0.363$, $\lambda_3 = 0.001$.

(Notice : $\lambda_3$ is arbitrary given by the value of 0.001)

**Step 6.**

$x^{11} = (3.88, 6.41, 1.65)$ is found. [refer to Figure 1]

**Step 7.**

$z^{11} = (8.94, 3, 7)$. Compare this with $z^*$.

**Step 8.**

Suppose the DM wishes to try to find an improved solution.

**Step 9.**

Suppose the DM responses that $J^* = \{1\}$, $\triangle_1 = 10$ and for $j \notin J^*$, $\triangle_2 = 4$, $\triangle_3 = 1$.

**Step 10.**

Since $J^*$ has only one element, $k = 1$.

**Step 11.**

$u^h = (0, 0.0076, 0.1843)$.

**Step 12.**

$\hat{\lambda}^h = (u^h + \lambda^h) = (0.45, 0.3706, 0.1853)$.

**Step 13.**

Problem $(P_1)$ finds an efficient solution $x^{12} = (4.5, 6, 1.75)$ by minimizing the 1st objective function over the subset of the efficient face (□ DEJK) containing $x^{11}$. [refer to Figure 1]

**Step 14.**

$z^{12} = z(x^{12}) = (7.5, 4.75, 7)$.

**Step 15.**

Suppose the DM wihes to try to find an improved solution.

By letting $x^{12} = x^h$, go to Step 4.

**Iteration 2.**

**Step 4.**

Suppose the DM choose $I^* = \{1\}$ and $\triangle_1 = 11$.

**Step 5.**

$h = 2$.

And $\lambda_1 = 0.001$, $\lambda_2 = 0.363$, $\lambda_3 = 0.187$.

(Notice $\lambda_1$ is arbitrary given by the value of 0.001)

**Step 6.**

$x^{21} = (7.1, 1.8, 0)$ is found. [refer to Figure 1]

**Step 7.**

$z^{21} = (-3.5, 12.4, 8.9)$. Compare this with $z^*$.

**Step 8.**

Suppose the DM is satisfied with all components of $z^{21}$.

Stop with $(z^{21}, x^{21})$ as the final solution.

## 5. CONCLUSIONS

In practical multiple objective linear programming applications, there is usually no need for the decision maker to consider solutions which are not efficient. Therefore, the interactive procedure presented here searches only among efficient solutions and terminates with a solution that is guaranteed to be efficient. It also can converge to nonextreme efficient final solutions rather than being restricted to only extreme efficient points of the feasible set. It does not require the DM to have a mathematical knowledge of the nature of efficient solutions or of the efficient set.

One of the most attractive feature of the procedure however, is that the method allows the DM to examine the efficient faces founded at each iteration. At each iteration, if the DM wants to find an improved solution, the subset of the efficient face containing the current solution is determined by the DM's responses. Then, the procedure finds an efficient solution which is preferred to the current solution by examing the subset determined by the DM's responses. Since the subset of the efficient face containing the current solution includes only the efficient solutions interesting to the DM, the procedure examining the subset of the efficient face containing the current solution gives the DM a better chance to find a satisfying solution. This feature would enable the DM to reach a satisfying solution in relatively few iterations. It also makes the DM possible to examine the region around the current solution. As iteration goes, the DM can explore a wide variety of efficient faces rather than efficient faces confined to only certain subregion of X since the efficient faces that the procedure finds need not be adjacent. This helps the DM explore the nature of the efficient set $X_E$ and also helps the DM have confidence with a final solution. For these reasons, I feel that the procedure offer significant promise in solving multiple objective linear programs rapidly and in a satisfying manner to the DM.

## REFERENCES

1. Benayoun, R., de Montgolfier, J., Tergny, J. and Laritchev, O., "Linear Programming with Multiple Objective Functions : Step Method (STEM)," Mathematical Programming, vol.1, pp. 366−375, 1971.
2. Benson, H. P., "Existence of Efficient Solutions for Vector Maximization Problems," Journal of Optimation Theory and Applications, vol.26, pp.569−580, 1978.
3. Benson, H.P., "Optimization over the Efficient Set," Journal of Mathematical Analysis and Applications, vol.98, pp.562−580, 1984.
4. Benson, H. P., and Sayin, S., "A Face Search Heuristic Algorithm for Optimizing over the Efficient Set," Discussion Paper, Department of Decision and Information Sciences, University of

Florida, Gainesville, Fl, 1991.
5. Dyer, J. S., "Interactive Goal Programming," Management Science, vol.19, pp.62-70, 1972.
6. Evans, G. W., "An Overview of Techniques for Solving Multiobjective Mathematical Programs," Management Science, vol.30, pp.1268-1282, 1984.
7. Fichefet, J., "GPSTEM : An Interactive Multiobjective Optimization Method," pp.317-332. In : A. Prekopa(Ed.), Progress in Operations Research, vol.1, 1976.
8. Grauer, M., Lewandowski, A., and Wierzbicki, A., "DIDASS-Theory, Implementation and Experiences," pp.22-30. In : Lecture Notes in Economics and Mathematical Systems, No.229, Springer, Berlin, 1984.
9. Geoffrion, A.M., J.S. Dyer and A. Feinberg., "An Interactive Approach for Multicriterion Optimization with an Application to the Operation of an Academic Department," Management Science, vol.19, pp.357-368, 1972.
10. Huckert, K., R. Rhode, O. Roglin and R. Weber., "On the Interactive Solution to a Multicriteria Scheduling Problem," Zeischrift fur Opns. Res, vol.24, pp.47-60, 1980.
11. Kendall, K. E., and S. M. Lee., "Formulating Blood Rotation Policies with Multiple Objectives," Management Science, vol.26, pp.1145-1157, 1980.
12. Kok, M., and F. A. Lootsma., "Pairwise-Comparison Methods in Multiple Objective Programming, with Applications in a Long-Term Energy Planning Model," European Journal of Operational Research, vol.22, pp.44-45, 1985.
13. Marcotte, O., and R. M. Soland., "An Interactive Branch-and-Bound Algorithm for Multiple Criteria Optimization," Management Science, vol.32, pp.61-75, 1986.
14. Masud, A. S., and Hwang, C. L., "Interactive Sequential Goal Programming," Journal of Operational Research Society, vol.32, pp.391-400, 1981.
15. Murty, K.G., Linear Programming, Wiley, New York, 1983.
16. Ross, G. T., and R. M. Soland., "A Multicriteria Approach to the Location of Public Facilities," European Journal of Operational Research, vol.4, pp.307-321, 1980.
17. Spronk, J., and Telgen, J., "An Ellipsoidal Interactive Multiple Goal Programming Method," pp.380-387, Lecture Notes in Economics and Mathematical System, No.190, Springer, Berlin, 1981.
18. Steuer, R. E., "An Interactive Multiple Objective Linear Programming Procedure," TIMS Studies in the Management Sciences, vol.6, pp.225-239, 1977.
19. Steuer, R.E., Multiple Criteria Optimization : Theory, Computation, and Application. John Wiley, New York, 1986.
20. Steuer, R. E., and Choo, E. U., "An Interactive Weighted Tchebycheff Procedure for Multiple Objective Programming," Mathematical Programming, vol.26, pp.326-344, 1983.

21. Tabot, F. B., "Resource-Constrained Project Scheduling with Time Resource Tradeoffs : The Nonpreemptive Case," Management Science, vol.28, pp.1197-1210, 1982.
22. Yu, P.L., Multiple-Criteria Decision Making, Plenum, New York, 1985.
23. Zionts, S., and D. Deshpande., A Time Sharing Computer Programming Application of a Multiple Criteria Decision Method to Energy Planning-A Progress Report. In Multiple Criteria Problem Solving, pp.549-560, S.Zionts(ed.). Springer-Verlag, New York, 1978.
24. Zionts, S., and Wallenius, J., "An Interactive Programming Method for Solving the Multiple Criteria Problem," Management Science, vol.22, pp.652-663, 1976.