Error Probability of a DS/SSMA System with Hard-Limiting Correlators

부호상관기를 쓰는 DS/SSMA 계통의 오류확률

Jin Seon Yun*, Iickho Song**, Seong Ill Park**
윤 진 선*, 송 익 호**, 박 성 일**

ABSTRACT

In this paper the performance of a DS/SSMA system with hard-limiting correlation receivers is analyzed over nonselective Rician fading channels with impulsive noise. More specifically, computation of the average bit-error probabilities of the DS/SSMA system has been accomplished in this paper by exact computation for short spreading sequences and by approximation for long spreading sequences.

요 약

이 논문에서는 충격성 잡음이 있는 비선택적 라이시안 페이딩 채널에서 부호 상관 수신기를 쓰는 DS/SSMA 계통의 성능을 분석하였다. 이 계통의 평균 이진 오류 확률을 확산 수열이 짧을 때에는 정확한 계산으로, 확산 수열이 길 때에는 근사적인 방법으로 얻었다.

Ⅰ. Introduction

In several previous studies [e.g., 1-3], the performance of DS/SSMA communication systems has been investigated in a variety of circumstances under which we have, for example, Gaussian noise and fading channels.

In many practical communication problems, the usual Gaussian noise assumption sometimes becomes inadequate. For instance, the impulsive component of the interference in several communication porblems has been found to be significant and thus may not be ignored in such cases. In [2] a hard-limiting correlation receiver is considered as an alternative to linear receiver for reception of transmitted signals, and the performance of a DS/SSMA system with linear correlation receivers is analyzed in [3] when the channels show nonselective fading and impulsive noise characteristics.

In this paper, we will consider the multiple-access model and use a DS/SSMA system which makes use of hard-limiting correlation receivers, as an extension of the studies considered in [1-3]. An expression to compute the exact bit error probability of the DS/SSMA system using hard-limiting correlation receivers in impulsive noise is derived to analyzed the performance of the system for short speading sequences over nonselective Rician fading channels. We will then obtain an apprixmation to the bit error probability for

* 현대전자
** 한국과학기술원 전기 및 전자공학과
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long spreading sequences.

II. The system and channel model

Let us consider an asynchronous system which allows K users to share a common channel, for which the transmitted signal modulated by binary phase shift keying (BPSK) for the k th user is

$$s_k(t) = \mathrm{Re}\{\sqrt{2P_k}\, b_k(t)a_k(t)\exp(j\theta_k)\exp(j\omega_c t)\}. \tag{1}$$

In (1) P_k is the power of the k th transmitted signal, ω_c is the carrier frequency, and θ_k is the phase of the k th signal. The data signal $b_k(t)$ consists of rectangular pulses $V_T(t)$ of duration T which takes on values $+1$ and -1 with equal probability. That is,

$$b_k(t) = \sum_{m=-\infty}^{\infty} b_m^{(k)}\, V_T(t-mT), \tag{2}$$

where $b_m^{(k)} \in \{-1, +1\}$ is the m th data bit of the k th user. The code waveform $a_k(t)$ generated by a spreading sequence assigned to the k th signal is

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)}\, V_{T_c}(t-jT_c), \tag{3}$$

where T_c is the chip length and the sequence $\{a_j^{(k)}\}$ is a periodic sequence with period $N = T/T_c$ for each $a_j^{(k)} \in \{-1, +1\}$. Here N is also the number of chips.

The received signal $r(t)$ at a given receiver over nonselective Rician fading channels with impulsive noise can be expressed as

$$r(t) = \sum_{k=1}^{K} y_k(t-\tau_k) + n(t), \tag{4}$$

where, for $lT \le t < (l+1)T$ and $k = 1, 2, \cdots, K$,

$$y_k(t) = s_k(t) + \sqrt{2P_k}\, b_k(t)a_k(t)\gamma_k A_l^{(k)} \cos(\omega_c t + \theta_l^{(k)} + \theta_k), \tag{5}$$

$n(t)$ represents the channel noise, and τ_k is the delay of the k the signal. The attenuation of the signal due to the fading is represented by $\gamma_k A_l^{(k)}$, and the phase shift due to the fading is denoted by $\theta_l^{(k)}$. Here, the attenuation parameter γ_k is a nonnegative number, the nonnegative random variable $A_l^{(k)}$ satisfying the normalization constraint $E\{[A_l^{(k)}]^2\} = 1$ is assumed to have a Rayleigh distribution, and the phase shift $\theta_l^{(k)}$ is assumed to be uniform over $[0, 2\pi]$. We assume that $\theta_1 = 0$ and $\tau_1 = 0$.

The first-order probability density function (pdf) of the noise sample η_j is

$$f_{\eta_j}(x) = (1-\varepsilon) f_N(x) + \varepsilon f_I(x), \tag{6}$$

where $\varepsilon \in [0, 1]$ and f_N and f_I are the pdf of nominal and impulsive components, respectively. This model is called the ε-mixture noise model [4]. The ratio of the variance of impulsive component to that of the nominal component, defined as $\upsilon^2 = \sigma_I^2/\sigma_N^2$, is usually assumed to be in the order between 1 and 100 [4].

The test statistic Y_N in Figure 1 is

$$Y_N = \sum_{j=0}^{N-1} Z_j^{(1)} = \sum_{j=0}^{N-1} a_j^{(1)} sgn(Z_j), \tag{7}$$

where $sgn(\cdot)$ denotes the signum function.

Assuming equally likely bit polarities, the average bit-error probability using the test statistic of (7) can be written as [2]

$$\overline{P}_e = \frac{1}{2}\, Pr[Y_N \ge 0 \mid b_0^{(1)} = -1] + \frac{1}{2}\, Pr[Y_N \le 0 \mid b_0^{(1)} = +1]. \tag{8}$$

Our statistical assumptions on the multiple access interference are as follows. The variables $b^{(k)}$ $(b_{-1}^{(k)}, b_0^{(k)})$, τ_k, $\phi_k \triangleq \theta_k - \omega_c\tau_k$, $A_l^{(k)}$, and $\theta_l^{(k)}$, $2 \le k \le K$, are mutually independent random variables

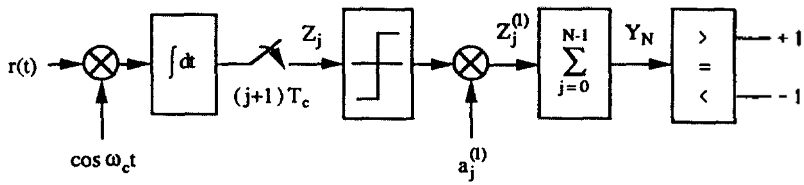

Figure 1. The DS/SSMA hard-limiting correlation receiver

with $\tau_k \sim U[0, T]$ and $\phi_k \sim U[0, 2\pi]$. It is assumed that $b_m^{(k)}$, $-\infty < m < +\infty$, is a sequence of independent data bits for each k and that $P_r(b_m^{(k)} = -1) = P_r(b_m^{(k)} = +1) = 1/2$ for each k and m. We also assume that the above random variables are independent of the channel noise and of $b_m^{(1)}$ for all m.

Ⅲ. Analysis of the DS/SSMA communication system

3.1 Single user case

The noise samples can be expressed as

$$\eta_j = \int_{jT_c}^{(j+1)T_c} n(t)\cos\omega_c t \, dt, \tag{9}$$

where the noise samples, $\eta_0, \eta_1, \cdots, \eta_{N-1}$, are assumed to be independent and identically distributed (i.i.d.) random variables with mean zero and variance $N_0T_c/4$.

Since $sgn(x) \in \{-1, +1\}$, the test statistic Y_N is an odd integer assuming that N is odd. Because of the symmetry of the system, $\overline{P}_e$ for the hard-limiting correlator can be written as

$$\overline{P}_e = \sum_{m=1}^{N} Pr[Y_N = m | b_0^{(1)} = -1]. \tag{10}$$

When $K=1$ (single-user), it is easy to see that the random variables $(Y_N + N)/2$ is binomially distributed under either bit condition. Therefore we have

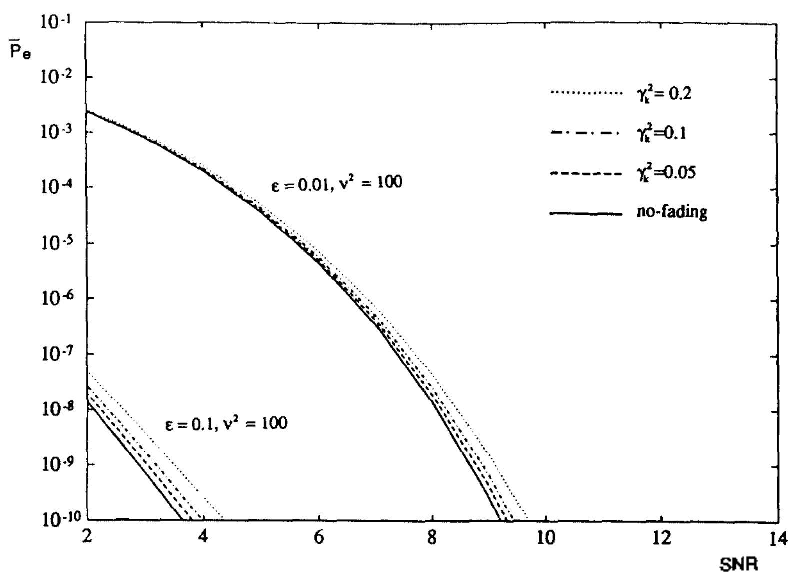

Figure 2. Single-user error probability over nonselective Rician fading channels with the ε mixture noise, $N = 31$.

$$\overline{P}_e = \sum_{j=(N+1)/2}^{N} \binom{N}{j} p^j (1-p)^{N-j}, \tag{11}$$

where $p \underline{\Delta} P_r[\eta_j^{(1)} - \delta_0^{(1)} \geq 1]$ with

$$\eta_j^{(1)} = \frac{a_j^{(1)} \eta_j}{\sqrt{P_1/2}\ T_c}, \tag{12}$$

and

$$\delta_0^{(1)} = \gamma_1 A_0^{(1)} \cos\theta_0^{(1)}. \tag{13}$$

In Figure 2 we plot the single-user performance (11) for several channels when $N=31$. It is seen that the performance over fading channels becomes worse as the effects of fading becomes larger. It is also shown that the average bit error probability decreases as the contribution of impulsive noise increases at the same SNR.

3.2 Multiuser case

From the symmetry of the system, it follows that the average bit-error probability for the receiver can be written as, using (8),

$$\overline{P}_e = E\{\sum_{m=1}^{(N+1)/2} P_r(Y_N = 2m-1 | b_0^{(1)} = -1, b, \tau, \phi, \theta, A)\}, \tag{14}$$

where $b = (b_{-1}^{(2)}, b_0^{(2)}, b_{-1}^{(3)}, b_0^{(3)}, \cdots, b_{-1}^{(K)}, b_0^{(K)})$, $\tau = (\tau_2, \tau_3, \cdots, \tau_K)$, $\phi = (\phi_2, \phi_3, \cdots, \phi_K)$, $\theta = (\theta_{-1}^{(2)}, \theta_0^{(2)}, \cdots, \theta_{-1}^{(K)}, \theta_0^{(K)})$, and $A = (A_{-1}^{(2)}, A_0^{(2)}, \cdots, A_{-1}^{(K)}, A_0^{(K)})$.

It is straightforward to see that the quantity $(Y_N+N)/2$ has a multinomial distribution which is related to Bernoulli trials with variable probabilities. The probability of $Y_N = m$ for odd m's between 1 and N can thus be written as

$$P_r[Y_N = m | b_0^{(1)} = -1, \rho_l] = q(N, \frac{N+m}{2}), \tag{15}$$

where $q(j, n)$ denotes the probability that n of $Z_0^{(1)}, Z_1^{(1)}, \cdots, Z_{j-1}^{(1)}$ are $+1$'s and satisfies [5] $q(j, n) = p_{j-1} q(j-1, n-1) + (1-p_{j-1}) q(j-1, n)$ with p_l denoting the probability that the j th input of the accumulator is $+1$: i.e., $p_j \underline{\Delta} P_r[Z_j^{(1)} = +1 | b_0^{(1)} = -1, \rho_l]$ for $j = 0, 1, \cdots, N-1$, where ρ_l is used to denote the collection of the random variables $\{b, \tau, \phi, \theta, A\}$.

Now the probabilities p_j, $j = 0, 1, \cdots, N-1$, can be written as [6]

$$p_j = Pr[\eta_j^{(1)} + I_j^{(1)}(\rho_l) - \delta_0^{(1)} \geq 1 | \rho_l], \tag{16}$$

where the jth sample $I_j^{(1)}(\rho_l)$ of the interference is given as

$$I_j^{(1)}(\rho_l) = a_j^{(1)} \sum_{k=2}^{K} \frac{\sqrt{\varepsilon_{k,1}}}{T_c} [B(j, m_k) a_{j-m_k-1}^{(k)} \tau_k + \hat{B}(j, m_k) a_{j-m_k}^{(k)} (T_c - \tau_k)]. \tag{17}$$

In (17), $\tau_k \underline{\Delta} \tau_k - m_k T_c$, $m_k \underline{\Delta} [\tau_k / T_c]$, $\varepsilon_{k,1} \underline{\Delta} P_k / P_1$,

$$B(j, m_k) \underline{\Delta} \begin{cases} H(-1) & 0 \leq j \leq m_k, \\ H(0) & m_k + 1 \leq j \leq N-1, \end{cases} \tag{18}$$

and

$$\hat{B}(j, m_k) \underline{\Delta} \begin{cases} B(j, m_k) & m_k \neq j \\ H(0) & m_k = j \end{cases} \tag{19}$$

where

$$H(n) = [\cos\phi_k + \gamma_k A_n^{(k)} \cos(\theta_n^{(k)} + \phi_k)] b_n^{(k)}. \tag{20}$$

Starting from (16) we first calculate $q(\cdot, \cdot)$, which is then used to evaluate the exact value of the average bit-error probabilities of the hard limiting correlation receiver over fading channels with impulsive noise from (14) and (15).

Let us consider the two user case employing the max-SNR auto optimal least sidelobe energy (AO LSE) sequences chosen from [7, Figure A. 1]. In Figure 3 we plot the average bit error probabilities obtained by simulation using (14) versus SNR for several channels under the assumption that the two users have the same power when $N=31$. The effects of fading and impulsive noise on the performance are similar to those for the single user case. Figure 3 clearly shows the effects of adding a second user are more substan-

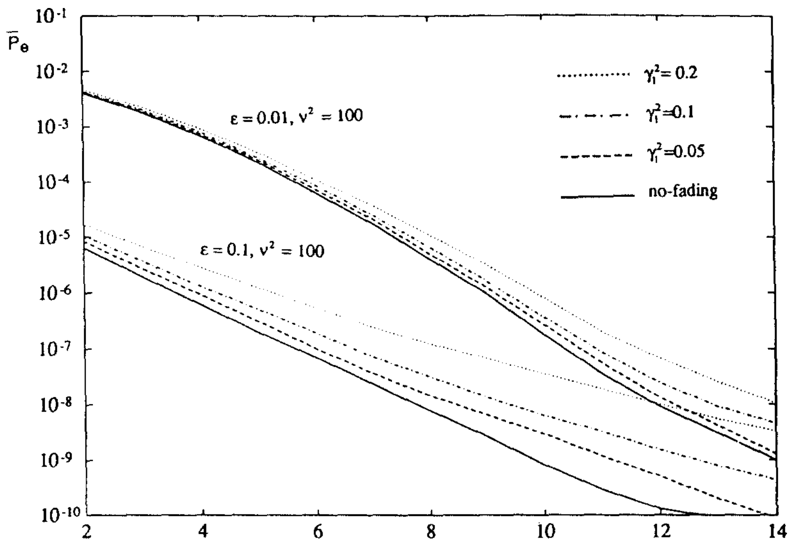

Figure 3. Two-user error probability over nonselective Rician fading channels with the ε mixturenoise, $N=31$.

tial for high SNR.

3.3 Binomial approximation

We can approximate the multinomial distribution by binomial distribution for large values of N. In particular, let us approximate (14) as

$$\overline{P}_e \approx E\{ \sum_{j=(N+1)/2}^{N} \binom{N}{j} q_l^j (1-q_l)^{N-j} \mid \rho_l \}, \tag{21}$$

where q_l is the "success" probability associated with the approximating binomial distribution. Thus we have

$$q_l = \frac{1}{N} \sum_{j=0}^{N-1} Pr[\eta_j^{(1)} - \delta_0^{(1)} \geq 1 - I_j^{(1)}]. \tag{22}$$

Using (6), (12), and (13), and noting that $\delta_0^{(1)}$ is a Gaussian random variable, q_l is given as

$$q_l = \frac{1}{N} \sum_{j=0}^{N-1} [(1-\varepsilon) Q\left[\frac{1-I_j^{(1)}}{\sqrt{\sigma_n^2 + \gamma_1^2/2}} \right] + \varepsilon Q\left[\frac{1-I_j^{(1)}}{\sqrt{v^2\sigma_n^2 + \gamma_1^2/2}} \right]], \tag{23}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left\{ -\frac{y^2}{2} \right\} dy, \tag{24}$$

and

$$\sigma_n^2 = \frac{N_0 N}{2E_b^{(1)}(1-\varepsilon+\varepsilon v^2)} \tag{25}$$

As we can see from Table 1, which is obtained when $K=2$, $\varepsilon=0.01$, $v^2=100$, and $SNR=8$dB, the value obtained by the approximation (21) is closer to the exact value as the number of the chips becomes larger or as the fading becomes less important.

Table 1. Binomial approximation to the average bit-error probability when $\varepsilon=0.01$, $v^2=100$, $SNR=8.0$dB, and $K=2$.

N		$\gamma_k=0.5$	$\gamma_k=0.1$	$\gamma_k=0.2$
31	P_e^B	3.369e-5	4.399e-5	7.072e-5
	$\overline{P}_e$	5.000e-6	6.617e-6	1.091e-5
63	P_e^B	1.967e-6	2.448e-6	3.701e-6
	$\overline{P}_e$	4.655e-7	5.584e-7	8.043e-7
127	P_e^B	2.309e-7	2.714e-7	3.690e-7
	$\overline{P}_e$	8.770e-8	1.002e-7	1.305e-7

Ⅳ. Conclusion

In this paper, we investigated the performance of a DS/SSMA system with hard-limiting correlation receiver over fading channels with impulsive noise.

We first considered the single-user performance of the hard-limiting correlation receiver. An expression for the average bit-error probability was obtained.

The two-user performance of the hard-liming correlation receiver was then considered as a special case of multiuser performance. Based on an expression for the average bit-error probability of the hard-limiting correlation receiver, we calculated the average bit error probability in various cases of fading, impulsive noise, and multiple-access interference.

The performance of the hard-limiting correlation receiver was shown to be generally better when the noise is more impulsive. The effects of fading on the performance was shown to be more clear for multiuser case than for single-user case. Multiple-access interference was clearly shown to degrade the performance considerably.

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▲尹 眞 鮮(Jin Seong Yun) 12권 2호 참조
현대전자 산업전자연구소 연구원

▲宋 翊 鎬(Iickho Song) 12권 2호 참조
한국과학기술원 전기 및 전자공학과 부교수

▲朴 聖 日(Seong Ill Park) 12권 2호 참조
한국과학기술원 전기 및 전자공학과 박사과정