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조파저항 계산을 위한 자유표면 조건의 비교

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Comparison of Free-Surface Boundary Conditions for Computing Wave Resistance

by

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요 약

Poisson[1], Ogilvie[2], Dawson[3] 등에 의해 사용된 자유표면에서의 경계조건을 2차원 몰수체를 대상으로 택하여 동일한 패널법을 이용, 비교검토하였다. 또한 Poisson 경계조건을 사용하여 몰수된 구와 Wigley 선형에 대해 구한 결과도 나타내었다. 결론적으로 검토된 경계조건들보다는 더 정확한 결과를 줄 수 있고, 또 엄밀한 비선형 경계조건 보다는 보다 실용성있는 경계조건을 새로이 유도할 필요가 있는 것으로 생각된다.

Abstract

In computing the wave resistance numerically, satisfying the boundary condition(BC) on the body surface is not so difficult, and then what form of the BC on the free surface(FS) be used is a crucial question. To shed some light on this, we examine the various BC's on the FS, namely, the Poisson's[1], the Ogilvie's[2] and the Dawson's[3] BC, using the same panel method for submerged bodies in two-dimension. We also show the performance of the Poisson's BC for a submerged sphere and the Wigley hull. It seems that we are still in need of a theory which gives a BC on the FS more accurate than those tested, and more practically applicable than the exact nonlinear BC.

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INTRODUCTION

Wind waves on the surface of lakes, ocean waves coming endlessly towards the beach and the waves generated by moving bodies have ever attracted human minds. The Cauchy-Poisson problem, in which the displacement and the velocity of the FS at the initial instant are given and the evolution of the free surface at the later time is sought, was perhaps the first one analyzed in terms of hydrodynamics. We may regard Poission[1] as the first who used the linearized free surface boundary condition(FSBC).

Kelvin[4] started successfully the investigation on the FS waves generated by a disturbance moving either on or beneath the FS. And it was Michell[5] who derived for the first time an integral formula, which now bears his name, giving the wave resistance of a ship that can be approximated as thin. In the first half of this century, most studies on the wave resistance of ships were done around the Michell's integral. However, as it became evident that the prediction of the Michell's integral is inaccurate for the practical range of the Froude number(F_n) of most ships, the search for the more accurate method for computing the wave resistance began around early 1960's.

The boundary value problem for the wave resistance acted on a ship moving with a uniform speed(U) on the FS of an inviscid fluid consists of the Laplace equation, which is the law of mass conservation for an irrotational flow of an incompressible fluid, subject to the suitable BC's on the FS, on the body surface and on infinity. Since the displacement of the FS is not known *a priori*, the BC's on the FS is essentially nonlinear, and there are one kinematic and one dynamic BC. The kinematic BC says that the fluid particle once on the FS should remain there all the time, and the dynamic BC demands the constancy of the pressure on the FS. On the body surface must hold the condition of impermeability, which requires that the normal component of the fluid velocity vanish there. The BC at infinity, often called the radiation condition, is due to the assumption of reaching the steady state in a finite time, and imposes limitations on the asymptotic behavior of the solution both far upstream and far downstream.

Michell's thin ship theory assumes that the ratio of beam to length of a ship is so small that the BC on the body surface can be applied at the centerplane of the ship and that the BC on the FS may be applied at the undisturbed FS both in the linearized form. Therefore, the effort for seeking more refined methods than the thin ship theory was focused upon how to satisfy the above two BC's more accurately, and there have been considerable developments since 1960's along this line.

At first, naturally there were higher order thin ship theories, but people soon realized that going for the higher order does not improve the situation and that the higher order corrections yielded even worse results for the Fn's as was pointed out by Ogilvie[6]. One of the significant lessons we learned form the higher order theory seems due to Eggers [7]. He asserted that the second order effect of the FSBC is much smaller than that of the BC on the body surface for a full ship at low Fn's. This finding appears to have given an encouraging support for the later developments of the Neumann-Kelvin approach. Satisfying the exact BC of the Neumann type on the wetted surface of a ship in her static equilibrium state, along with the FSBC in linearized form, sometimes called the Kelvin equation, Brard[8] derived the so-called Neumann-Kelvin problem for computing the wave resistance of a ship. Though this approach is inconsistent from the viewpoint of the perturbation method, it received a broad acceptance as giving a practical tool for the numerical computation of the wave resistance(see Baar & Price[9]). The low Fn problem was taken up by Ogilvie[2], who made use of the double-body flow as the basic one in deriving the FSBC for the perturbation potential. Using the double-body flow not the uniform flow as the basic one was at least then rather an unusual idea, which was to give a strong impact upon the way how to compute the wave resistance afterwards. Furthermore, it was timely because obtaining the double-body flow around a body of arbitrary shape became easier and less expensive owing to the fast development of computer and of the panel method(see Hess & Smith[10]). Baba & Takekuma[11] extended Ogilvie's argument to a three-dimensional flow around a ship, and claimed to get good results. However, Inui & Kajitani [12] pointed out that the Baba's method failed to eliminate the exaggerated humps and hollows of the wave resistance curve.

By the end of 1970's many different methods were competing as a means of computing the wave resistance of a ship, and the need to compare them in a unified fashion resulted in the Workship at the DTNSRDC(see Bai[13]). Two outcomes of the Workship are noteworthy. Firstly, the scattering between the participants' results was too big even when the same theoretical formula, for instance the Michell's integral for the Wigley hull, was used. Secondly, Dawson's method looked most promising because of its simplicity in implementation and of its excellent performance for various hull forms in the wide range of Fn's. Taking the double-body flow as the basic one, Dawson derived a new form of the FSBC and used it in his panel code which was evolved form the works of Hess & Smith[10]. Throughout the 1980's Dawson's method became very popular, but there were very few basic studies on the characteristics of the Dawson's FSBC until very recently.(see Raven[14].)

Furthermore, Raven[15] also reported that he obtained almost the same values of the wave resistance and the surface elevation, when he implemented in the same panel code the Dawson's FSBC and the classical Poisson's FSBC, which is derived by taking the uniform flow as the basic one and will be named after Poisson, not Kelvin, in this study. This result is rather contrary to the common belief, and it seems necessary on this occasion that we compare the performance of the various FSBC's to attain more basic understandings. We chose the Poisson's, the Ogilvie's and the Dawson's FSBC, and implemented them in a panel code for two-dimensional submerged bodies. We also show the results given by Poisson's BC for a submerged sphere and the Wigley parabolic hull.

In the sequel, we first derive the above FSBC's not only for completeness but for showing their differences as well as similarities. Then follows a numerical aspect of the panel code we used, the numerical results and the discussion, consecutively.

FREE SURFACE BOUNDARY CONDITIONS

As to the coordinate system we take the x-axis on the calm water surface in the direction of the uniform flow, the z-axis vertically upward, and the y-axis so as to form a right-handed system. In this section for convenience we restrict ourselves to the two-dimensional case. We assume the existence of the total velocity potential $\Phi(x,z)$, then the kinematic BC on the FS $z=\zeta(x)$ is given by

$$\Phi_x\zeta' - \Phi_z = 0, \tag{1}$$

and the dynamic BC on the FS by

$$\zeta = -\frac{1}{2g}(\Phi_x^2 + \Phi_z^2 - U^2). \tag{2}$$

Here, the superscript $'$ to ζ denotes the differentiation with respect to x, and the subscripts x, z the partial differentiation. g is the acceleration of gravity. Eliminating ζ from the above two conditions, we get

$$\Phi_x^2\Phi_{xx}+2\Phi_x\Phi_z\Phi_{zx}+\Phi_z^2\Phi_{zz}+g\Phi_z=0$$
$$\text{on } z=\zeta(x), \tag{3}$$

which may be called the combined FSBC.

Poisson's FSBC

We decompose the total velocity potential into the velocity potential of the uniform flow, $\bar{u}_x$, and that of the perturbed flow, $\phi(x,z)$, i.e.,

$$\Phi(x,z) = Ux + \phi(x,z). \tag{4}$$

With this decomposition, equations (1) and (2) are rewritten as

$$(U+\phi_x)\zeta'-\phi_z=0 \qquad \text{on} \quad z=\zeta(x),$$
$$\zeta=-\frac{1}{2g}(2U\phi_x+\phi_x^2+\phi_z^2) \quad \text{on} \quad z=\zeta(x),$$

respectively. If we expand the above equations around z=0, and neglect nonlinear terms in ϕ and ζ, we obtain

$$U\zeta'-\phi_z=0 \qquad \text{on} \quad z=0, \qquad (5)$$
$$\zeta=-\frac{1}{g}U\phi_x \qquad \text{on} \quad z=0. \qquad (6)$$

Eliminating ζ from the above, we find the Poission's FSBC as

$$U^2\phi_{xx}+g\phi_z=0 \qquad \text{on} \quad z=0. \qquad (7)$$

Once ϕ is obtained, ζ can be computed from (6).

Ogilvie's FSBC

If the double-body flow, given in advance, is taken as the basic one, the total velocity potential may be decomposed as

$$\Phi(x,z)=\bar{\phi}(x,z)+\hat{\phi}(x,z), \qquad (8)$$

where $\bar{\phi}$ is the velocity potential of the double-body flow, and $\hat{\phi}$ a new perturbation potential. With this decomposition, equations (1) and (2) are given in the form

$$(\bar{\phi}_x+\hat{\phi}_x)\zeta'-(\bar{\phi}_z+\hat{\phi}_z)=0 \qquad (9)$$
$$\text{on} \quad z=\zeta(x),$$
$$\zeta=-\frac{1}{2g}\{(\bar{\phi}_x+\hat{\phi}_x)^2+(\bar{\phi}_z+\hat{\phi}_z)^2-U^2\}$$
$$\text{on} \quad z=\zeta(x), \qquad (10)$$

respectively. We may also decompose the FS elevation, $\zeta(x)$, into two parts

$$\zeta(x) = \bar{\zeta}(x)+\hat{\zeta}(x), \qquad (11)$$
$$\bar{\zeta}(x) = -\frac{1}{2g}\{\bar{\phi}_x^2(x,0)-U^2\}, \qquad (12)$$

where $\bar{\zeta}$ may be regarded as the FS elevation resulting from the basic double-body flow. Ogilvie [2] treated the low-speed limiting case, that is $U\rightarrow 0$, others being fixed, and derived a linear FSBC for the lowest order problem. Following him, we assume that

$$\bar{\phi}=O(U),\ \hat{\phi}=O(U^5),\ \bar{\zeta}=O(U^2),$$
$$\hat{\zeta}=O(U^4),$$
$$(\frac{\partial}{\partial x},\frac{\partial}{\partial z},\frac{d}{dx})$$
$$=\begin{cases} O(1), & \text{when operated on } \bar{\phi},\bar{\zeta}, \\ O(U^{-2}), & \text{when operated on } \hat{\phi},\hat{\zeta}. \end{cases}$$

Substituting the decomposition of ζ given in (11) into (9) and (10), and expanding the partial derivaties of $\bar{\phi}$ around $z=0$, while those of $\hat{\phi}$ around $z=\bar{\zeta}$, and taking only the lowest order terms of $O(U^3)$, we get

$$\bar{\phi}_x(x,0)\hat{\zeta}'-\hat{\phi}_z(x,\bar{\zeta})=h', \qquad (13)$$
$$\hat{\zeta}=-\frac{1}{g}\bar{\phi}_x(x,0)\hat{\phi}_x(x,\bar{\zeta}), \qquad (14)$$
$$h(x)=-\bar{\phi}_x(x,0)\bar{\zeta}(x). \qquad (15)$$

In deriving the above we made use of $\bar{\phi}_z(x,0)=0$ and $\bar{\phi}_{zz}=-\bar{\phi}_{xx}$. There is a correspondence between U in (5), (6) and $\bar{\phi}_x(x,0)$ in the above. We note that $h(x)$ is equivalent to the pressure distribution on the FS due to the basic double-body flow. Eliminating ζ from the above, and neglecting the higher order terms than $O(U^3)$, we find

$$\bar{\phi}_x^2(x,0)\hat{\phi}_{xx}(x,\bar{\zeta}) + g\hat{\phi}_z(x,\bar{\zeta}) = f', \quad (16)$$

$$f(x) = \frac{1}{2}\{U^2 - \bar{\phi}_x^2(x,0)\}\bar{\phi}_x(x,0). \quad (17)$$

Since the derivatives of the unknown function $\hat{\phi}$ are computed at $z=\bar{\zeta}$, the above equation is not a convenient form to apply. However, because of the assumption on the order of magnitude of the operator $\frac{\partial}{\partial z}$ applied on $\hat{\phi}$, it is not good to simply expand (16) around $z=0$. Ogilvie got around this difficulty by introducing the coordinate transformation

$$(x^*, z^*) = (x, z - \bar{\zeta}). \quad (18)$$

Then the relation between the partial derivatives in the new and the old variables is

$$(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}) = (\frac{\partial}{\partial x^*} - \bar{\zeta}'\frac{\partial}{\partial z^*}, \frac{\partial}{\partial z^*}). \quad (19)$$

Considering $\bar{\zeta}'=O(U^2)$, the governing equation in terms of the new variables to the leading order is the same Laplace equation. And the FSBC is given by

$$\bar{\phi}_x^2\hat{\phi}_{xx} + g\hat{\phi}_z = \frac{1}{2}[U^2 - 3\bar{\phi}_x^2(x-\bar{\xi})]\bar{\phi}_{xx}(x-\bar{\xi})$$

$$\text{on} \quad z = 0, \quad (20)$$

where the superscript * to the new independent variables is deleted for brevity. Here, the new transformed coordinate system is used only for obtaining $\hat{\phi}$, not for $\bar{\phi}$. We shall call (20) the Ogilvie's FSBC. Given the double-body potential $\bar{\phi}$, $\bar{\zeta}$ can be easily computed using (12). Then once the perturbation potential $\hat{\phi}(x,z)$ is obtained, $\zeta(x)$ can be found using (11) and

$$\hat{\zeta} = -\frac{1}{g}\bar{\phi}_x(x,0)\hat{\phi}_x(x,\xi) \quad (21)$$

Dawson's FSBC

Dawson[3] also took the double-body flow as the basic one. Unlike Ogilvie, who obtained his FSBC as a BC of the lowest order problem for the low-speed limiting case, Dawson tried to develop a practical numerical procedure for solving the boundary value problem of the wave resistance of a ship. This may explain why he need not make any assumption on the order of magnitude of the terms neglected in deriving his FSBC. As shown by Kim[16] unwittingly, Dawson's solution is equivalent to the result of the first iteration of Kim's iterative procedure which was set up to satisfy the exact nonlinear FSBC's (9) and (10).

Since $z=0$ is to be taken as the starting value of the unknown FS elevation, (9) and (10) are now satisfied on $z=0$. Then from these, eliminating ζ, we get the same equation as (3) but now satisfied on $z=0$. For $\Phi=\bar{\phi}$ is to be the initial guess of the unknown total potential, neglecting all the nonlinear terms in the derivatives of $\hat{\phi}$, we obtain

$$(\bar{\phi}_x^2\hat{\phi}_x)_x + g\hat{\phi}_z = -\bar{\phi}_x^2\bar{\phi}_{xx} \qquad \text{on} \quad z = 0. \quad (22)$$

Alternatively, replacing $\hat{\phi}$ by $\Phi-\bar{\phi}$, we get

$$(\bar{\phi}_x^2\Phi_x)_x + g\Phi_z = 2\bar{\phi}_x^2\bar{\phi}_{xx} \qquad \text{on} \quad z = 0, \quad (23)$$

which is the Dawson's FSBC for two-dimensional problems. For three-dimensional problems, he introduced the directional derivative along the streamline on the FS due to the double-body flow, and if we replace x-derivative by the streamline derivative in the obove equation, we obtain the equation (14) in Dawson[3]. Once Φ is known, we can get the FS elevation using (2) but now computed on $z=0$.

Comparison of the Ogilvie's FSBC (20) with the Dawson's (22) shows that the latter has on the left an additional term proportional to $\hat{\phi}_x$, and that the right of the former, if U is equated approximately to $\bar{\phi}_x(x,0)$, is equal to that of the latter.

Nevertheless, on the whole, effects of the basic double-body flow are represented differently in the two BC's. Which BC works better for a given problem remains to be seen.

NUMERICAL METHODS

A panel method in the spirit of Hess & Smith [10] and of Dawson[3] was used to solve the flow around two- and three-dimensional bodies. Discretizing the FSBC, we followed the fashion initiated by Dawson. In this section all lengths are non-dimensionalized by the length of the body L, and all velocities by U, thus

$$(x,z)=(x^*,z^*)L,\ (u,w)=(\phi_x,\phi_z)$$
$$=(u^*,w^*)U,\ \frac{U^2}{gL}=Fn^2=\frac{1}{KL}, \tag{24}$$

where K is the characteristic wave number. In the sequel the superscript * to the dimensionless variables is deleted for brevity. In Dawson[3], he described two operators of four-point finite difference for u_x. One of them has no errors from u_{xx} and u_{xxxx}, called the first kind four-point upwind differencing(4PUD) and gave a reasonable conservation of the wave amplitude. The other has no errors from u_{xx} and u_{xxx}, called the second kind 4PUD and caused the excessive growth of the wave amplitude downstream. For it is not so messy to show why such amplification occurs, we present a numerical error analysis on this using the modified equation method(Fletcher[17]) in the following(see also Van[18]). For convenience, we take the Poisson's FSBC, and rewrite it as

$$u_x+Fn^{-2}w=0 \qquad \text{on} \quad z=0. \tag{25}$$

We assume that the panel size h on the FS is constant, then the second kind 4PUD operator is defined by

$$\mathcal{D}_2u_j=\frac{1}{6h}(11u_j-18u_{j-1}+9u_{j-2}-2u_{j-3}), \tag{26}$$

$$u_j=u(x_j)=u(x_0+jh), \tag{27}$$

where x_0 is the x-coordinate of the upstream boundary of the computational domain. The discretized FSBC using $\mathcal{D}_2$ can be written as

$$\mathcal{D}_2u_j+Fn^{-2}w_j=0$$
$$\text{on}\quad z=0,\ \ j\in[1:N_f], \tag{28}$$

where N_f is the number of panels on the plane $z=0$. u_x with the leading order error term is expressed as

$$u_{xj}=\mathcal{D}_2u_j+\frac{1}{4}u_{xxxx}h^3. \tag{29}$$

And we may suppose at far downstream that

$$u(x)\sim exp(iKx),\quad \frac{\partial^n u}{\partial x^n}=(iK)^n u, \tag{30}$$

where $i=\sqrt{-1}$. Substituting (29) into (28), and making use of (30), we obtain

$$u_{xj}+Fn^{-2}w_j-\frac{1}{4}h^3K^4u_j=0$$
$$\text{on}\quad z=0. \tag{31}$$

According to Lamb[19](see § 242), the last term on the left hand side of the above equation corresponds to the negative damping, proportional to h^3, of the Rayleigh's type, which explains the amplification reported by Dawson.

The first kind 4PUD operator is definded by

$$\mathcal{D}_1u_j=\frac{1}{6h}(10u_j-15u_{j-1}+6u_{j-2}-u_{j-3}) \tag{32}$$

and if we carry out the similar analysis, we can find

$$u_{xj} + Fn^{-2}w_j + i(\frac{1}{6}h^2K^3 + \frac{11}{120}h^4K^5)u_j + \frac{1}{12}h^5K^6u_j = 0 \qquad \text{on } z = 0. \tag{33}$$

The last term on the left hand side of (33) has now the positive damping, proportional to h^5, of the Rayleigh's type, which shows why the first kind 4PUD resulted in a 'reasonable conservation' of the wave amplitude downstream. It should be clear that by the reasonable conservation what Dawson really meant was a reasonable damping. On the other hand, as the errors from the odd order derivatives, being the imaginary part of the left hand side in (33), affect the phase of the waves, the first kind 4PUD may change the phase of the solution more than the second kind.

In order to conserve the wave amplitude downstream as much as possible, we may use the linear combination of the first and the second kind 4PUD operators. We define $D_3=\alpha D_1+(1-\alpha)D_2$, where α is so determined that the resulting error in the wave growing downstream is minimized. The leading error for the combined operator D_3 is

$$\alpha(\frac{1}{12}h^5K^6) + (1-\alpha)(-\frac{1}{4}h^3K^4) = 0. \tag{34}$$

Therefore, we obtain

$$\alpha = \frac{1}{1+\beta}, \quad \beta = \frac{1}{3}h^2K^2 = \frac{4\pi^2}{3N_w^2}, \tag{35}$$

where N_w is the number of panels per a characteristic wavelength. α depends only on N_w, and for most two-dimensional computations reported here we used the combined 4PUD operator and $N_w=16$. Then the corresponding value of α predicted by (35) is 0.951, and from our computations we confirmed that the damping is the least when $\alpha=0.94$. Furthermore, through the simple analysis above, we see why N_w is an important parameter determining the wave damping characteristic of the numerical solution for two-dimensional problems, and also why it is crucial to use a 'good' panel arrangement for three-dimensional problems which include waves of all wavelengths.

Discretizing the Ogilvie's FSBC (20) and the Dawson's (23), we also utilized the combined 4PUD operator with a proper value of α given by (35).

RESULTS & DISCUSSIONS

Making use of the panel method described in the previous section, we first discuss the result for the two-dimensional submerged bodies, namely a circular cylinder and a foil used by Salvesen[19]. Then follows the discussion on two three-dimensional bodies, which are a submerged sphere and the parabolic Wigley hull.

Submerged circular cylinder

We consider the flow around a circular cylinder whose center is at $(0, -d)$ and diameter is L. According to Lamb[19](see § 247, 249), a linear solution satisfying the Poisson's FSBC along with an approximate body BC renders

$$R = \frac{1}{4}\rho g A^2, \quad A = \pi L Fn^{-2} exp(-Kd), \tag{36}$$

$$C_W = \frac{R}{\frac{1}{2}\rho U^2 L} = \frac{\pi^2}{2}Fn^{-6}exp(-2Kd), \tag{37}$$

where R is the wave resistance, A the wave amplitude far downstream, ρ the water density. Fig.1 shows C_W as a function of Fn given by (37) and the numerical Poisson-solution when $d=L$. C_W in the numerical computation is obtained by integrating C_p multiplied by the negative of the x-component of the unit outward normal vector along the body surface, and

$$C_p = \frac{p}{\frac{1}{2}\rho U^2}, \tag{38}$$

$$p = \frac{1}{2}\rho(U^2 - \Phi_x^2 - \Phi_z^2), \tag{39}$$

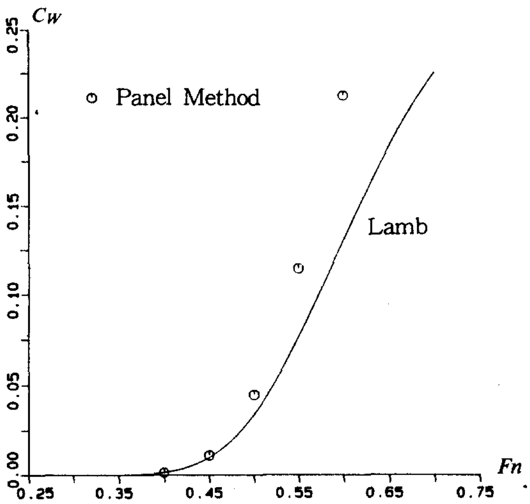

Fig.1 For a submerged circular cylinder whose submerged depth is the same as its diameter, C_W by the Poisson's FSBC is compared with the analytical results by the linear theory of Lamb[19].

i.e., p is the dynamic pressure. Noting that the numerical solution satisfies the BC on the body surface exactly with the possible numerical error in discretization only, the agreement between two predictions for $Fn<0.5$ and the disparity for $Fn>0.5$ is understandable. Because of the shallow submerged depth, for higher Fn's we can anticipate strong nonlinear effects which may cause the increase of C_W in the numerical results. To compare the performance of the various FSBC's, we show for $Fn=0.4$ and $d=L$ in Fig.2 the surface elevations predicted by them, and in Fig.3 the values of $-C_p$ on the body surface. The difference between the results is much bigger than that we might expect, even if we take into account that the Dawson's FSBC as well as Ogilvie's are supposed to work well for low Fn's. In Table 1 the values of A and C_W are also compared. When we take the Poisson-solution as a reference, both the Ogilvie- and the Dawson-solution are too big. There is an inconsistency in the sense that A of the

Table 1. For a submerged circular cylinder whose $Fn=0.4$ and submerged depth is the same as its diameter, A/L and C_W by three models are compared.

FSBC	A/L	C_W
Poisson	0.037	0.13E-2
Ogilvie	0.24	0.52
Dawson	0.46	0.28

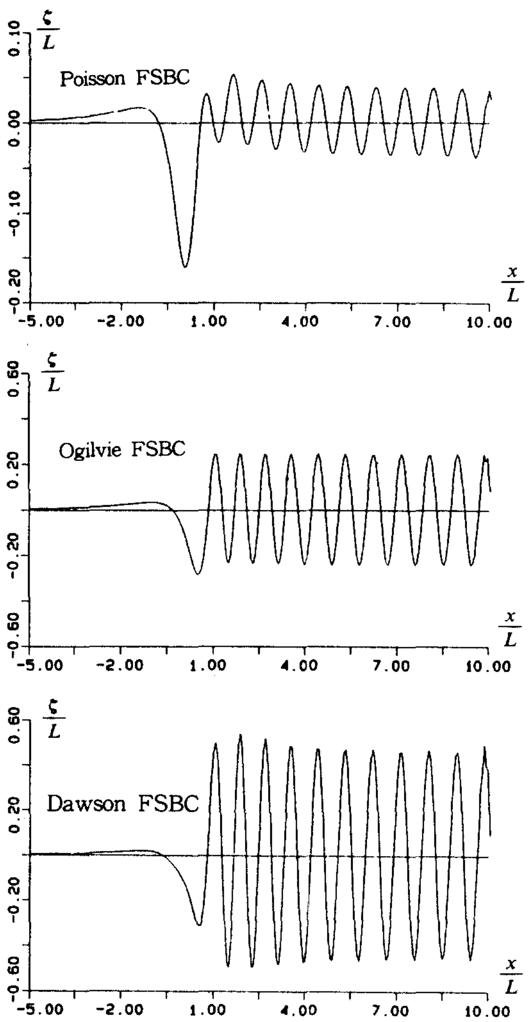

Fig.2 Surface elevations given by three FSBC's due to a submerged circular cylinder, whose $Fn=0.4$ and submerged depth is one diameter.

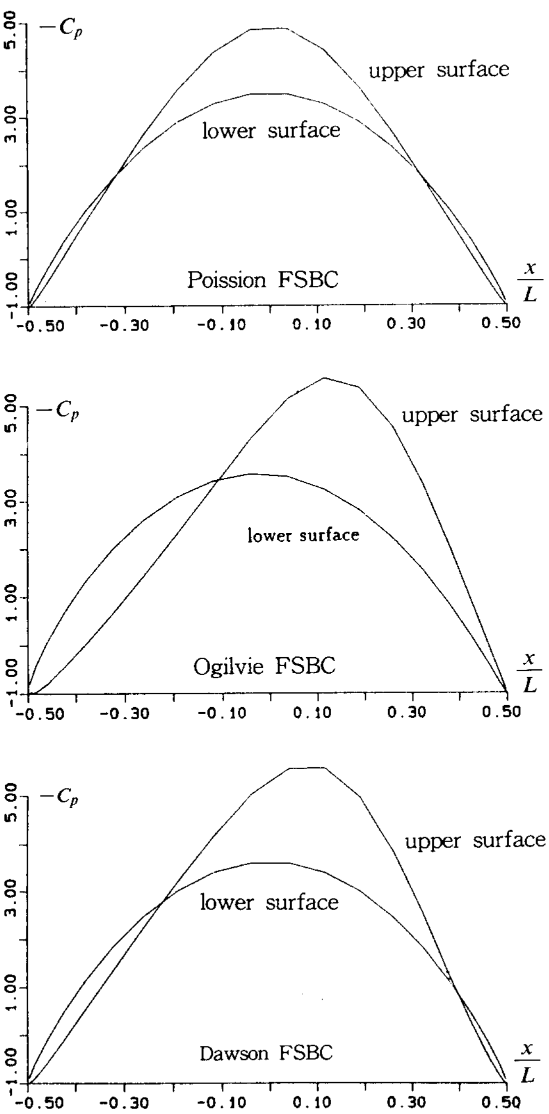

Fig.3 $-C_p$ given by three FSBC's due to a submerged circulary cylinder whose Fn=0.4 and submerged depth is one diameter.

Ogilvie's model is smaller than that of the Dawson's, while C_W of the former is large. Consequently they do not satisfy the relation that C_W be proportional to A^2. Though this relation is based upon the Poisson's FSBC, it is correct upto the third order, thus the inconsistency is still hard to accept. No reasonable explanation can be afforded, as long as the solutions are obtained without error.

The numerical code used was checked by various means so that a possibility for making error seemed minimized. We only note that the C_p curve by the Ogilvie-solution has the least symmetry and results in the largest value of C_W.

Submerged foil

Salvesen[20] measured surface elevations around a foil, shown in Fig.4, obtained by a source-sink combination. To compare with his results, we show in Fig.5 surface elevations computed using various FSBC's(see Van[18]) for Fn=0.422 and d=0.017 43 L, where d is the submerged depth of the

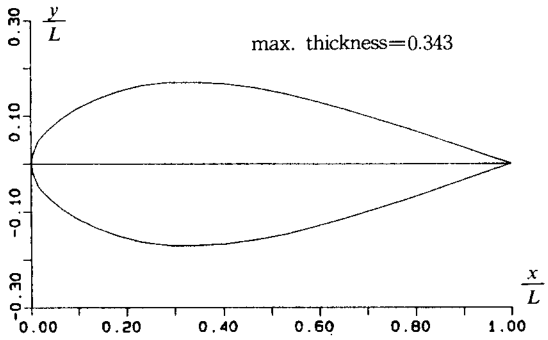

Fig.4 Shape of the foil used by Salvesen[20].

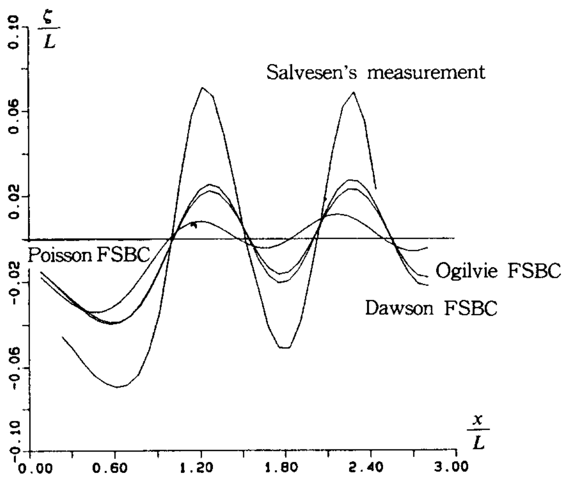

Fig.5 Comparison of surface elevations given by three models and the experimental data by Salvesen[20], due to the submerged foil, shown in Fig.4, whose Fn=0.422 and submerged depth is 0.9174L.

symmetric line of the foil being parallel to the x-axis. First, surface elevations given by three FSBC's are not so different as for the circular cylinder. Since the Poisson model satisfies the body BC more accurately than the classical linear one, one may expect some improvements, but the Poisson-solution is almost the same as the first-order result of Salvesen's. We may interprete this as higher-order effects of the body BC is not as important as those of the FSBC, which was in fact Salvesen's main theme. However, all of the FSBC's tested do show large differences from the measured results. Numerical solutions are worse than te Salvesen's second-order solution, though the Dawson's and the Ogilvie's model work better than the Poisson's. We show A and C_W in Table 2 by three models. The Ogilvie's model yields again smaller A but larger C_W than the Dawson's, although now the differences are not so great.

Table 2. For a submerged foil of Salvesen[20] whose $Fn=0.422$ and submerged depth is $0.9174L$, A/L and C_W by three models are compared.

FSBC	A/L	C_W
Poisson	0.008	0.151E-2
Ogilvie	0.020	0.174E-2
Dawson	0.024	0.131E-2

Submerged sphere & Wigley hull

As reproted by Raven[15], and as we have seen in the above, the Poisson's FSBC may have the equal applicability as the Dawson's in solving wave resistance problems. So in the following we present numerical results obtained by using the Poisson's FSBC. We consider the flow around a submerged sphere whose center is at(0, 0, $-d$) and diameter is L. For this, Havelock[21] gave a formula for C_W using a linear solution satisfying the Poisson's FSBC along with an approximate body BC as

$$C_W = \frac{R}{\frac{1}{2}\rho U^2 S_w} \tag{40}$$

$$= \frac{1}{16}\sqrt{\frac{\pi}{2d}} Fn^{-7} exp(-\delta)\{1 + \frac{3}{4\delta} + \frac{9}{32\delta^2} + O(\delta^{-3})\}, \quad \text{for } \delta = 2Kd \gg 1. \tag{41}$$

Fig.6 compares values of C_W obtained numerically with those by the above formula. In numerical computations, the number of panels on the FS was 10 per wavelength both in the x- and in the y-direction, and that on the body surface where $y>0$ was 16 both in the polar and in the azimuthal direction. The number of total panels on the FS was varied from 600 to 800 for various Fn's. The agreement between two results is even better than for the case of the circular cylinder, which is comprehensible because the nonlinearity due to the shallow submergence for high Fn's may affect less in the three-dimensional flow.

Next, we take the Wigley's parabolic hull whose hull surface, $y=\pm f(x,z)$, is given by

$$f(x,z) = \frac{B}{2}\{1-(\frac{2x}{L})^2\}\{1-(\frac{z}{T})^2\}, \tag{42}$$

$$(B, T, S_w) = (\frac{L}{10}, \frac{L}{16}, 0.14836L^2). \tag{43}$$

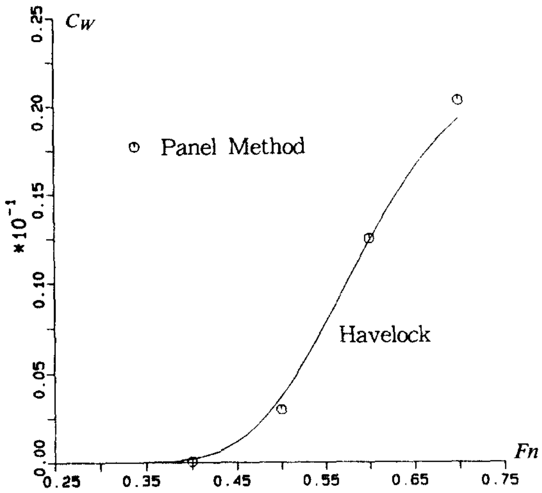

Fig.6 For a submerged sphere whose submerged depth is the same as its diameter, C_W by the Poisson's FSBC is compared with the analytical results by the linear theory of Havelock[21].

Typical arrangement of panels for the computation is shown in Fig.7, and in Fig.8 the comparison of C_W obtained numerically with the thin ship results by Lee[22]. The agreement is almost the same as that of the Fig.7 in Dawson[3], who computed for the Wigley model 1805A.

After we completed our computations, we became aware of the further developments on the FSBC of Ogilvie's, namely Ogilvie & Chen[23] and Chen & Ogilvie[24]. We believe their new findings are in accordance with ours reported above.

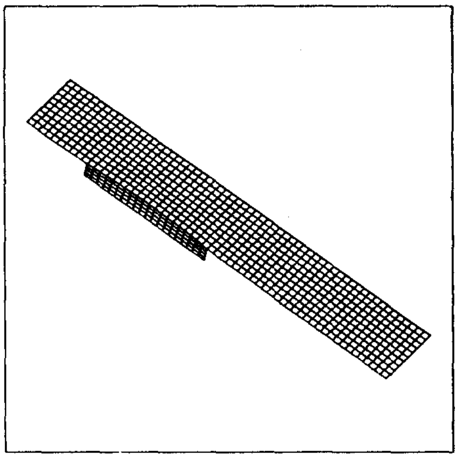

Fig.7 Typical arrangement of panels for the Wigley parabolic hull.

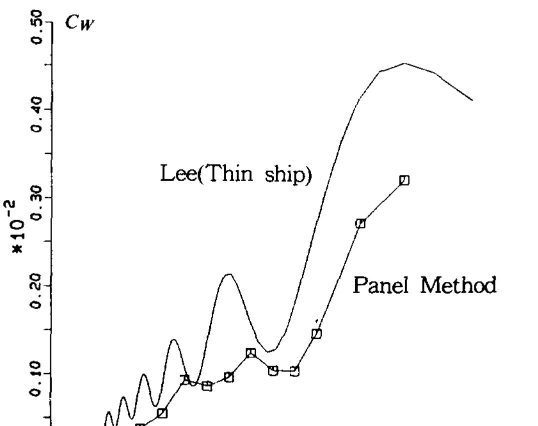

Fig.8 For the Wigley parabolic hull, C_W by the Poisson's FSBC is compared with the thin ship results by Lee[22].

CONCLUSION

As evidenced by the discussion presented so far, the linearized FSBC's tested here, which can be regarded as the most well-known and widely used ones, are not accurate enough to cover the whole range of body forms and speeds. There have been some attempts to solve numerically the exact nonlinear FSBC's, but such a solution method may not be regarded as practically useful because of its long computation time. In order to make the numerical computational method a reliable and helpful tool in the field of wavemaking resistance, it seems that we need develop a theory capable of yielding a FSBC giving more accuracy than those tested here and producing results with less efforts and time than the exact nonlinear model.

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