Inventory Model with Uncertain Backorders
- 부재고 양이 불확실한 경우의 재고 모형 -

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요 약

본 논문은 품절기간 중 수요의 일부는 부재고되고 나머지 일부는 유실되는 재고 모형을 제시한다. 부재고되는 수요량은 불확실하다. 따라서 부재고되는 수요의 양을 확률변수로 놓고, 부재고되는 수요량의 표준편차가 누적품절량의 크기에 비례하는 경우를 다루었다. 투영에 의한 분해법(decomposition by projection)을 사용하여 최적 재고 정책을 도출하였다.

1. Introduction

The application of mathematical ideas for inventory control is one of the most widely researched topics in the literature. Aggarwal[1] and Schwarz[6] reported comprehensive surveys on a variety of models developed on the basis of different assumptions, operational environments and solution proceduers. In most of these models, one common characteristic is that the demand during the stockout period is either entirely backordered or entirely lost. But in many cases, such as the retail business, the typical situation is one where only some of the customers agree to wait until the stock is replenished, while the others decide to buy eleswhere, namely, a situation of partial backordering. The formulation of inventory replenishment policies for such a situation has received relatively little attention. Jelen[2] described inventory models which considered a mixture of backorders and lost sales but no detail solution procedure was reported. Montgomery et al.[3] developed an infinite-time horizon lot size model in which the demand during the stockout period is partially backordered.

In view the nonconvexity of the cost function obtained, a complicated two stage minimization procedure using non-singular transformation of the cost function was suggested to determine the optimal order quantity. Rosenberg[5] reformulated the model in terms of the fictitious demand rate and obtained the optimal solution using decomposition by projection in the stepwise manner specified. Park[4] reformulated by defining a time propotional backorder cost and a fixed penalty cost per unit lost. This problem is then solved directly using the results from a different choice of model paramaters and the unimodality of the cost model.

In this paper, a mathematical model is developed for an inventory system in which the demand during the stockout period is partially backordered. Assuming that the standard deviation of quantity backordered is proportional to the cumulative shortage, the optimal order quantity and the optimal shortage level permitted per cycle are determined.

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2. Model development

The single-echelon, single item, constant-demand case is considered. We first define a set of notation which is used throughout this paper

x = a random variable, representing the quantity backordered.

$f_{x/b}(x)$ = probability density function of x, given that the cumulative shortage per cycle is b units

$\sigma^2_{x/b}$ = variance of x, given that the cumulative shortage per cycle is b units

$E(x/b)$ = The expected value of x, which is equal to βb

Q = order quantity

d = Demand rate

β = A constant which indicates the average proportion of the demand during the stockout period that can be backordered

b = Total demand during the stockout period in a cycle

A = Ordering cost per cycle

h = Inventory holding cost per unit per unit time

π_1 = Shortage cost per unit backordered per unit time

π_2 = Penalty cost of a lost sale including the lost profit per unit

The behavior of the inventory level of the system is depicted in Figure 1. It can be seen that quantity backordered per inventory cycle is x units and the quantity lost is (b-x) units. Thus, the inventory level at the beginning of a cycle is reduced to (Q-x)units. Inventory depletion occurs at the constant rate d and the next replenishment lot arrives when the total demand during the stockout period reachs b units. Consequently, the length of each inventory cycle depends on the amount of unsatisfied demand which can be backordered. The quantity backordered is uncertain and depends on the cumulative shortage b.

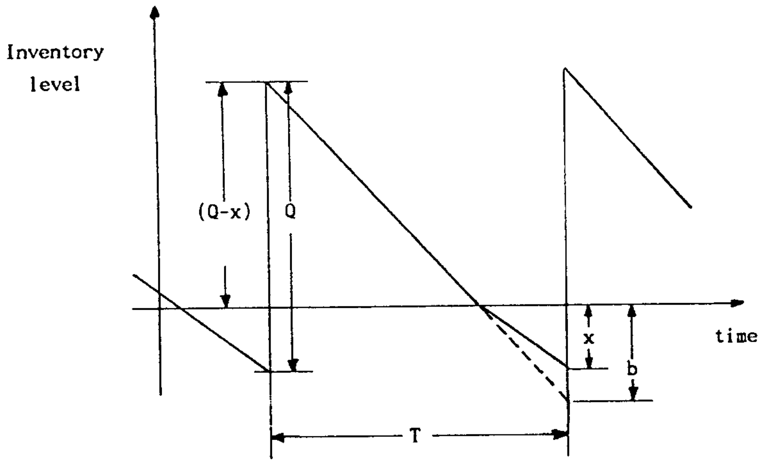

Fig 1. The behavior of the inventory system

If the expected number of units backordered is

$$E(x/b) = \int_0^\infty x f_{x/b}(x) dx = \beta b \quad (1)$$

the variance of the quantity backordered is then given by

$$\sigma^2_{x/b} = \int_0^\infty (x - E(x/b))^2 f_{x/b}(x) dx = E(x^2/b) - \beta^2 b^2 \quad (2)$$

It is also evident that the cycle length is

$$\frac{Q-x}{d} + \frac{b}{d} \quad (3)$$

and the total cost per cycle is

$$A + \frac{h(Q-x)^2}{2d} + \frac{\pi_1 xb}{2d} + \pi_2(b-x) \quad (4)$$

in which the first term represents the ordering cost, the second one the inventory holding cost, the third one the shortage cost and the last one the penalty cost due to lost sales. The mean cycle length then becomes

$$\frac{1}{d}\int_0^\infty (Q+b-x) f_{x/b}(x) dx \quad (5)$$

which can be reduced to

$$\frac{1}{d}[Q + (1-\beta)b] \quad (6)$$

The expected total cost per cycle is therefore given by

$$\int_0^{\infty}\left[A+\frac{h(Q-x)^2}{2d}+\frac{\pi_1 xb}{2d}+\pi_2(b-x)\right]f_{x/b}(x)dx \qquad (7)$$

which can be reduced to

$$A+\frac{h}{2d}(Q^2-2\beta Qb+\sigma^2_{x/b}+\beta^2b^2)+\frac{\pi_1}{2d}\beta b^2+\pi_2(1-\beta)b \qquad (8)$$

In view of (6) and (8), it can be shown that the expected total cost per unit time satisfies the equation

$$TC(Q,b)=\frac{Ad}{Q+(1-\beta)b}+\frac{hd(Q-\beta b)^2}{2d[Q+(1-\beta)b]}+\frac{hd\sigma^2_{x/b}}{2d[Q+(1-\beta)b]}$$

$$+\frac{\pi_1\beta db^2}{2d[Q+(1-\beta)b]}+\frac{\pi_2(1-\beta)db}{Q+(1-\beta)b} \qquad (9)$$

3. Optimal Inventory policy

In this inventory model, we assume that the standerd deviation of the quantity backordered is proportional to the cumulative shortage, i.e, $\sigma_{x/b} = \alpha b$.

The expected total cost per unit time becomes as follows from equation (9)

$$TC(Q,b)=\frac{Ad}{Q+(1-\beta)b}+\frac{hd(Q-\beta b)^2}{2d[Q+(1-\beta)b]}+\frac{d[h\alpha^2+\pi_1\beta]b^2}{2d[Q+(1-\beta)b]}$$

$$+\frac{\pi_2(1-\beta)db}{Q+(1-\beta)b} \qquad (10)$$

As the cost function given by equation (10) is not convex, the optimal values of Q and b can be determined by using decomposition by projection in a stepwise manner as described by Rosenberg[5].

Hence, it follows from equation (10) that the expected total cost per unit time can be reduced to

$$TC(Q,b)=\frac{A}{T}+\frac{h(Q-\beta b)^2}{2dT}+\frac{(h\alpha^2+\pi_1\beta)b^2}{2dT}+\frac{\pi_2(1-\beta)b}{T} \qquad (11)$$

where T is the mean cycle length given by (6).

A new inventory decision variable D , called the fictitious demand rate is then introduced in accordance with the transformation

$$D=\frac{Q-\beta b}{T}$$

to simplify the analysis.