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수리가능제품의 최적재고수준의 결정기법

Algorithm to Determine the Optimal Spare Inventory Level for Repairable-Item Inventory System

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Abstract

This article concerns the problem of determining the optimal spare inventory level for multiechelon repairable-item inventory system. The system we are concerned has several bases and a central depot. When an item fails, it is dispatched to a repair facility and, a spare, if available, is plugged in immediately. When the failed item is repaired, it is sent to the base and either used to fill a backorder or stored at a spare inventory point. We develop an optimal algorithm to find the spare inventory level at each base spareswhich minimizes the total expected cost and, simultaneously, satisfies a specified minimum service rate. The algorithm is applied to examples with good results,

The repairable items such as engines of a fighter plane or a ship are expensive, critically important, and subject to infre-quent failure. Since inventory investment and shortages are significant factors determining the efficiency of the overall system containing the repairable-items, there has been considerable interest in multiechelon repairable-item inventory system. The researches on the system are focused upon the trade-offs among spare inveritory level and repair capacities, as well as analyses of the underlying stochastic process that might be used to describe the system. In this article, we investigate the problem of datermining the spare inventory level satisfying minimum fill rate at minimum cost.(Fill rate(service rate) is the percentage of failed items immediately replaced by a spare item. The minimum fill rate is a predetermined fill rate which should be achieved for the efficient operation of a given system.) Before presenting the details of the model developed, we summarize the previous work in repairable-item multiechelon inventory system.

There have been two parallel streams of research in this area. The first is based on

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the METRIC model. The model developed by Sherbrooke [12] consists of several bases and a central depot. When an item failure occurs, it is dispatched to a repair facility and a spare, if available, is plugged in. If the spare is not available, it is backordered. When the failed item is repaired, it is sent to a base to fill a backorder or is stored at a spare inventory point if there is no backorder outstanding. The research in this area includes papers by Sherbrooke [12], Feeney and Sherbrooke [4], Musckstadt [9. 10], and Muckstadt and Thomas [11]. By simplifying the models with the ample-server, no-queueing-for-repair assumption, these researchers have been able to work with large, complex systems and able to focus on optimization, particularly with respect to the stockage levels at the bases and the central depot. However, as Albright [1] points out in his paper, a key assumption of these METRIC models, ample -service assumption, leads to consistent underestimation of the amount of congestion in the system and, consequently, results in fewer spares than are really needed to achieve a specified backorder level. Thus the results of METRIC models are difficult to apply to real problems where repair facilities have finite capacities.

The other stream of research is based on the similar model but has not made the ample-repair-capacity, constant-failure-rate assumptions. Instead, they study machine-repair queueing models with finite repair capacities. Gross et al. [7] present an implicit enumeration algorithm to calculate the capacities of the base and depot repair facilities as well as the spare inventory level which together guarantee a specified service rate at minimum cost for a two-echelon(two levels of repair, one level of supply) system. Unfortunately, the enumeration scheme of the method requires considerable computer running times even for problems with a relatively small numbers of items and bases. Gross et al. [5, 6], Albright and Soni [2] present methods for calculating the stationary distribution of a multidimensional Markovchain process to find the operating characteristics of a given system. Later Albright and Soni [3] apply the similar approach to a two-echelon repairable inventory system. The models in this stream are more realistic than the comparable METRIC models, and are certainly more difficult to solve due to the huge multidimensional state spaces involved. More recently, Albright [1] develops an algorithm for calculating an approximation to the model with a single type of item stocked and repaired by several bases and a central depot. His approach yields an accurate approximation for reasonably large versions of the system. The main purpose of the methods in this stream is to analyze the current status of a given system and, consequently, they are impractical to apply to optimization pro-blems.

In this paper, using the results from queueing theory and the special properties of a cost function, we are able to develop an efficient method to find the optimal spare inventory level at each base which minimizes expected holding plus shortage costs and, simultaneously achieve a specified minimum service rate for large real world problems.

The rest of this article is organized as follows. In Section 1 we describe the model and, in Section 2, introduce the optimal algorithm for the model. We present an example to explain the algorithm and conclude with some remarks.

1. MODEL FORMULATION

We introduce the following notation :

i = index for base, $i=1, \cdots, I$.

j = index for depot.

λ_i = failure rate at base i.

μ_i = repair rate of a repair channel at base repair center i.

μ_j = repair rate of a repair channel depot repair center.

c_i = number of repair channel at base repair center i.

c_j = number of repair channel at depot repair center.

α_i = probability that a failure at base i is base repairable.

f_i = actual fill rate(service rate) at base i.

F_i = minimum fill rate at base i.

s_i = spare items level at base i.

n = number of failed items being repaired at base repair center.

N = number of failed items being repaired at depot repair center.

k_i = number of failed items from base i at depot repair center.

κ_i = number of items being transported from depot repair center to base i

t_i = travel time from depot repair center to base i

z_i = total number of failed items at base i

As shown in figure 1, we consider a system with a number of bases and a central depot. The depot stocks no spares and only repairs the failed items from bases. When a repair job is completed, the item is returned to the base where it originated.

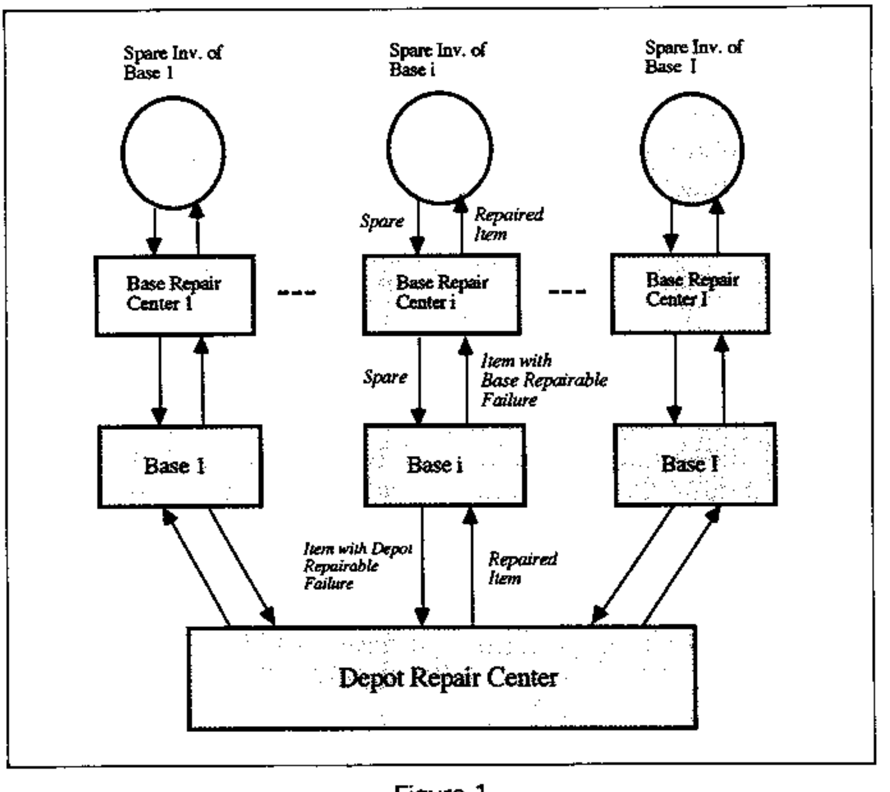

Figure 1.

Item failure rate of the base i is exponentially distributed with parameter λ_i. At failure, the item is replaced by a base spare if one is available. If spare is not available, the item is backordered. the failed item is classified as base repairable with probability α_i. Otherwise, with probability $1-\alpha_i$, the item must be sent to the depot for repair. Additionally, we assume that travel times are positive only in the depot-to-base direction ; in the base-to-depot direction they are assumed to be zero.(This assumption can be relaxed without difficulty.) Then the items of a base which are currently at the base repair center or at the depot repair center or in transit from the depot to the base after repair can be considered as the *total failed items* of the base since they are not available for replacement at the base. Thus, after steady-state has been reached, the probability distribution of the total failed items of a base can be calculated by convolution of the probability distributions of the items at the base repair center and at the depot repair center and in transit from the depot to the base. Now we derive the above mentioned three probability distributions.

Probability distribution of items at base repair center

We introduce the following notation for a few steady-state probability distributions to be introduced.

$P_i(n)$ = probability distribution that there are n items at base repair center i.

$P_j(N)$ = probability distribution that there are N items at depot repair center.

$P_{ij}(k_i)$ = probability distribution that there are k_i items from base i at depot repair center.

$P_{ji}(\kappa_i)$ = probability distribution that there are κ_i items in transit from depot to base i.

$P(z_i)$ = probability distribution that total number of items of base i at base repair center and depot repair center and in transit from depot to base i is z_i.

Since we assume that the item failure rate of the base i is exponentially distributed with λ_i, and that the failed item is base repairable with probability α_i, the base repairable failure rate of the base i is exponentially distributed with parameter $\alpha_i\lambda_i$. When the time to repair a failed item at the base repair center i is exponential with parameter μ_i, the probability distribution that there are n items at the base repair center i, $P_i(n)$, can be obtained from Equations of $M/M/c_i$ queueing model as in Equations (1) and (2).

$$P_i(n) = \begin{cases} (\sigma_i^n/n!)\ P_i(0) & \text{for } 1 \leq n \leq c_i, \\ \sigma_i^n/(c_i!\ c_i^{n-c_i}) & \text{for } n \geq c_i, \end{cases} \tag{1}$$

and

$$P_i(0) = 1/[\sum_{n=0}^{c_i-1} \sigma_i^n/n! + (\sigma_i^{c_i}/c_i!)(1/1-\rho_i))], \tag{2}$$

where $\sigma_i = \alpha_i\lambda_i/\mu_i$ and $\rho_i = \alpha_i\lambda_i/c_i\mu_i$. Note that the above probability distributions do not exist unless the steady-state condition is satisfied, i.e., $\rho_i < 1$ is satisfied.

Probability distribution of items at depot repair center

Since depot repairable failure rate of the base i is exponentially distributed with parameter $(1-\alpha_i)\lambda_i$, and failure rate of each base has independent exponential distribution, interarrival time at depot repair center has exponential distribution with parameter $\sum_{i=1}^{I}(1-\alpha_i)\lambda_i$. Thus the probability that there are N items at the depot repair center, $P_j(N)$, is derived from Equations of $M/M/c_j$ queueing model.

$$P_j(N)=\begin{cases}(\sigma_j^N/N!)\ P_j(0) & \text{for } 1\le N\le c_j,\\ \sigma_j^N/(c_j!\ c_j^{N-c_j}) & \text{for } N\ge c_j.\end{cases} \quad (3)$$

and

$$P_j(0)=1/[\sum_{N=0}^{c_j-1}\sigma_j^N/N!+(\sigma_j^{c_j}/c_j!)$$
$$(1/(1-\rho_j))], \quad (4)$$

where $\sigma_j=\sum_{i=1}^{I}(1-\alpha_i)\lambda_i/\mu_j$ and $\rho_j=\sum_{i=1}^{I}(1-\alpha_i)\lambda_i/c_j u_j$.

Above probabilities exist if the steady-state condition is met, i.e., $\rho_j<1$ is satisfied.

When we define $\theta_i=(1-\alpha_i)\lambda_i/\sum_{i=1}^{I}(1-\alpha_i)\lambda_i$, that is the proportion of items from the base i at depot repair center, the conditional probability that there are $\kappa_1 \cdots \kappa_I$ tiems from base 1, ⋯ I, respectively, given that N items are at the depot repair center, is

$$P(K/N)=\binom{N}{K}\prod_{i=1}^{I}\theta_i^{k_i} \text{ where } K=(k_1,\cdots k_I). \quad (5)$$

Thus the probability that there are k_i items from the base i given that N items are at the depot repair center is

$$P(k_i\mid N)\equiv P_{ij}(k_i\mid N)=\binom{N}{k_i}\theta_i^{k_i}\ (1-\theta_i)^{N-k_i} \quad (6)$$

To find out $P_{ij}(k_i)$, we condition on the number of items at the depot repair center as follows :

$$P_{ij}(k_i)=\sum_{N=0}^{\infty}P_{ij}(k_i \mid \text{total number of items at the depot is } N)\ P(\text{total number of items at the depot is } N)$$

$$=\sum_{N=0}^{\infty}P_{ij}(k_i/N)\ P_j(N) \text{ (By definition of } P_j(N))$$

$$=\sum_{N=0}^{\infty}\binom{N}{k_i}\theta_i^{k_i}\ (1-\theta_i)^{N-k_i}P_j(N) \quad (7)$$

Probability distribution of items in transit.

By the equivalence property of queueing system (see, for example, p. 641 of Hillier and Lieberman [8]), the probability distribution of the number of items in transit from the depot repair center to the base i is same as the probability distribution of depot repairable failure rate of the base i. Therefore the probability distribution of number of items in transit has poisson distribution with parameter $(1-\alpha_i)\lambda_i t_i$ as in Equations (8).

$$P_{ji}(\kappa_i)=[((1-\alpha_i)\lambda_i t_i)^{k_i}\quad \exp(-(1-\alpha_i)\lambda_i t_i)]/\kappa_i!. \quad (8)$$

Probability distribution of total failed items

Since the total failed items of a base is composed of the items which are currently at the base repair center and at the depot repair center and in transit from the depot repair center to the base, the probability distribution of the total failed items of the base i can be obtained as in Equation (9) by convolution of the previously derived probability distributions.

$$P(z_i)=\sum_{\kappa_i}\sum_{\kappa_i} P_{ji}(\kappa_i)^* \; P_j(k_i)^* \; P_i(z_i-k_i-\kappa_i) \tag{9}$$

Expected cost and minimum fill rate

We define additional notation.

h_i = unit holding cost per unit time of base i

b_i = unit shortage cost per unit time of base i.

$TC(s_i)$ = expected total cost per unit time at base i with spare inventory level s_i

If the total failed items of a base is greater than the spare inventory level, then the shortage cost is incurred. Otherwise the holding cost is incurred. Thus the total expected cost of the base i, which is the sum of the expected shortage and holding costs, can be written as in Equation (10).

$$TC(s_i)=h_i\sum_{z_i=0}^{s_i}(s_i-z_i)\,P(z_i)+b_i\sum_{z_i=s_i+1}^{\infty}(z_i-s_i)\,P(z_i). \tag{10}$$

Theorem 1. The expected total cost function, $TC(s_i)$, is unimodal on the interval $[0, \infty)$.

The proof of the theorem is contained in Appendix.

Before introducing another Theorem, we show an Equation expressing relationship between the minimum fill rate(F_i) and actual fill rate (f_i) below.

$$f_i \equiv Pr\{z_i \leq s_i\}=\sum_{z_i=0}^{s_i} P(z_i) \geq F_i \tag{11}$$

Theorem 2. Let s_i^* be the inventory level satisfying equations A(1.1) and A(1.2)(see Appendix for reference), that is the minimum point of the cost function. Let $\bar{s}_i$ be the minimum of s_i values satisfying equation (11). Then the spare inventory level to achieve the minimum fill rate at minimum cost is $Max\{s_i^*, \bar{s}_i\}$.

Proof. Consider graphs in Figure 2. As shown in case (a) of Figure 2, if $s_i^* \geq \bar{s}_i$, then the inventory level satisfying the minimum fill rate at minimum cost should be s_i^*. On the other hand, if the reverse is true as in case (b) of Figure 2, then the inventory level should be $\bar{s}_i$.

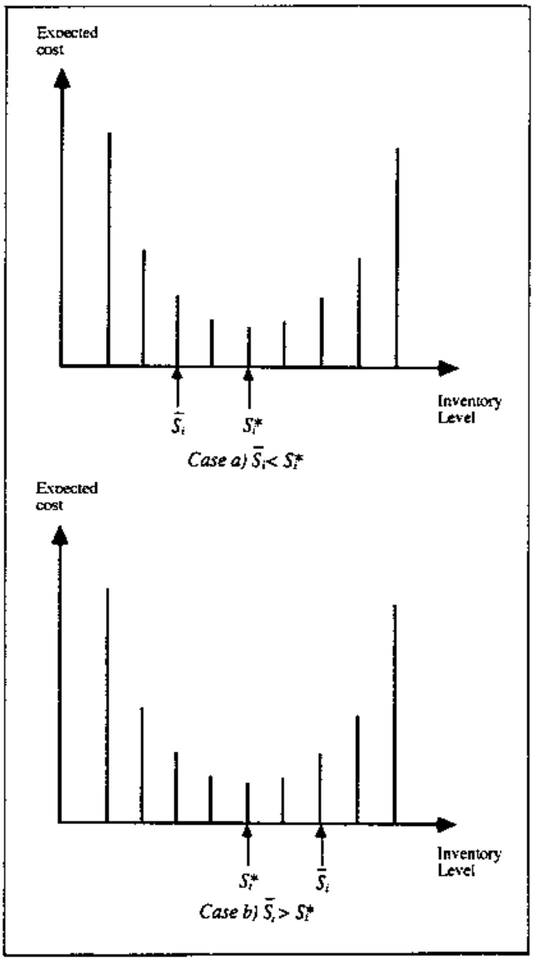

Figure 2. total expected cost function

2. THE ALGORITHM

We now formally present the algorithm

which determines the spare inventory level to achieve a predetermined minimum fill rate at minimum cost.

Step 1. Verify the following steady-state conditions are satisfied.

$$\rho_j=\sum_{i=1}^{I}(1-\alpha_i)\lambda_i/c_j u_j < 1 \text{ and}$$

$$\rho_i=\alpha_i\lambda_i/c_i\mu_i < 1 \text{ for } i=1, \cdots, \text{I}.$$

If the conditions are met, go to Step 2. Otherwise stop.

Step 2. Calculate $P_j(N)$ untile the probability becomes less than $\varepsilon=10^{-4}$.

Step 3. Calculate $P_i(n)$, $P_{ij}(k_i)$, $P_{ji}(\kappa_i)$ for $i=1$, $\cdots$, I until each probability becomes less than $\epsilon=10^{-4}$.

Step 4. Calculate the minimum point of the cost function, s_i^*, by the Bisection Method.

Step 5. Calculate the minimum inventory level satisfying the minimum fill rate, $\bar{s}$.

Step 6. Choose the maximum of s_i^* and $\bar{s}$ as the answer for a given system.

In Step 1, if the steady-state conditions are not met, no probability distribution exists and the optimal inventory level does not exist. Steps 2 and 3 are to calculate probability distributions previously introduced. In Steps 4 and 5, we find the minimum point of the cost function using the Bisection Method and calculate the minimum inventory level satisfying the specified minimum fill rate. Using the results of Steps 4 and 5, we are able to find the desired answer in Step 6.

Example

To illustrate the implementation and performance of the Algorithm, we employ the following example. Consider a multiechelon inventory system with two bases and a depot as an example. The shortage costs for each base are 20 and the holding costs are set to 30. Other relevant data can be found in Table Ⅰ. The proposed algorithm is programmed in C and tested on a 80386 based IBM compatible P.C. system. Table Ⅱ shows the desird inventory levels when we consider the total cost alone. For base 1, when we neglect the minimum fill rate, the minimum point(inventory level of the minimum total expected cost) is 11 items at the minimum cost of 38.58. For base 2, it is 20 items at the cost of 50.38. The solution of the example, inventory levels satisfying the minimum fill rate at minimum cost, is shown in Table Ⅲ. The first column is for the given minimum fill rates and the second column is for the achieved(actual) fill rates of the base. As we expected, the optimal spare level and the optimal cost are decreased until the minimum point is reached as we gradually lower the minimum fill rate. When the optimal inventory level arrives at the minimum point, it remains at that point despite further decrease of the minimum fill rate. The example problem took less than 10 minutes to find the optimal solution. Therefore, we may safely conclude that the method is capable of solving real problems in reasonable time.

Table 1. data for the example

Parameters / Base/Depot	λ_i	α_i	t_i	c_i	μ_i
Base 1	10	0.6	2	2	25
Base 2	20	0.75	3	2	30
Depot	–	–	–	4	3

Table 2. minimum cost inventory level

Base	Inventory Level	Minimum Cost
Base 1	11	38.58
Base 2	20	50.38

Table 3. output of the algorithm for the example

Minimum fill rate	Actual fill rate	Optimal spare level	Optimal cost
0.99	0.994(0.992)	20(30)	95.85(120.40)
0.95	0.954(0.959)	16(26)	60.87(83.06)
0.90	0.927(0.916)	15(24)	53.03(67.32)
0.85	0.888(0.883)	14(23)	46.38(60.32)
0.80	0.833(0.840)	13(22)	41.39(55.62)
0.75	0.759(0.786)	12(21)	38.60(52.03)
0.70	0.759(0.721)	12(20)	39.60(50.38)
0.65	0.667(0.721)	11(20)	38.58(50.38)
0.60	0.667(0.721)	11(20)	38.58(50.38)

3. CONCLUDING REMARKS

In this article we developed a method to calculate the optimal spare inventory level which satisfies a predetermined minimum service rate at minimum cost. With this approach, we are able to solve large problems very quickly. We believe that the method could be used for efficient management of real repairable item inventory system.

Possible extensions to our model include incorporation of more sophisticated cost structures such as nonlinear holding or shortage costs. Furthermore, it may be possible to enhance the model accuracy by relaxing the implicit assumption of infinite number of items operating at each base, which enables us to make use of the formulas from M/M/s model.

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APPENDIX

Proof of Theorem 1

For the given cost function to be unimodal, there should exists an inventory level (integer point), s_i, satisfying the following Equations (A.1.1) and (A.1.2).

$$TC(s_i+k+1) > TC(s_i+k) \text{ for } k=0, 1, \cdots, \infty \quad (A.1.1)$$

$$TC(s_i-k-1) > TC(s_i-k) \text{ for } k=0, 1, \cdots, s_i-1 \quad (A.1.2)$$

Now we prove the Theorem for $k=0$ first.

1) Case k=0

Plugging $k=0$ into Equations (A.1.1) and (A.1.2) gives.

$$TC(s_i+1) > TC(s_i) \quad (A.2.1)$$

$$TC(s_i+1) > TC(s_i) \quad (A.2.2)$$

Now Equation (10) can be expressed as in Equations (A.3), (A.4), and (A.5).

$$TC(s_i) = h_i \sum_{z_i=0}^{s_i-1} (s_i-Z_i)\ P(Z_i) + b_i \sum_{z_i=s_i+1}^{\infty} (Z_i-s_i)\ P(Z_i) \quad (A.3)$$

$$TC(s_i+1) = h_i \sum_{z_i=0}^{s_i} (s_i+1-Z_i)\ P(Z_i) + b_i \sum_{z_i=s_i+2}^{\infty} (Z_i-s_i-1)\ P(Z_i) \quad (A.4)$$

$$TC(s_i-1) = h_i \sum_{z_i=0}^{s_i} (s_i-1-Z_i)\ P(Z_i) + b_i$$

$$\sum_{z_i=s_i+2}^{\infty} (Z_i-s_i-1)\ P(Z_i) \quad (A.5)$$

Plugging Equations (A.3) and (A.4) into Equations (A.2.1) and (A.2.2), respectively, gives

$$h_i \sum_{z_i=0}^{s} P(Z_i)=b_i \sum_{z_i=s_i}^{\infty} P(Z_i)>0 \quad (A.6)$$

$$-h_i \sum_{z_i=0}^{s_i-1} P(Z_i)=b_i \sum_{z_i=s_i}^{\infty} P(Z_i)>0 \quad (A.7)$$

Using (A.6) and (A.7), (A.2.1) and (A.2.2) can be wirtten as in (A.8).

$$\sum_{z_i=s_i+1}^{\infty} P(Z_i)/\sum_{z_i=0}^{s_i} P(Z_i)<h_i/b_i<\sum_{z_i=s_i}^{\infty} P(Z_i)/\sum_{z_i=0}^{s_i-1} P(Z_i) \quad (A.8)$$

Since $\sum_{z_i=0}^{s_i} P(Z_i) > \sum_{z_i=0}^{s_i-1} P(Z_i)$ and $\sum_{z_i=s_i-1}^{\infty}$

$$P(Z_i) < \sum_{z_i=s_i}^{\infty} P(Z_i), \quad (A.8)$$

We obtain following Equation (A.9).

$$\sum_{z_i=s_i+1}^{\infty} P(Z_i)/\sum_{z_i=0}^{s_i} P(Z_i) < \sum_{z_i=s_i}^{\infty} P(Z_i)/\sum_{z_i=0}^{s_i-1} P(Z_i). \quad (A.9)$$

Thus there exists a real number that is between the values of the left and the right hand sides of (A.9). In other words, there exists h_i/b_i satisfying(A.8), which is the equivalent form of the original unimodality condition in (A.2.1) and (A.2.2).

This establishes the Theorem for $k=0$.

2) Case $k \fallingdotseq 0$

Equation (10) can be modified as in Equations (A.10) and (A.11).

$$TC(s_i+k)=h_i \sum_{z^i=0}^{s_i+k-1} (s_i+k-Z_i)\ P(Z_i)$$

$$+b_i \sum_{z_i=s_i+k=1}^{\infty} (Z_i-s_i-k)\ P(Z_i) \quad (A.10)$$

$$TC(s_i+k+1)= \sum_{Zi=0}^{s_i+k} (s_i-Z_i+k+1)\ P(Z_i)$$

$$+b_i \sum_{z_i=s_i+k+2}^{\infty} (Z_i-s_i-k)\ P(Z_i) \quad (A.11)$$

Using Equations (A.10) and (A.11), the unimodality condition in Equation (A.1.1) can be expressed as in Equation (A.12).

$$h_i \sum_{z_i=0}^{s_i+k} (s_i-Z_i+k+1)\ P(Z_i)-h_i \sum_{z_i=0}^{s_i+k} (s_i-Z_i+k)\ P(Z_i)+b_i \sum_{z_i=s_i+k+1}^{\infty} (Z_i-s_i-k-1)\ P(Z_i)-b_i \sum_{z_i=s_i+k+1}^{\infty} (Z_i-s_i-k)\ P(Z_i)=h_i \sum_{z_i=0}^{s_i+k} P(Z_i)-b_i \sum_{z_i=s_i+k+1}^{\infty} P(Z_i)>0 \quad (A.12)$$

(A.12) can be simplified further as in Equation (A.13) below.

$$h_i/b_i > \sum_{z_i=s_i+k+1}^{\infty} P(Z_i)/\sum_{z_i=0}^{s_i+k} P(Z_i) \quad (A.13)$$

Similarly Equation (10) can be written as in Equations (A.14) and (A.15).

$$h_i \sum_{zi=0}^{s_i-k-2} (s_i-Z_i-k-1)\ P(Z_i)-h_i \sum_{z_i=0}^{s_i-k-1} (s_i-Z_i-k)\ P(Z_i)+b_i \sum_{z_i=s_i=k}^{\infty} (Z_i-s_i+k+1)$$

$$P(Z_i) - b_i \sum_{z_i=s_i-k+1}^{\infty} (Z_i - s_i + k)\ P(Z_i) = -h_i$$

$$\sum_{z_i=0}^{s_i} P(Z_i) - b_i \sum_{z_i=s_i-k}^{\infty} P(Z_i) > 0 \quad (A.16)$$

(A.16) can be simplified further as in (A.17)

$$h_i/b_i < \sum_{z_i=s_i-k}^{\infty} P(Z_i) / \sum_{z_i=0}^{s_i-k-1} P(Z_i) \quad (A.17)$$

Now a real number h_i/b_i satisfying (A.8) satisfies (A.13) and (A.17) automatically. Since(A.13) and (A.17) comprise the unimodality condition for $k>0$, this establishes the Theorem for $k>0$ and, together with the proof for $k=0$, completes the proof for Theorem 1.

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