APPROXIMATION OF THE QUEUE LENGTH DISTRIBUTION OF GENERAL QUEUES

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CONTENTS

ABSTRACT

In this paper we develop an approximation formalism on the queue length distribution for general queueing models. Our formalism is based on two steps of approximation; the first step is to find a lower bound on the exact formula, and subsequently the Chernoff upper bound technique is applied to this lower bound. We demonstrate that for the *M*/*M*/1 model our formula is equivalent to the exact solution. For the *D*/*M*/1 queue, we find an extremely tight lower bound below the exact formula. On the other hand, our approach shows a tight upper bound on the exact distribution for both the *ND*/*D*/1 and *M*/*D*/1 queues. We also consider the $M+\Sigma N_jD_j/D/1$ queue and compare our formula with other formalisms for the $\Sigma N_jD_j/D/1$ and $M+D/D/1$ queues.

I. INTRODUCTION

Statistical analysis of queueing problems associated with the Asynchronous Transfer Mode (ATM) network which will be required to carry a wide variety of traffic types is extremely important for efficient operation of switching systems.

When a large number of various traffic sources are superposed on a link, the probability distributions of statistical parameters such as the queue length and average waiting time must be analyzed carefully to provide high quality services. In the ATM network, the loss rate is required to be of below the order of 10^{-10}.

One of the most important information one need to find in a traffic analysis is the queue length distribution (QLD), which is equivalent to the probability $P[Q>r]$ that the queue length Q is larger than r. However, for many traffic models, exact analytic formulas of the QLD are not available. For the analysis of QLDs of various traffic models, numerous analytic approaches and direct numerical simulations have been studied in the last few years [1-8].

In analytic approaches, Roberts and Virtamo [5] found the upper and lower bounds on the QLD of the $\Sigma_i N_i D_i/D/1$ queue where heterogeneous groups of constant bit rate (CBR) sources are superposed at the queue of a CBR server. Their formalism was suitable for a numerical calculation when the number of traffic sources are small. However, in the situation that a large number of input streams are superposed, the computation time required by their formula can be excessive. Furthermore, the application of such an approach may be limited to a few traffic models.

It is desirable to have a formalism which has the following advantages; (i) the formalism is favorable to a fast numerical computation, (ii) it can be applicable to arbitrary type of superposition of traffic sources, and (iii) the error in the approximation should be small. We note that, for general queueing problems, Nakagawa [6] made an approach based on the fundamental recursion formula and the Chernoff bound technique. However, his formalism was not rigorous in a modification that his calculated result is divided by a constant number to match the exact solution for $P[Q>0]$.

In this paper, we develop an approximation formalism on the QLD for general queueing problems. Our formalism is based on two steps of approximation. The first step in our approach is to find a lower bound below the exact formula. Subsequently, the Chernoff bound technique is used to find an upper bound on the distribution function obtained in the previous step. We note that the bound characteristics is lost in our proposed formula for the queue length distribution, because both the lower and upper bound techniques are used in the formalism. However, our philosophy on the problem is that this approach gains a better approximation than others using multiple bounds in the same direction.

To demonstrate advantages of the proposed method, we make a few comparisons of our formula with well known exact formulas of various traffic models such as *M/M/*1, *D/M/*1, *ND/D/*1, and *M/D/*1 queues. As an example of complex queueing problems, we derive an approximate formula of the QLD of the $M+\Sigma N_i D_i/D/1$ queue and compare our formula with other calculations for the $\Sigma N_i D_i/D/1$ and *M+D/D/*1 queues.

Our approach developed in this paper has all the advantages mentioned above. In this paper, application of our formalism is limited to a few queueing problems which have simple analytic solutions available. Application of our formalism to more complex queueing problems with non-existing analytic solutions also showed good agreements with the direct numerical simulation.

II. FORMALISM

We consider a queue with the customer arrival producing a stable queue. The stable queue means that a finite time interval T exists such that the event of the zero queue length occurs at least twice with probability one for $t > T$. A stable queue is guaranteed when the service load, which is defined as the ratio of average arrival rate against service rate, is less than one. Hence, for a stable queue, T is the sufficient interval for statistical analysis of the QLD. In other words, the queue length at the present time t_1 is affected by the arrival and transmission events occurred within T in the past. It is noted that T is an important parameter for the analysis of the dynamic processes of general queueing systems, but in this paper we need not to find the value of T in the calculation of the QLD.

In the analysis of continuous queueing process, we divide the time interval (t_1 -T, t_1) into a sufficiently large number of intervals which are not necessarily uniform. These intervals are labeled as 1, 2, 3... starting from t_1. Without loss of generality, the queue length is measured at the end of each interval. Then we have the following property of queue length q_i at end of the i-th interval.

$$q_i \geq q_{i+1} + a_i - s_i, \tag{1}$$

where q_i and a_i are the queue length and number of arriving cells in the i-th interval, while s_i refers to the number of cells that can be transmitted from an infinite queue in the same interval. Using the recursion relation in Eq. (1), we find that the queue length at t_1 has the following relation for all $i \geq 1$,

$$Q = q_1 \geq A_i - S_i, \tag{2}$$

where $A_i = \sum_{j=1}^{i} a_j$ is the number of arriving cells for i intervals from to, while $S_i = \sum_{j=1}^{i} s_j$ is associated with the number of cells that can be transmitted from an infinite queue for the same intervals. In other words, S_i represents the full transmission capacity of the server for i intervals and consequently is independent of A_i. Eq. (2) implies

$$P\left[Q > r\right] \geq P\left[A_i - S_i > r\right]. \tag{3}$$

Since Eq. (3) is true for all $i \geq 1$, we have

$$P\left[Q > r\right] \geq \max_{i\geq 1}\left\{P\left[A_i - S_i > r\right]\right\}. \tag{4}$$

In the continuous time process the size of an interval can be set to be infinitesimal, and we have the following relation;

$$P\left[Q > r\right] \geq \max_{t\geq 0}\left\{P\left[A_t - S_t - r - 1 \geq 0\right]\right\}. \tag{5}$$

Now, we apply the Chernoff upper bound technique to Eq. (5) to obtain (see appendix for proof)

$$P\left[A_t - S_t - r - 1 \geq 0\right] \leq \min_{z\geq 1}\left\{\Psi_{A_t}(z)\Psi_{S_t}\left(z^{-1}\right)z^{-(r+1)}\right\}, \tag{6}$$

where the probability generating function (PGF) $\Psi_U(z)$ of the random variable U taking on integral values n = 0, 1, 2... is defined by

$$\Psi_U(z) = E\left[z^U\right] = \sum_n P\left[U = n\right]z^n. \tag{7}$$

From Eqs. (5) and (6), our proposed formula for an approximation of the QLD of general queues in the continuous process is given as

$$P'[Q>r] = \max_{t>0}\left\{\min_{z\geq 1}\left\{\Psi_{A_t}(z)\Psi_{S_t}\left(z^{-1}\right)z^{-(r+1)}\right\}\right\}. \quad (8)$$

One can easily see that approximate value of the QLD for a given r corresponds to the magnitude of the saddle point of the function in the parenthesis of Eq. (8).

In the discrete time queueing processes, A_t, S_t, and T can be replaced by A_i, S_i, and I, respectively. I and i represent integer numbers. Hence for the discrete time queueing processes, our formula for the QLD is given as

$$P'[Q>r] = \max_{i\geq 1}\left\{\min_{z\geq 1}\left\{\Psi_{A_i}(z)\Psi_{S_i}\left(z^{-1}\right)z^{-(r+1)}\right\}\right\}. \quad (9)$$

It is noted $P[Q>r]$ can be measured in several different ways such as by the server, arriving customers, or other observers outside the system. In general, $\Psi_A(z)$ and $\Psi_S(1/z)$ are dependent on the scheme it is measured. For instance, if the queue length is measured by the server, $s_1 = 0$ in Eq. (1) because it is measured before the service. This is not important for the Poisson service process because of its lack-of-memory property, but is important for the deterministic service process.

III. APPLICATION

In this section we apply our formalism to a few queueing models whose solutions are known in analytic forms. We will demonstrate that our formula is in fact equivalent to the exact formula for the *M/M/*1 queue, and shows very good approximations for other queues such as the *D/M/*1, *ND/D/*1, *M/D/*1, $\Sigma N_jD_j/D/1$, and *M+D/D/*1 queues.

1. THE M/M/1 QUEUE

For the M/M/1 queue, we will prove that Eq. (8) is equivalent to the exact formula [7]

$$P'[Q>r] = \left(\frac{\lambda}{\mu}\right)^{r+1}, \quad (10)$$

where λ and μ represent the mean arrival and service rates of the Poisson processes, respectively. For a stable queue, $\lambda < \mu$ is assumed. The PGFs of the cell numbers in the arrival and service processes of the *M/M/*1 queue are given as

$$\Psi_{A_t}(z) = e^{\lambda t(z-1)} \quad (11)$$

and

$$\Psi_{S_t}\left(z^{-1}\right) = e^{\mu t(1/z-1)}. \quad (12)$$

Substituting Eqs. (11) and (12) in Eq. (8), we have

$$P'[Q>r] = \max_{t>0}\left\{z_t^{-(r+1)}e^{\lambda t(z_t-1)}e^{\mu t(1/z_t-1)}\right\}, \quad (13)$$

where a real number z_t satisfies $z_t \geq 1$ and is the unique solution that minimizes the function in the parenthesis of Eq. (13) for a given t. For the calculation of z_t, one may transform the function in the parenthesis of Eq. (13) into the semi-logarithmic space and take a derivative of the function with respect to z to find the z_t. Subsequently, a similar derivation is carried on the same function at z_t with respect to t to find the condition that maximizes the function. In this calculation, one must consider that z_t is a function of t. After some lines of simple calculations, one can easily show that Eq. (13) is equivalent to Eq. (10). Furthermore, the time interval t_m that maximizes the right-hand side of Eq. (5) is calculated as

$$t_m = \frac{r+1}{\mu - \lambda}, \tag{14}$$

and the corresponding z_{t_m} is

$$z_{t_m} = \frac{\mu}{\lambda}. \tag{15}$$

2. THE D/M/1 QUEUE

We consider a queue where a periodic arrival stream is served by a server following the Poisson process. Without loss of generality, it is assumed that the inter-arrival time is a unit interval and the service rate satisfies $\mu > 1$ for a stable queue. In this subsection, we assume that the queue length is measured by the arriving cells and the interval t can be restricted to integers $i = 1, 2, \dots$. Hence, the PGF of the number of arriving cells from a CBR source for an interval i is given as

$$\Psi_i(z) = z^i, \quad i = 1,2,\dots \tag{16}$$

For PGF for the service process, Eq. (12) is rewritten as

$$\Psi_{S_i}\left(z^{-1}\right) = e^{\mu i(1/z-1)}, \quad i = 1,2,\dots \tag{17}$$

Substituting Eqs. (16) and (17) in Eq. (8), we obtain our approximate formula for the QLD of the *D/M/*1 queue,

$$P'[Q > r] = \max\left\{e^{-\mu(r+1)}, \ z_i^{i-r-1} e^{\mu i(1/z_i - 1)}\right\}, \quad i > r+1, \tag{18}$$

where $z_i = \mu i/(i - r - 1)$. When the condition $\mu < e^{\mu-1}/(r+2)$ is satisfied, Eq. (18) is simply $P'[Q > r] = e^{-\mu(r+1)}$. This condition can be satisfied by high μ values and corresponding low service load values.

Though the interval i is restricted to integers, we take the derivative of Eq. (18) with respect to i to get an approximate maximum value. Surprisingly, one can get the exact formula [7] of $P[Q>r]$ of the *D/M/*1 queue from this calculation:

$$P'[Q > r] = \xi^{r+1}, \tag{19}$$

where ξ is the solution of the following equation

$$ln(\xi) = \mu(\xi - 1). \tag{20}$$

In fact, results of our numerical calculation showed that Eq. (18) is an extremely tight lower bound below the exact formula. To demonstrate this, we calculated the error ratio $\epsilon(r)$ defined by $1 - P'[Q>r]/P[Q>r]$. For instance, $\epsilon(5) = 1.9 \times 10^{-1}$, 5.0×10^{-4}, 7.8×10^{-6}, and 2.2×10^{-8} for $\mu = 5.0$, 2.0, 1.5, and 1.2, respectively.

On the other hand, if the queue length is measured by the server, the problems can be considered in terms of the continuous time process and Eq. (16) must be modified by $\Psi_t(z) = \eta z^{[t]+1} + (1-\eta) z^{[t]}$, where $\eta = t - [t]$ and $[t]$ is the integral part of t. Calculated results obtained in a similar calculation showed a good agreement with the direct simulation, though we do not plot the results in this paper. It is noted that $\Psi_t(z)$ measured by the server is greater than Eq. (16) measured by arriving customers. This implies that $P[Q>r]$ measured by the server is greater than that obtained by the arriving customers in our approximation approach.

3. THE ND/D/1 QUEUE

We consider a queue where periodic arrival streams are multiplexed and one cell can be served at every one unit time. With N sources, each transmitting one cell every D service time units, the service load is N/D. To compare our approximation for queueing systems such as the $ND/D/1$, $M/D/1$, and $M + \Sigma N_j\, D_j\, /D/1$ queues with other analytic formalisms in the rest of this paper, we will only consider a measuring scheme that the server measures the queue length. Then for an interval t > 0, $[t/D]+1$ cells arrive from a CBR source with probability

$$\eta_t = \frac{t}{D} - \left[\frac{t}{D}\right], \qquad (21)$$

while $[t/D]$ cells with probability $1-\eta_t$, where $[t/D]$ denotes the integral part of t/D. Then, the PGF associated with a CBR source is given as

$$\Psi(z) = \eta_t z^{\left[\frac{t}{D}\right]+1} + (1-\eta_t) z^{\left[\frac{t}{D}\right]} \qquad (22)$$

Considering all N sources, we have

$$\Psi_{A_t} = \Psi(z)^N. \qquad (23)$$

It is noted that for a constant rate service t can be restricted to integer numbers $i = 1, 2, \ldots$, if the service interval is a unit time. As i-1 cells are served for the interval i, the PGF for the service process is given as

$$\Psi_{S_i}\left(z^{-1}\right) = z^{-i+1}. \qquad (24)$$

Our approximation formula of the QLD for the $ND/D/1$ queue is obtained by substituting Eqs. (23) and (24) in Eq. (8),

$$P'[Q>r] = \max_{1\le i<N}\left\{z_i^{-k}\left(\eta_i z_i + 1 - \eta_i\right)^N\right\}, \qquad (25)$$

where $k = r + i - N[t/D]$ and $z_i = k\,(1-\eta_i)\,/\,\eta_i\,(N-k)$. The interval that maximizes the right-hand side of Eq. (25) is not given in a simple form. So, Eq. (25) is compared numerically with well-known formula [3] of the QLD for the $ND/D/1$ queue

$$P[Q>r] = \sum_{i=1}^{N-r}\binom{N}{r+i}\left(\frac{i}{D}\right)^{r+i}\left(1-\frac{i}{D}\right)^{N-r-i}\frac{D-N+r}{D-i},$$

$$for\ 0 \le r < N. \qquad (26)$$

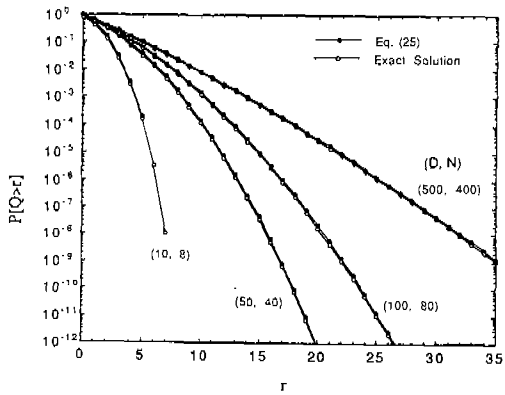

Fig.1. The queue length distribution of the ND/D/1 queue with service load 0.8. The closed circles denote results of Eq. (25), while the open circles refer to the exact formula in Eq.(26). The solid lines are guides to the eye.

Fig. 1 displays a few calculated results of QLD of the $ND/D/1$ queue for different CBR traffics, but at the same service load 0.8. Complementary to Figure 1, QLD for various service load with a CBR traffic is plotted in Figure 2. Both figures demonstrate that Eq. (25) is in a good agreement with Eq. (26). We find that Eq. (25) is an upper bound on the exact solution. We note that the error ratio of Eq.(25) increases as the service load decreases. But the error in the estimation of the buffer length is still less than one.

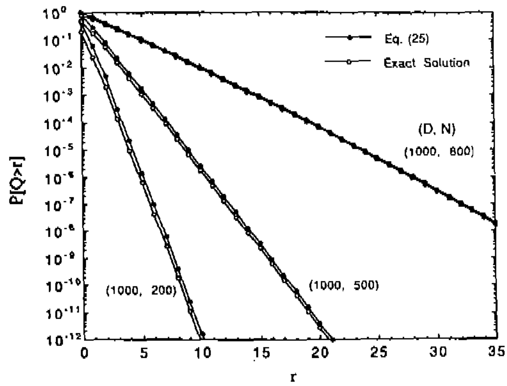

Fig. 2. The queue length distribution of the ND/D/1 queue for various load values. The inter-arrival time of a cell stream was chosen to be 1000 for this calculation.

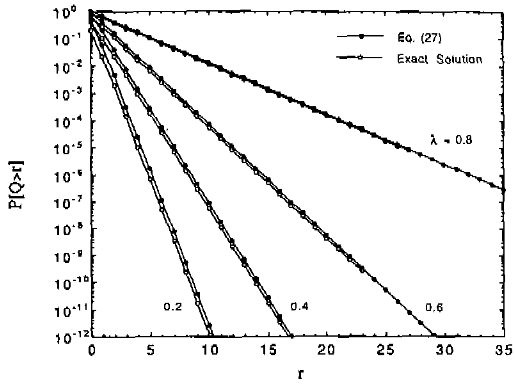

Fig. 3. The queue length distribution of the M/D/1 queue for various mean arrival rate λ. The closed circles denotes results of Eq. (27), while the open circles refer to the exact formula in Eq. (28).

4. THE M/D/1 QUEUE

In this section, our approximation method is applied to the *M/D/*1 queue with the Poisson arrival process and single server transmitting one cell per unit interval. As discussed in previous sections, PGFs for the arrival and service processes in i units of service interval are $\Psi_A(z) = e^{\lambda i(z-1)}$, and $\Psi_S(z^{-1}) = z^{1-i}$, respectively, if the QLD is measured by the server. The mean arrival rate of cells is denoted by λ. Hence, we have the approximation formula of the QLD of the M/D/1 queue

$$P'[Q>r] = \max_{i\geq 1}\left\{z_i^{-(r+i)} e^{\lambda i(z_i-1)}\right\}, \qquad (27)$$

where $z_i = (r+i)/(\lambda i)$.

In Fig. 3 we make a comparison of Eq. (27) with the exact formula [5] for the M/D/1 queue, which is given as

$$P[Q>r] = 1-(1-\rho)\sum_{i=0}^{r}\frac{(-\lambda\, i)^{r-i}}{(r-i)}e^{\lambda i}. \qquad (28)$$

Calculated results of QLD using Eq. (27) show a good agreement with the exact formula given in Eq. (28). We find that Eq. (27) is an upper bound on the exact formula. It is noted that the relative error from the exact solution increases for decreasing the service load value, but it is still not big enough to create one unit of error in the queue length.

5. THE M+Σ N_iD_i/D/1 QUEUE

In this section, we suppose that the multiplex handles a group of heterogeneous sources in addition to a stream following the Poisson process with the mean arrival rate λ. The transmission capacity of the server is assumed to be one cell per unit time. We consider m types of sources; there are N_i sources of type i generating cells at the rate of one per D_i time units. For time interval t, the PGF of the arrival numbers is obtained as

$$\Psi_{A_i} = e^{\lambda i(z-1)} \prod_{j=1}^{m} \Psi_j(z)^{N_j}, \tag{29}$$

where

$$\Psi_j(z) = \eta_{ij} z^{\left[\frac{t}{D_j}\right]+1} + (1-\eta_{ij}) z^{\left[\frac{t}{D_j}\right]}, \tag{30}$$

$$\eta_{ij} = \frac{t}{D_j} - \left[\frac{t}{D_j}\right]. \tag{31}$$

As we consider the QLD measured by the server, the time interval can be restricted to integers i= 1, 2... . The PGF for the service process is given as $\Psi_s(1/z) = z^{-i+1}$. Hence, we have our formula of the QLD of the $M+\Sigma N_jD_j/D/1$ as follows:

$$P'[Q>r] = \max_{i\geq 1}\left\{ z_i^{-k} e^{\lambda i(z_i-1)} \prod_{j=1}^{m} \left(\eta_{ij} z_i + 1 - \eta_{ij}\right)^{N_j} \right\}, \tag{32}$$

where z_i satisfies both $z_i \geq 1$ and the following equation

$$z_i = \frac{k}{\lambda i + \sum_{j=1}^{m} \frac{N_j \eta_{ij}}{\eta_{ij} z_i + 1 - \eta_{ij}}} \tag{33}$$

and

$$k \equiv r + i - \sum_{j=1}^{m} N_j \left[\frac{i}{D_j}\right]. \tag{34}$$

As our formulas in previous sections showed upper bounds on the exact solutions of both the $ND/D/1$ and $M/D/1$ queue, we believe Eq. (32) for the $M+\Sigma N_jD_j/D/1$ queue has the same characteristics with respect to the exact solution whose analytic formula is not available.

As a special case of $M+\Sigma N_jD_j/D/1$ queue, we display our calculated results of the QLD of the $\Sigma N_jD_j/D/1$ queue in Fig.4. In the figure we also make a comparison between our formula and Roberts and Virtamo's (RV) formula (Eq. (13) in Ref. 5) for an upper bound on the QLD of the $\Sigma N_jD_j/D/1$ queue. Though RV did not mention

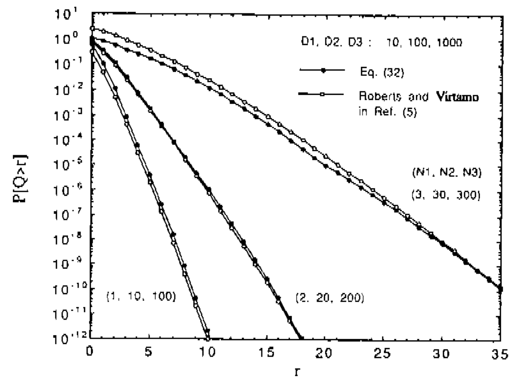

Fig.4. The queue length distribution of the $\Sigma N_jD_j/D/1$ queue with service loads 0.3, 0.6, and 0.9. The closed circles denote results of Eq. (32), while the open circles refer to the Roberts and Virtamo's formula (Eq.(13) in Ref. 5) for an upper bound on the QLD of the $\Sigma N_jD_j/D/1$ queue.

Table 1. Queue length r of the M+D/D/1 queue satisfying $P[Q>r] \cong 10^{-10}$. D denotes the interarrival time of the CBR source in the unit of service interval. For given D and service load values, the mean arrival rate of the Poisson process is determined.

Load \ D	2	5	10	15	30	50	70	100
0.8	24	42	48	50	52	52	53	53
0.5	*	14	16	17	18	18	18	18
0.2	*	*	7	8	8	8	9	9

the measuring scheme in their formalism, we consider that their formula is for the QLD measured by the server, because their formula is based on the expansion of series in terms of integer numbers of the deterministic service time. In our calculation, the inter-arrival times of three types of CBR stream, D_1, D_2, and D_3 were chosen to be 10, 100, and 1000, and $(N_1, N_2, N_3)=$ (1, 10 , 100), (2, 20, 200), and (3, 30, 300) which correspond to the service load 0.3, 0.6, and 0.9, respectively. Comparison of our formula with RV's shows a general agreement. For lower service load values, RV's formula shows a tighter upper bound than Eq. (32), but for high service load values the reverse is true.

As another example, we calculated the queue length of the $M+D/D/1$ queue satisfying $P[Q>r] = 10^{-10}$. Table 1 shows calculated queue length r with respect to various values of inter-arrival time of the CBR source for service loads 0.8, 0.5, and 0.2. Our calculation is in a good agreement with other reports [8].

IV. CONCLUSION

In this paper we developed an approximation formalism for the QLD of general queueing models. Our approximation method consists of two steps of bound techniques, one lower bound and a subsequent upper bound. We proved that our formula for the *M/M/1* queue is in fact equivalent to the exact solution. For the *D/M/1* queue, our approach finds an extremely tight lower bound below the exact formula. On the other hand, for the *ND/D/1* and *M/D/1* queues, our formula shows tight upper bounds on the exact formula. Our calculation of the QLD for the $\Sigma N_i D_i/D/1$ and $M+D/D/1$ queues showed a good agreement with those using other formalisms.

Though in this paper the application of our formalism is limited to a few queueing problems which have simple analytic solutions available, we believe that this approach can be extended to other complicated systems. As the numerical calculation using our formalism needs only $O(N)$ operations, it can be completed within a second using a 80387 math-coprocessor-equipped IBM personal computer. Hence, the algorithm based on our formalism may be implemented in ATM switch system for the real-time analysis of various traffics.

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APPENDIX

In this appendix, we prove that

$$P\left[A_t - S_t - r - 1 \geq 0\right] \leq \min_{z\geq 1}\left\{\Psi_{A_t}(z)\Psi_{S_t}\left(z^{-1}\right)z^{-(r+1)}\right\}. \tag{A1}$$

Proof

At first, for random variables N and M > 0, we have

$$P\left[N \geq 0\right] = P\left[z^N \geq 1\right],\ z \geq 1 \tag{A2}$$

and

$$P[M \geq 1] \leq E[M], \tag{A3}$$

where $E[\ *\ ]$ denotes the expectation value. From these two equations, we have

$$P[N \geq 0] \leq E\left[z^N\right],\ z \geq 1. \tag{A4}$$

Hence, we have the following relation

$$P\left[A_t - S_t - r - 1 \geq 0\right] \leq \min_{z \geq 1}\left\{E\left[z^{A_t - S_t - r - 1}\right]\right\}. \qquad (A5)$$

Finally, it is noted that $E\left[z^{A}\right] = \Psi_A(z)$ and $E\left[z^{S}\right] = \Psi_S(z^{-1})$. This completes our proof.

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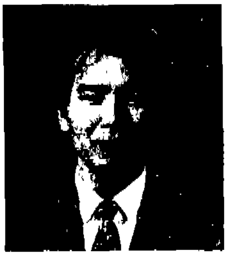

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