Fuzzy Pre-Irresolute Mappings

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1. Introduction and preliminaries

Weaker forms of fuzzy continuity have been considered by many authors [1, 2, 6, 8, 9] using the concepts fuzzy semiopen sets [1], fuzzy regularly open sets [1] and fuzzy preopen sets [2]. J. H. Park et al. [11] showed that fuzzy precontinuity and fuzzy almost continuity, due to Mukherjee and Sinha [8] is equivalent concepts.

In Section 2 of this paper we define and study fuzzy pre-irresolute mapping which is stronger than fuzzy precontinuous, and show that the concepts of fuzzy continuous and fuzzy pre-irresolute mappings are independent. In Section 3, we introduce and study concepts of fuzzy pre-separation axioms of fuzzy topological spaces.

Throughout this paper, by \((X, \tau)\) (or simply \(X\)) we mean a fuzzy topological space in Chang's [3] sense. A fuzzy point in \(X\) with support \(x \in X\) and value \(\alpha\) \((0 < \alpha \leq 1)\) is denoted by \(x_\alpha\). For a fuzzy set \(A\) in \(X\), \(\text{Cl}A\), \(\text{Int}A\), \(1 - A\) and \((A)_0\) will respectively denote the closure, interior, complement and support of \(A\), whereas the constant fuzzy sets taking on the values 0 and 1 on \(X\) are denoted by \(0_X\) and \(1_X\), respectively. A fuzzy set \(A\) of \(X\) is said to be q-coincident with a fuzzy set \(B\), denoted by \(AqB\), if there exists \(x \in X\) such that \(A(x) + B(x) > 1\) [7]. It is known [7] that \(A \leq B\) if and only if \(A\) and \(1 - B\) are not q-coincident, denoted by \(Aq(1 - B)\). For definitions and results not explained in this paper, the reader is referred to [1, 2, 7] in the assumption they are well known. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd' and 'fts', respectively.

**Definition 1.1** [1,2]. A fuzzy set \(A\) in \(X\) is said to be

(a) fuzzy semiopen (fuzzy semiclosed) if \(A \leq \text{ClInt}A\) (resp. \(\text{IntCl}A \leq A\)),

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(b) fuzzy preopen (fuzzy preclosed) if \( A \leq \text{IntCl} A \) (resp. \( A \geq \text{ClInt} A \)).

**Theorem 1.1** [2]. (a) An arbitrary union of fuzzy preopen sets is a fuzzy preopen set,

(b) any intersection of fuzzy preclosed sets is a fuzzy preclosed set.

**Theorem 1.2** [2]. Let \( X \) and \( Y \) be fts's such that \( X \) is product related to \( Y \). Then the product \( U \times V \) of a fuzzy preopen set \( U \) in \( X \) and a fuzzy preopen set \( V \) in \( Y \) is a fuzzy preopen set in the fuzzy product spaces \( X \times Y \).

**Definition 1.2** [11]. A fuzzy set \( A \) in a fts \( X \) is said to be fuzzy pre-q-nbd (fuzzy pre-nbd) of fuzzy point \( x_\alpha \) if there exists a fuzzy preopen set \( B \) such that \( x_\alpha B \leq A \) (resp. \( x_\alpha \in B \leq A \)).

**Theorem 1.3** [11]. A fuzzy set \( A \) is a fuzzy preopen if and only if for each fuzzy point \( x_\alpha q A, A \) is a fuzzy pre-q-nbd of \( x_\alpha \).

**Definition 1.3** [2]. Let \( A \) be any fuzzy set of a fts \( X \). Then fuzzy pre-closure (pCl) and pre-interior (pInt) of \( A \) are defined as follows:

\[
p\text{Cl} A = \bigwedge \{ B \mid B \text{ is fuzzy preclosed and } A \leq B \},
\]

\[
p\text{Int} A = \bigvee \{ B \mid B \text{ is fuzzy preopen and } B \leq A \}.
\]

**Theorem 1.4** [11]. Let \( A \) be a fuzzy set in \( X \) and \( x_\alpha \) be a fuzzy point in \( X \). Then \( x_\alpha \in p\text{Cl} A \) if and only if for each fuzzy pre-q-nbd \( U \) of \( x_\alpha, U q A \).

**Theorem 1.5.** Let \( A \) be a fuzzy set in a fts \( X \). Then \( A \) is fuzzy semiopen set if and only if \( p\text{Cl} A = \text{ClInt} A \).

*Proof.* Let \( A \) be a fuzzy semiopen set in \( X \). Then \( p\text{Cl} A \) is fuzzy preclosed and so \( \text{ClInt} p\text{Cl} A \leq p\text{Cl} A \leq p\text{Cl} p\text{Cl} A \). Since \( A \) is fuzzy semiopen set, \( p\text{Cl} A \leq p\text{Cl} \text{ClInt} A = \text{ClInt} A \). Hence \( p\text{Cl} A = \text{ClInt} A \).

Conversely, let \( A \) be a fuzzy set with \( p\text{Cl} A = \text{ClInt} A \). Then \( A \leq p\text{Cl} A = \text{ClInt} A \) and hence \( A \) is fuzzy semiopen.
2. Fuzzy pre-irresolute mappings

**Definition 2.1** [2]. A mapping \( f : X \rightarrow Y \) is said to be fuzzy precontinuous if \( f^{-1}(V) \) is a fuzzy preopen set for each fuzzy open set \( V \) in \( Y \).

**Theorem 2.1.** For a mapping \( f : X \rightarrow Y \) the following are equivalent:

(a) \( f \) is fuzzy precontinuous.
(b) \( \text{ClInt}f^{-1}(B) \leq f^{-1}(\text{Cl}B) \) for each fuzzy set \( B \) in \( Y \).
(c) \( f(\text{ClInt}A) \leq \text{Cl}f(A) \) for each fuzzy set \( A \) in \( X \).

**Proof.** (a)\( \Rightarrow \) (b): Let \( B \) be a fuzzy set in \( Y \). Then by Theorem 3.7 of [11], \( f^{-1}(\text{Cl}B) \) is fuzzy preclosed in \( X \). Since \( \text{ClInt}A \leq A \) for each fuzzy preclosed set \( A \) in \( X \), \( \text{ClInt}f^{-1}(B) \leq \text{ClInt}f^{-1}(\text{Cl}B) \leq f^{-1}(\text{Cl}B) \).

(b)\( \Rightarrow \) (c): Straightforward.

(c)\( \Rightarrow \) (a): Let \( V \) be a fuzzy closed set in \( Y \). By hypothesis, we have

\[
\text{Cl}f^{-1}(V) = f(\text{ClInt}f^{-1}(V)) \leq f^{-1}(\text{Cl}f^{-1}(V)) \leq \text{Cl}f^{-1}(V) = V,
\]

Then \( f^{-1}(V) \) is a fuzzy preclosed set and hence by Theorem 3.7 of [11], \( f \) is fuzzy precontinuous.

**Definition 2.2.** A mapping \( f : X \rightarrow Y \) is said to be fuzzy pre-irresolute if \( f^{-1}(V) \) is a fuzzy preopen set in \( X \) for each fuzzy preopen set \( V \) in \( Y \).

Clearly a fuzzy pre-irresolute mapping is fuzzy precontinuous, but the converse is not true by the following example.

**Example 2.1.** Let \( U_1 \), \( U_2 \), \( U_3 \) and \( U_4 \) be fuzzy sets in unit interval \( I \) defined as follows:

\[
U_1(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{2} \\ 2x - 1 & \frac{1}{2} \leq x \leq 1 \end{cases}, \quad U_3(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x - 1) & \frac{1}{4} \leq x \leq 1 \end{cases}
\]
Consider fuzzy topologies \( \tau_1 = \{0_I, U_3, 1_I\} \) and \( \tau_2 = \{0_I, U_1, U_2, U_1 \cup U_2, 1_I\} \). Define \( f : (I, \tau_1) \to (I, \tau_2) \) by \( f(x) = \frac{1}{2}x \) for each \( x \in I \). Then \( f \) is a fuzzy precontinuous but not fuzzy pre-irresolute.

**Theorem 2.2.** For a mapping \( f : X \to Y \) the following are equivalent:

(a) \( f \) is fuzzy pre-irresolute.

(b) \( f^{-1}(B) \) is fuzzy preclosed in \( X \) for each fuzzy preclosed set \( B \) in \( Y \).

(c) \( pCl f^{-1}(B) \leq f^{-1}(pCl B) \) for each fuzzy set \( B \) in \( Y \).

(d) \( f(pCl A) \leq pCl f(A) \) for each fuzzy set \( A \) in \( X \).

(e) \( f^{-1}(pInt B) \leq pInt f^{-1}(B) \) for each fuzzy set \( B \) in \( Y \).

**Proof.** (a)\( \Leftrightarrow \) (b): Clear.

(b)\( \Rightarrow \) (c): Let \( B \) be a fuzzy set in \( Y \). By (b), \( f^{-1}(pCl B) \) is fuzzy preclosed and so \( pCl f^{-1}(B) \leq f^{-1}(pCl B) \).

(c)\( \Rightarrow \) (d) and (d)\( \Rightarrow \) (c) can be easily seen.

(c)\( \Rightarrow \) (e): Let \( B \) be any fuzzy set in \( Y \). By (c), we have

\[
1 - pInt f^{-1}(B) = pCl f^{-1}(1 - B) \leq f^{-1}(pCl(1 - B)) = 1 - f^{-1}(pInt B).
\]

Thus \( f^{-1}(pInt B) \leq pInt f^{-1}(B) \).

(e)\( \Rightarrow \) (a): Let \( B \) be any fuzzy pre-open set in \( Y \). Then \( B = pInt B \). By (e), we have \( f^{-1}(B) = f^{-1}(pInt B) \leq pInt f^{-1}(B) \). Then \( f^{-1}(B) \) is a fuzzy pre-open set and hence \( f \) is fuzzy pre-irresolute.

**Theorem 2.3.** A mapping \( f : X \to Y \) is fuzzy pre-irresolute if and only if for each fuzzy point \( x_\alpha \) in \( X \) and each fuzzy pre-nbd \( V \) of \( f(x_\alpha) \), there exists a fuzzy pre-nbd \( U \) of \( x_\alpha \) such that \( f(U) \subseteq V \).

**Proof.** The proof is easy and hence omitted.

**Theorem 2.4.** A mapping \( f : X \to Y \) is fuzzy pre-irresolute if and only if for each fuzzy point \( x_\alpha \) in \( X \) and each fuzzy pre-open pre-q-nbd
V of \( f(x_\alpha) \), there exists a fuzzy preopen pre-q-nbd \( U \) of \( x_\alpha \) such that \( f(U) \leq V \).

**Proof.** Let \( x_\alpha \) be a fuzzy point in \( X \) and \( V \) be a fuzzy preopen pre-q-nbd of \( f(x_\alpha) = f(x) \). Since \( V(f(x)) + \alpha > 1 \), there exists a positive real number \( \beta \) such that \( V(f(x)) > \beta > 1 - \alpha \), so that \( V \) is a fuzzy preopen pre-nbd of \( f(x) \). By Theorem 2.3, there exists a fuzzy preopen set \( U \) containing \( x_\beta \) such that \( f(U) \leq V \). Now, \( U(x) \geq \beta \) implies \( U(x) > 1 - \alpha \) and thus \( U \) is a fuzzy preopen pre-q-nbd of \( x_\alpha \).

Conversely, let \( V \) be a fuzzy preopen set and \( x_\alpha \in f^{-1}(V) \). Let \( m \) be a positive integer such that \( 1/m \leq f^{-1}(V)(x) \). For any positive integer \( n \geq m \), we put \( \alpha_n = 1 + 1/n - f^{-1}(V)(x) \). Then \( 0 < \alpha_n \leq 1 \) for all \( n \geq m \). Now, we have

\[
V(f(x)) + \alpha_n = V(f(x)) + 1 + \frac{1}{n} - f^{-1}(V)(x) = 1 + \frac{1}{n} > 1
\]

Thus \( V \) is a fuzzy preopen pre-q-nbd of \( f(x)\alpha_n \) for all \( n \geq m \). By hypothesis, there exists a fuzzy preopen set \( U_n \) in \( X \) such that \( x_\alpha \in U_n \) and \( f(U_n) \leq V \) for all \( n \geq m \). We put \( U = \bigvee_{n \geq m} U_n \). Then by Theorem 1.1, \( U \) is a fuzzy preopen set in \( X \) such that \( f(U) = \bigvee_{n \geq m} f(U_n) \leq V \).

Next we will show that \( x_\alpha \in U \). Since \( U_n(x) + \alpha_n > 1 \) for all \( n \geq m \), we have \( U(x) > f^{-1}(V)(x) - 1/n \) for all \( n \geq m \) which implies \( U(x) \geq f^{-1}(V)(x) \geq \alpha \). Thus \( x_\alpha \in U \).

**Theorem 2.5.** Let \( f : X \to Y \) one-to-one and onto. \( f \) is fuzzy pre-irresolute if and only if \( p\text{Int}f(A) \leq f(p\text{Int}A) \) for each fuzzy set \( A \) in \( X \).

**Proof.** Let \( A \) be any fuzzy set in \( X \). Then clearly \( f^{-1}(p\text{Int}f(A)) \) is a fuzzy preopen set. By Theorem 2.2, we have

\[
f^{-1}(p\text{Int}f(A)) \leq p\text{Int}f^{-1}(f(A)) = p\text{Int}A,
\]

\[
f(f^{-1}(p\text{Int}f(A))) \leq f(p\text{Int}A).
\]

Since \( f \) is onto, \( p\text{Int}f(A) = f(f^{-1}(p\text{Int}f(A))) \leq f(p\text{Int}A) \).

Conversely, let \( B \) be any fuzzy preopen set in \( Y \). Then \( B = p\text{Int}B \).

By hypothesis, \( f(p\text{Int}f^{-1}(B)) \geq p\text{Int}f(f^{-1}(B)) = p\text{Int}B = B \). This
implies that \( f^{-1}(f(pIntf^{-1}(B))) \geq f^{-1}(B) \). Since \( f \) is one-to-one, \( pIntf^{-1}(B) \geq f^{-1}(B) \). Hence \( f^{-1}(B) = pIntf^{-1}(B) \).

**Theorem 2.6.** Let \( X_1, X_2, Y_1 \) and \( Y_2 \) be sets such that \( X_1 \) is product related to \( X_2 \), and \( f_1 : X_1 \rightarrow Y_1, f_2 : X_2 \rightarrow Y_2 \) be mappings. If \( f_1 \) and \( f_2 \) are fuzzy pre-irresolute, then so is \( f_1 \times f_2 \).

**Proof.** Let \( V = \bigvee_{i,j}(G_i \times H_j) \), where \( G_i \)'s and \( H_j \)'s are fuzzy pre-open sets in \( Y_1 \) and \( Y_2 \) respectively, be a fuzzy preopen set in \( Y_1 \times Y_2 \). Using Lemmas 2.1 and 2.3 of [1], we have

\[
(f_1 \times f_2)^{-1}(V) = \bigvee_{i,j}(f_1 \times f_2)^{-1}(G_i \times H_j) = \bigvee_{i,j}[f_1^{-1}(G_i) \times f_2^{-1}(H_j)].
\]

Since \( f_1 \) and \( f_2 \) are fuzzy pre-irresolute, \( f_1^{-1}(G_i) \) and \( f_2^{-1}(H_j) \) are fuzzy preopen sets, and because of Theorems 1.1 and 1.2, it follows that \( (f_1 \times f_2)^{-1}(V) \) is a fuzzy preopen set, which implies that \( f_1 \times f_2 \) is fuzzy pre-irresolute.

**Theorem 2.7.** Let \( f : X \rightarrow Y \) be a mapping and \( g : X \rightarrow X \times Y \) be the graph of \( f \). If \( g \) is fuzzy pre-irresolute, then \( f \) is fuzzy pre-irresolute.

**Proof.** It follows from Lemma 2.4 of [1].

**Theorem 2.8.** Let \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) be mappings.

(a) If \( f \) and \( g \) are fuzzy pre-irresolute, then \( g \circ f \) is fuzzy pre-irresolute.

(b) If \( f \) is fuzzy pre-irresolute and \( g \) is fuzzy precontinuous, then \( g \circ f \) is fuzzy precontinuous.

**Proof.** Straightforward.

The following Example 2.2 shows that fuzzy continuous and fuzzy pre-irresolute mappings are independent.

**Example 2.2.** Let \( U_1, U_2 \) and \( U_3 \) be fuzzy sets in \( X = \{a, b, c\} \) defined as follows:

\[
U_1(a) = 0.4, \quad U_1(b) = 0, \quad U_1(c) = 0;
U_2(a) = 0, \quad U_2(b) = 0.4, \quad U_2(c) = 0;
U_3(a) = 0.4, \quad U_3(b) = 0.4, \quad U_3(c) = 0.
\]
Consider the fuzzy topologies \( \tau_1 = \{0_X, 1_X, U_1, U_2, U_1 \lor U_2\} \) and \( \tau_2 = \{0_X, 1_X, U_1, U_3\} \).

(a) If a mapping \( f : (X, \tau_2) \to (X, \tau_1) \) defined by \( f(a) = b, f(b) = a, f(c) = c, \) then \( f \) is fuzzy continuous but not fuzzy pre-irresolute.

(b) If a mapping \( f : (X, \tau_1) \to (X, \tau_2) \) defined by \( f(a) = b, f(b) = a, f(c) = c, \) then \( f \) is fuzzy pre-irresolute but not fuzzy continuous.

**Theorem 2.9.** If \( f : X \to Y \) is fuzzy precontinuous and fuzzy open, then \( f \) is fuzzy pre-irresolute.

**Proof.** It follows from Theorem 4.3 of [12].

3. Separation axioms

**Definition 3.1.** A fts \( X \) is said to be fuzzy pre-\( T_0 \) if for every distinct two fuzzy points \( x_\alpha \) and \( y_\beta \), the following conditions are satisfied:

(a) When \( x \neq y \), either \( x_\alpha \) has a fuzzy pre-nbd which is not q-coincident with \( y_\beta \), or \( y_\beta \) has a fuzzy pre-nbd which is not q-coincident with \( x_\alpha \).

(b) When \( x = y \) and \( \alpha < \beta \) (say), there is a fuzzy pre-q-nbd of \( y_\beta \) which is not q-coincident with \( x_\alpha \).

**Definition 3.2.** A fts \( X \) is said to be fuzzy pre-\( T_1 \) if for every distinct two fuzzy points \( x_\alpha \) and \( y_\beta \), the following conditions are satisfied:

(a) When \( x \neq y \), \( x_\alpha \) has a fuzzy pre-nbd \( U \) and \( y_\beta \) has a fuzzy pre-nbd \( V \) such that \( x_\alpha \overline{q} V \) and \( y_\beta \overline{q} U \).

(b) When \( x = y \) and \( \alpha < \beta \) (say), then there exists a fuzzy pre-q-nbd \( V \) of \( y_\beta \) such that \( x_\alpha \overline{q} V \).

**Definition 3.3.** A fts \( X \) is said to be fuzzy pre-\( T_2 \) if for every distinct two fuzzy points \( x_\alpha \) and \( y_\beta \), the following conditions are satisfied:

(a) When \( x \neq y \), \( x_\alpha \) and \( y_\beta \) have fuzzy pre-nbds which are not q-coincident.

(b) When \( x = y \) and \( \alpha < \beta \) (say), then \( x_\alpha \) has a fuzzy pre-nbd \( U \) and \( y_\beta \) has a fuzzy pre-q-nbd \( V \) such that \( U \overline{q} V \).

Obviously, fuzzy pre-\( T_2 \) \( \Rightarrow \) fuzzy pre-\( T_1 \) \( \Rightarrow \) fuzzy pre-\( T_0 \). Also, fuzzy \( T_1 \) axiom [6] \( \Rightarrow \) fuzzy pre-\( T_i \) axiom, for \( i = 0, 1, 2 \).
THEOREM 3.1. A fts $X$ is fuzzy pre-$T_0$ if and only if for every pair of distinct $x_\alpha$ and $y_\beta$, either $x_\alpha \notin p\text{Cl}(y_\beta)$ or $y_\beta \notin p\text{Cl}(x_\alpha)$.

Proof. The proof is easy and hence omitted.

THEOREM 3.2. A fts $X$ is fuzzy pre-$T_1$ if and only if for every fuzzy point $x_\alpha$ is fuzzy preclosed in $X$.

Proof. The proof is easy and hence omitted.

THEOREM 3.3. A fts $X$ is fuzzy pre-$T_2$ if and only if for every fuzzy point $x_\alpha$ in $X$, $x_\alpha = \bigwedge\{p\text{Cl}V \mid V$ is fuzzy pre-nbd of $x_\alpha\}$ and for every $x, y \in X$ with $x \neq y$, there is a fuzzy pre-nbd $U$ of $x_1$ such that $y \notin (p\text{Cl}U)_0$, where $(p\text{Cl}U)_0$ is support of $p\text{Cl}U$.

Proof. Let $x_\alpha$ and $y_\beta$ be fuzzy points in $X$ such that $y_\beta \notin \{x_\alpha\}$. If $x \neq y$, then there are fuzzy preopen sets $U$ and $V$ containing $y_1$ and $x_\alpha$ respectively such that $U \q V$. Then $V$ is a fuzzy pre-nbd of $x_\alpha$ and $U$ is a fuzzy pre-q-nbd of $y_\beta$ such that $U \q V$. Hence $y_\beta \notin p\text{Cl}V$. If $x = y$, then $\alpha < \beta$, and hence there are a fuzzy pre-q-nbd $U$ of $y_\beta$ and a fuzzy pre-nbd $V$ of $x_\alpha$ such that $U \q V$. Hence $y_\beta \notin p\text{Cl}V$.

Finally, for distinct two point $x$, $y$ of $X$, since $X$ is fuzzy pre-$T_2$, there exist fuzzy preopen sets $U$ and $V$ such that $x_1 \in U$, $y_1 \in V$ and $U \q V$. Since $1 - V$ is fuzzy preclosed set containing $U$, $p\text{Cl}U \leq 1 - V$. Hence $y \notin (p\text{Cl}U)_0$.

Conversely, let $x_\alpha$ and $y_\beta$ be distinct fuzzy points in $X$.

When $x 
eq y$, we first suppose that at least one of $\alpha$ and $\beta$ is less than 1, say $0 < \alpha < 1$. Then there exists a positive real number $\lambda$ with $0 < \alpha + \lambda < 1$. By hypothesis, there exists a fuzzy pre-nbd $U$ of $y_\beta$ such that $x_\alpha \notin p\text{Cl}U$. Then there exists a fuzzy pre-q-nbd $V$ of $x_\alpha$ such that $V \q U$. Since $\alpha < 1 - \lambda < V(x)$, $V$ is fuzzy pre-nbd of $x_\alpha$ such that $U \q V$.

Next if $\alpha = \beta = 1$, by hypothesis there exists a fuzzy pre-nbd $U$ of $x_1$ such that $p\text{Cl}U(y) = 0$. Then $V = 1 - p\text{Cl}U$ is a fuzzy pre-nbd of $y_1$ such that $U \q V$.

When $x = y$ and $\alpha < \beta$ (say), then there exists a fuzzy pre-nbd $U$ of $x_\alpha$ such that $y_\beta \notin p\text{Cl}U$. Hence there exists a fuzzy pre-q-nbd $V$ of $y_\beta$ such that $U \q V$. Therefore, $X$ is fuzzy pre-$T_2$. 
THEOREM 3.4. Let $f : X \to Y$ be one-to-one mapping.

(a) If $f$ is fuzzy precontinuous and $Y$ is fuzzy $T_i$, then $X$ is fuzzy pre-$T_i$ for $i = 0, 1, 2$.

(b) If $f$ is fuzzy pre-irresolute and $Y$ is fuzzy pre-$T_i$, then $X$ is fuzzy pre-$T_i$ for $i = 0, 1, 2$.

Proof. We give a proof for $i = 1$ only; the other cases being similar, are omitted. Let $x_\alpha$ and $y_\beta$ be distinct two fuzzy points in $X$.

When $x \neq y$, we have $f(x) \neq f(y)$, and by the fuzzy $T_1$ property of $Y$, there exist fuzzy nbds $U$ and $V$ of $f(x)_\alpha$ and $f(y)_\beta$ respectively such that $f(x)_\alpha qV$ and $f(y)_\beta qU$. Since $f$ is fuzzy precontinuous, $f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy pre-nbds of $x_\alpha$ and $y_\beta$ respectively such that $y_\beta qf^{-1}(U)$ and $x_\alpha qf^{-1}(V)$.

When $x = y$ and $\alpha < \beta$ (say), then $f(x) = f(y)$. Since $Y$ is fuzzy $T_1$, there exists a fuzzy q-nbd $V$ of $f(y)_\beta$ such that $f(x)_\alpha qV$. Then $f^{-1}(V)$ is fuzzy pre-q-nbd of $y_\beta$ such that $x_\alpha qf^{-1}(V)$. Hence $X$ is fuzzy pre-$T_1$.

(b): The proof is similar to (a).

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