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HYPERSURFACE WITH UNIT NORMAL VECTOR FIELD OF $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$

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1.Introduction

Yano [1] introduced the (f, g, u, v, λ)-structure on $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ as a submanifold of codimension 2 of a $(2n+2)$-dimensional Euclidean spcae E^{2n+2} or hypersurface of a $(2n+2)$-dimensional unit sphere $S^{2n+1}(1)$, that is, there exist a $(1,1)$ type tensor field f_j^k,two vector fields u^k, v^k, two 1-forms u_i, v_j a function λ and a Riemannian metric g_{ji} satifying the conditions:

$$(1.2)\begin{cases} f_j^t f_t^i = -\delta_j^i + u_j u^i + v_j v^i, \\ u_t f_j^t = \lambda v^j, \quad f_t^h u^t = -\lambda v^h, \quad v_t f_i^t = -\lambda u_j, \\ f_t^h v^t = \lambda u^h, \quad u_t u^t = v_t v^t = 1 - \lambda^2, \quad u_t v^t = 0, \\ f_j^t f_i^s g_{ts} = g_{ji} - u_j u_i - v_j v_i. \end{cases}$$

In 1982 S.S. Eum ,U-H. Ki amd Y.H. Kim [2] prove the following theorems.

THEOREM A [2]. *Let M be a hypersurface of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ $(n > 1)$ with the $(f, g, u, v, w, \lambda, \mu, \nu)$-structure satisfying $\lambda^2 + \mu^2 + \nu^2 = 1$. If we take v^h as the unit normal vector,then M as a submanifold of codimension 3 of a Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$-dimensional sphere $S^{2n+1}(1)$.*

In this paper we improve Theorem A as follows:

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THEOREM B. *Let M be a hypersurface of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})(n > 1)$. If we take v^h as the unit normal vector, then M as a submanifold of codimension 3 of a Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$-dimensonal sphere $S^{2n+1}(1)$.*

2.Structure equations of hypersurfaces of $S^n(1/\sqrt{2})\times S^n(1/\sqrt{2})$

Let M be a hypersurfaces immersed isometrically in $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ and suppose that M is covered by the system of coordinate neighborhoods $\{\overline{V};\overline{x}^a\}$, where here and in the sequel, the indices $a, b, c, d, \cdots$ run over the range $\{1, 2, \cdots, 2n-1\}$.

We put

(2.1) $B_c^h = \partial_c x^h, \qquad \partial_c = \partial/\partial y^c.$

Then B_c^h are $2n-1$ linearly independent vectors of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ tangent to M. And denote by N^h the unit normal vector to M. Since the immersion $i : M \longrightarrow S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$ is isometric, the induced metric g_{cb} on M is given by $g_{cb} = g_{ji}B_c^j B_b^i$. Next transformating B_c^j and N^j by f_j^h, we can express then respectively as follows:

(2.2) $f_j^h B_c^j = f_c^a B_a^h + w_c N^h, \quad f_j^h N^h = -w^a B_a^h,$

where f_c^a denotes the components of a tensor field of type (1.1), we 1-form and w^a vector field assciated with w_a given by $w^a = w_c g^{ca}, g^{ca}$ being the contravariant components of the induced metric tensor g^{cb}.

We also express the vector field u^h and v^h respectively as follows:

(2.3) $u^h = u^a B_a^h + \mu N^h, \qquad v^h = v^a B_a^h + \nu N^h,$

where u^a and v^a are vector fields on M, μ and ν functions on M.

Applying the operator f_h^k to (2.2) and (2.3) respectively,
and making use of (1.1), we obtain the so-called $(f, g, u, v, w, \lambda, \mu, \nu)$-structure given by

$$f_b^e f_e^a = -\delta_b^a + u_b u^a + v_b v^a + w_b w^a,$$

$$(2.4)\begin{cases} f_e^a u^e = -\lambda v^a + \mu w^a, \\ f_e^a v^e = \lambda u^a + \nu w^a, \\ f_e^a w^e = -\mu u^a - \nu v^a, \end{cases}$$

or equivalently

$$u_e f_a^e = \lambda v_a - \mu w_a, \qquad v_e f_a^e = -\lambda u_a - \nu w_a, \qquad w_e f_a^e \ \mu u_a + \nu v_a,$$

$$(2.5)\begin{cases} u_e u_e = 1 - \lambda^2 - \mu^2, & u_e v^e = -\mu v, \qquad u_e w^e = -\lambda\mu, \\ v_e v^e = 1 - \lambda^2 - \nu^2, & v_e w^e = \lambda u \\ w_e w^e = 1 - \mu^2 - \nu^2, & \end{cases}$$

where u_a, v_a and w_a are 1-forms associated with u^a, v^a and w^a respectively.

3. Proof of Theorem B

Let M be a hypersurfaces of $S^n(1/\sqrt{2}) \times S^n(1/\sqrt{2})$. If we take v^h as the unit normal vector field, then we may put $v^h = \nu N^h$ by the second equation of (2.3). This assumption implies that

(3.1) $\quad v^a = 0, \qquad \nu^2 = 1,$

or, using (2.5) and $v_e v^e = 1 - \lambda^2 - \nu^2 = 0$, we find

(3.2) $\quad \lambda = 0.$

From the second equation of (2.4), (3.1), (3.2), we get

(3.3) $\quad w^a = 0$

or,using (2.5) , $w_e w^e = 1 - \mu^2 - \nu^2 = 0$ and $\quad \nu^2 = 1$, we have

(3.4) $\quad \mu = 0.$

So,(3.1), (3.2) and (3.4) show that

$$\lambda^2 + \mu^2 + \nu^2 = 0 + 0 + \nu^2 = 1$$

Hence, by the theorem A, M as a submanifold of codimension 3 of a Euclidean space E^{2n+2} is an intersection of a complex cone with generator C and a $(2n+1)$ - dimensional sphere $S^{2n+1}(1)$.

References

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