□ 論 文 □

도시공간상에서 교통시설에 대한 최적가격(요금) 구조에 관한 연구

-부분 최적해의 결과-

A Spatial Pattern of An Optimal Transportation Pricing Structure: -Based on the result of a local solution-

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목 차

국문요약

교통수요와 공급은 상호 밀접한 연관성을 지니고 있다. 교통수요는 교통시설 서비스수준 및 교통시설의 이용가격에 영향을 미치는 반면에 교통시설 공급비용과 서비스수준은 교통수요에 영향을 준다. 또한, 교통수요와 공급간의 상호작용은 도시공간상에서 발생하기 때문에 교통시스템의 공간적 구조 및 도시의 공간적 특성은 공급, 가격 및 수요를 통합한 교통균형모형에 있어서 매우 중요한 영향인자로 작용하게 된다. 그러나, 이제까지 개발된 교통균형모형에서는 통행인의 통행시간가치 및 교통체증의 도시공간상 변화가능성을 적절하게 반영하지 못하고 있다.본 연구에서는 도시공간상에서 통행인의 통행시간가치 변화패턴을 반영한 후 도시공간상에서의 교통시설의 최적 서비스수준, 교통수단별 최적요금체계를 도출할 수 있는 교통균형모형을 개발하였다. 실증분석을 통하여 교통균형모형에 있어서 공간성의 중요성을 재확인 할 수 있었다. 단핵도시구조를 지닌 도시공간상에서의 최적버스요금은 통행거리에 따라 할증되는 체계를 가져야 한다. 선행연구에서는 승용차에 대한 통행혼잡세 부과는 소득역분배적인 효과를 초래하는 것으로 알려 졌다. 그러나 버스의 요금구조나 서비스수준이 최적수준에서 제공된다면 통행혼잡세는 소득역분배적 결과를 초래하지 않는 것으로 나타났다.

I. Introduction

The road capacity investment solution to relieve ever-growing traffic congestion is no longer feasible alternative for many governments. As an alternative, many public agencies are considering to implement the congestion pricing policy. The idea behind congestion pricing is to charge for the use of underpriced or unpriced transportation facilities so as to reduce the excess transportation demand. However, congestion pricing is not just a matter of setting prices on the use of transportation facilities. Since the demand change induced by congestion pricing will influence the supply characteristics of the transportation system, congestion pricing should be dealt in the context of a transportation equilibrium analysis. In other words, congestion prices and the transportation service levels must be decided simultaneously so as to minimize the transportation system provision costs and user's travel time costs.

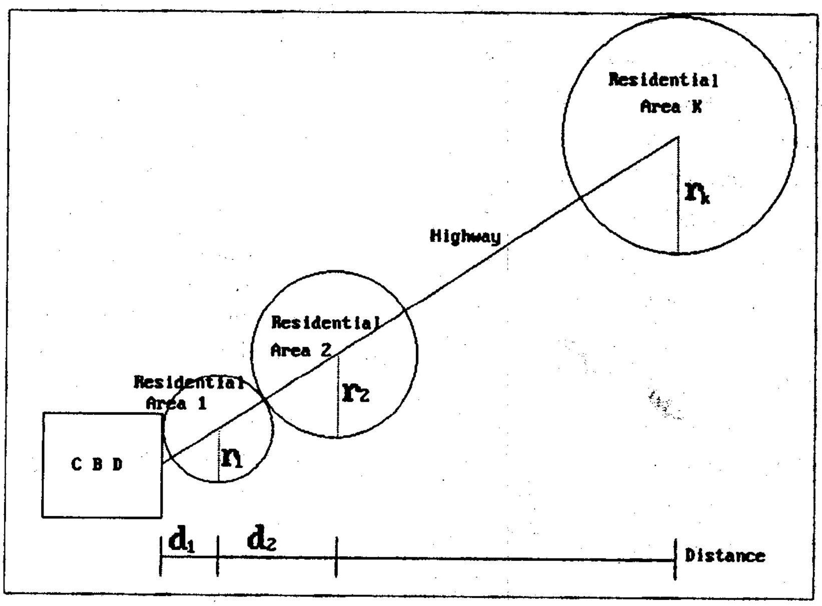
As the travel pattern over an urban space become more complex, the spatial structure of the transportation system .and spatial taste variation over an urban space becomes more important factors in the transportation equilibrium analysis. However, previously developed transportation equilibrium models (Keeler and Small 1977, Talvitie 1980, Viton 1983) have important limitations because that they failed to account for the spatial variability of traffic congestion and values of travel time over an urban space. There have been many confirmations, both theoretically and empirically, that the value of travel time varies with income (Hau 1986, Jara-D az and Videla 1989). Haurin's (1980) study reveals that there are strong interdependencies between population density, household location and income in an urban area. Since the population distribution, the transportation networks structure and the commuters' work places are not uniformly distributed over an urban space, a desirable transportation equilibrium model should incorporate those spatial variations which affect transportation equilibrium conditions.

The purpose of this study is to develop a transportation equilibrium model for an urban area where there is a significant spatial variation in taste and traffic congestion level. From the model, we can observe how the optimal investment levels of transportation facility, auto congestion pricing schedules and bus fare structure vary over an urban space. The remainder of this paper is organized as follows. Section II describes the urban spatial structure and assumptions under which the transportation equilibrium model is developed. Section III outlines the proposed supply model of the transportation equilibrium. Section IV explains the demand model to be incorporated into the transportation equilibrium model and outlines the demand and supply adjustment process. Section V explains the required empirical parameters drawn from previous works. The characteristic of urban transportation equilibrium under various pricing options and the result comparison between the model and previous works are presented in Section Ⅵ. Finally, section Ⅶ explains important findings of the model and some conclusion drawn from the findings.

Ⅱ. The Structure of Urban Transportation Corridor

This study extends Viton's(1983) urban space configuration so as to incorporates heterogeneous shadow prices of travel time in the urban area. Viton(1983) considered a simple urban transportation corridor with a single origin and destination pair under an assumption that homogeneous individuals live in the origin (the residential area) and work at the Central Business District (CBD) destination. The extension of an urban space configuration in this study is based on the "new urban economics" standard monocentric city, where the city is represented by a set of homogeneous concentric rings from the CBD to the urban boundary (Richardson 1977, Peterson 1985).

The study focuses on a typical urban transportation corridor taken from a monocentric city. The CBD is located in the center of the city as a workplace. Several residential areas are located along the highway. Commuters working in the CBD are assumed to live in residential areas.



⟨Figure 1⟩ A Proposed Transportation Corridor in An Urban Area

This study considers only morning peak-hour traffic to find the optimal highway capacity (CBD-bound only), bus service levels and congestion price. A typical morning peak hour work trip in the model is composed of a local journey over local residential streets, a line-haul trip over a fixed highway and another journey over local streets within the destination area. In each residential area, two modes (auto and a public bus transit) are available for residents' use. The bus mode runs as a local bus in residential areas and in the CBD area, and as an express bus on the highway. A bus commuter will walk to the nearest bus route and wait at the bus–stop until the next bus arrives. If a commuter takes his/her own auto, it is assumed that he/she drives directly from his/her home.

The location and route of the highway is assumed to be predetermined by engineering factors. The optimal highway lane capacity in the k–th residential segment(w_k) is to be determined by the optimization model. We assume that the highway is designed to handle traffic without queues (bottlenecks) at the highway interchanges. The node of highway corridor is located on the center of each circular residential area. Local streets in the residential area and the CBD area are placed radially from this node (highway interchange). The actual travel distance and the travel speed in the residential area and the CBD area are depend on the geometric layout of the urban arterial road network and on traffic volume. the investment problem of such roads is not considered in this model. The issue of the urban transportation network structure and of urban arterial roads investment are important and should be incorporated in the model. However, how to incorporate those sort of issues is remained to be resolved in the further study. The auto–obtainable speed on those urban arterial streets is assumed to be S^*(miles per hour). Since buses are required to stop frequently at bus–stops to load and unload passengers in both the residential and the CBD areas, buses are assumed to run at one half of the auto–obtainable speed in the residential and CBD areas.

Population density in each residential area is assumed to decline from the center[1]. Without specifying the exact population density function of each residential area, the representative resident in each residential area is assumed to located at rk/2 miles away from the center of the residential area [2]. The traffic density generated from each residential area depends on the its population size. We further assume that there is a predetermined number of peak–hour commuters(N_k) in the k–th residential area. It is also assumed that there is no peak–spreading or shifting[3]. Relaxing this assumption might be a subject for further research such as a flexible working hour schedule and a dynamic transportation equilibrium

1) This assumption is supported by both theoretical and empirical justifications. Well known population density functions are the quadratic exponential, the linear gamma, the negative square root exponential and log–normal function. See Alperovich (1983), Nairn and O'neil (1988).

2) Here, I assume that population density in residential area k declines from its center so that the average person lives a half of r_k away from the center. To satisfy this assumption all we need to do is to find a negative population density function which satisfy the following properties: $\int_0^{x=r_k} x \cdot f(x)dx = \frac{1}{2} r_k$, where f(x) is a population density function.

3) There is an empirical study supporting this assumption (Gordon, Kumar and Richardson 1990).

model.

The destinations of all residents are a one-mile long and one-mile wide CBD destination square area. the average commuter's destination is simply assumed to located 1/2 miles away from the CBD center. Of course, it is relatively easy to relax this assumption if the distribution of residents' destination in the CBD area is known. There is no bus terminal or bus storage facility available in the CBD area. Once a bus reaches its destination within the CBD, the bus agency must immediately send all buses back to their originating residential areas at the speed S^{**} miles per hour. This assumption enables us to disregard the major and minor direction considerations in Viton's model.

II The Supply Model

1. Highway Construction and Maintenance costs

Using Keeler and Small's(1977) highway construction and maintenance cost function definition, we can specify the highway rental cost $(C_k(w_k))$ of k-th link with road width (w_k) as follow: $C_k(w_k) = (\frac{\tau}{1-e^{-rL}})K(w_k)+M(w_k)+\tau A(w_k)$, (1)

where, τ is interest rate, L is the effective life-time of the highway, $K(w_k)$ is the construction cost of the highway link k with lane width w, $M(w_k)$ is the maintenance cost of link k, $A(w_k)$ is the land acquisition cost of link k.

2. The Full Costs of the Auto Mode

The full costs associated with an auto trip includes auto operating and maintenance costs, social external costs, and the user's cost. When the auto operating costs including gasoline and maintenance are A_{oh}(dollars per highway-mile) and A_{or}(dollars per local-street mile), the auto operating and maintenance cost(A_{mk}) of a vehicle leaving from area k becomes $A_{mk} = A_{oh}(L_k) + A_{or}[(r_k+1)/2]$. If the social external cost is A_e(dollars per auto-mile) and it is assumed to be invariant to road types, the external cost produced by an auto-vehicle departing from area k becomes $A_{ek} = A_e[L_k + (1+r_k)/2])$. The user's cost of the auto in-vehicle travel time (A_{tk}) for an average auto commuter living in the k-th residential area is $A_{tk} = V_{ak}[\frac{r_k+1}{2S^*} + \sum_{i=1}^{k}\frac{d_i}{S_i}]$, where V_{ak} is the value of auto travel time of an average commuter in the k-th residential area.

When passenger load factor per auto vehicle is ρ, the sum of auto operating, external and user cost (TAC) of transporting E1, E2,..., and E_k passengers from the 1-st, the 2-nd, ..., the K-th residential area becomes

$$TAC= \sum_{j=1}^{K}[\ (A_{mj}+A_{ej})(E_j/\rho)+A_{tj}E_j]$$
$$= \sum_{j=1}^{K}[\ A_{oh}(L_j)+A_{or}(\frac{r_j+1}{2})+A_e(L_j+\frac{1+r_j}{2})]\ (\frac{E_j}{\rho}) + V_{aj}[\ \frac{r_j+1}{2S^*}+\sum_{i=1}^{j}\frac{d_i}{S_i}\ ]\ E_j\ . \quad (2)$$

3. The Full Cost of Bus Mode

Suppose there are R_k routes laid out along the radii in the k-th residential area. If the peak-hour bus headway in area k is h_k (hours), the bus agency sends a bus every h_k (hours) from the terminal to cover R_k feeder routes in the residential area k. On every feeder route in the residential area k, a bus runs every $R_k h_k$ hours as a local bus. During peak-hours, $f_k(=1/h_k)$ local buses per hour feed into the highway from the residential area k. A bus traverses $2(r_k+1)$ miles on local streets per round trip. The bus also has to run $d_1, d_2, \ldots, d_k$ miles at speeds of $S_1, S_2, \ldots, S_k$, respectively, and $L_k(=\sum_{i=1}^{k} d_i)$ miles at the attainable highway speed of S^{**} for the return trip.

Suppose that bus agency's operating costs including capital depreciation and relevant external costs of pollution and noise can be summarized by non-overlapping bus-mileage costs B_m (dollars per bus-mile) and bus operating time costs B_h (dollars per bus-hour), then, the agencies' operating cost for a single bus round trip from residential area k to the CBD (TDC_k) becomes

$$TDC_k = B_m[\ r_k+1+\sum_{j=1}^{k} d_j] \times 2 + B_h[\ \frac{r_k+1}{2S^*} + \sum_{j=1}^{k}\frac{d_j}{S_j}] + B_h[\ \frac{r_k+1}{2S^*} + \sum_{j=1}^{k}\frac{d_j}{S^{**}}]\ . \qquad (3)$$

The bus user's costs of time and effort vary with the level of bus service. From the study of travel demand model (McFadden 1974, Mohring 1987), a bus user's cost living in the k-th residential area is characterized by the following three values of time:

V_{bk} = value of bus in-vehicle time for commuters living in the k-th area,

V_{wk} = value of bus waiting time for commuters living in the k-th residential area,

V_{wwk} = value of walking time for commuters living in the k-th residential area.

In reality, the actual walking distance from home to bus stop depends on the road layout, bus routes and the location of bus stops in the residential area.

In this study, buses operate on the road which is radially laid out from the center of the residential area. If the bus agency is serving Rk bus routes for area k and a person walks 3-miles per hour along the circumference of the circle with $r_k/2$ mile radius, then the walk access time becomes $\frac{\pi r_k}{12R_k}$ hours. As a result, the cost of walking time for an average commuter in the k-th residential area becomes $V_{wwk}\frac{\pi r_k}{12R_k}$ dollars. Fisher and Viton (1974) found that half of the headway is a good approximation of the actual waiting time for relatively short headways. Given the half-of-the-headway approximation for waiting time, the cost of the waiting time of a commuter living in the k-th residential area becomes $V_{wk}[(0.5) R_k h_k]$ dollars. The cost of bus in-vehicle travel time for an average bus commuter departing from the k-th area is $V_{bk}\ [\frac{r_k+1}{4S^*} + \sum_{j=1}^{k}\frac{d_j}{S_j}]$.

Therefore, the total bus costs (TBC_k) of servicing f_k buses for transporting Q_k bus passengers in the residential area k is the sum of the bus agency costs, the bus users' walking time costs, the bus users' waiting time costs and the bus users' in-vehicle travel time costs.

$$TBC_k = f_k TDC_k + Q_k [\ V_{wuk}(\frac{\pi r_k}{12R_k}) + V_{wk}(R_k \frac{h_k}{2}) + V_{bk}(\frac{r_k+1}{4S^*} + \frac{d_1}{S_1} + \frac{d_2}{S_2} + \cdots\cdots + \frac{d_k}{S_k})] \quad (4)$$

Ⅳ. The Demand and Equilibrium Model

1. Urban Travel Demand Model

The supply model in the previous section requires as inputs aggregated auto and bus passenger volumes (E_k, Q_k) for each residential area k along the corridor and various value of time derived from the demand model. The logit model (see Ben-Akiva and Lermans 1985) is widely used for the analysis of transportation demand. However, most travel demand models developed so far implicitly assume that travel demands are spatially invariant. In this study, a spatial travel demand model developed by Jung (1991) using expansion method is adopted. In his model specification, the representative indirect conditional utility of an individual i for the auto alternative (I_{ai}) is defined as

$$\begin{aligned} I_{ai} = \ & \beta_1 \times (Auto-\text{in}-vehicle\ time) \\ & + \beta_2 \times [\ (Auto-\text{in}-vehicle\ time) \\ & \quad \div (Travel\ distance)] \\ & + \beta_3 \times (Cost \div Wage) \times (Inner-city\ dummy) \\ & + \beta_4 \times (Cost \div Wage) \times (Middle-city\ dummy) \\ & + \beta_5 \times (Cost \div Wage) \times (Suburban\ dummy) \\ & + \varepsilon \end{aligned} \quad (5)$$

and the representative indirect conditional utility of an individual i for the bus transit alternative (Ibi) is defined as

$$\begin{aligned} I_{bi} = \ & \beta_6 \times (Bus-\text{in}-vehicle\ time) \\ & + \beta_7 \times [(Bus-\text{in}-vehicle\ time) \div (\text{Income})] \\ & + \beta_8 \times (Bus-walk-access\ time) \\ & \quad \times (Inner-city\ dummy) \\ & + \beta_9 \times (Bus-walk-access\ time) \\ & \quad \times (\text{Not}-inner-city\ dummy) \\ & + \beta_{10} \times (Bus\ headways) \\ & \quad \times (\text{Not}-suburban\ dummy) \\ & + \beta_{11} \times (Bus\ headways) \times (Suburban\ dummy) \\ & + \beta_{12} \times (Cost \div Wage) \times (Inner-city\ dummy) \\ & + \beta_{13} \times (Cost \div Wage) \times (Middle-city\ dummy) \\ & + \beta_{14} \times (Cost \div Wage) \times (suburban\ dummy) \\ & + \beta_{15} \times (Bus-dummy) \\ & + \varepsilon \end{aligned} \quad (6)$$

In each residential area, residents' socio-economic characteristics and their taste of the transportation system characteristics are assumed to be homogeneous. From the above travel demand model, we are able to estimate the bus mode choice probability (P_{ik}(bus)) that an individual i living in area k choose a bus mode. The aggregate passenger volume for bus mode from area k becomes $Q_k(= P_{ik}(\text{bus})\ N_k)$, where N_k denotes the traffic density (per hour) from the k-th area.

2. Transportation Equilibrium Model

1) Optimal Peak-Load Investment and Bus Service Levels

For any given peak-time traffic level and values of time, the peak-time investment level and service-level which minimize the total social costs are called the optimal. The total social cost associated with the urban transportation is composed of bus agency's operating costs, bus users' costs for time and effort, auto users' auto operating costs, auto users' costs for time and effort, social external costs, and highway construction and maintenance costs. If the peak period lasts T hours per day, the following minimization problem (equation 7) yields estimates of the optimal highway capacity and bus service levels. In this minimization problem, the highway capacity and bus service levels are determined simultaneously so as to minimize the total social cost associated with urban transportation system.

$$\begin{aligned} \underset{w_k, R_k, h_k}{MIN}\ TC = \sum_{k=1}^{K} T \Big[& TDC_k\left(\frac{1}{h_k}\right) \\ & + Q_k\Big[V_{wwk}\left(\frac{\pi r_k}{12R_k}\right) + V_{wk}\left(\frac{R_k h_k}{2}\right) \\ & + V_{bk}\left(\frac{r_k+1}{4S^*} + \sum_{j=1}^{k}\frac{d_j}{S_j}\right)\Big] \\ & + E_k\Big[V_{ak}\left(\frac{r_k+1}{2S^*} + \sum_{j=1}^{k}\frac{d_j}{S_j}\right)\Big] \\ & + \Big[A_{oh}(L_k) + A_{or}\left(\frac{r_k+1}{2}\right)\Big]\left(\frac{E_k}{\rho}\right) \\ & + A_e\left(L_k + \frac{1+r_k}{2}\right)\left(\frac{E_k}{\rho}\right)\Big] + \sum_{k=1}^{K} C_i(W_i) \end{aligned} \qquad (7)$$

where,

T = the peak period (in hours)

TDC_k = the agency's bus operating cost for a single bus round trip from residential area k to the CBD (in dollars), k=1,...,K,

h_k = the bus headway in residential area k (in hours), k=1,...,K,

r_k = the radius of the residential area k (in miles), k=1,...,K,

R_k = the number of bus routes in the residential area k, k=1,...,K,

S^* = the travel speed on local streets in residential areas and the CBD (in miles per hour),

S_j = the travel speed on the j−th link of the highway (in miles per hour), j=1,...,K,

d_j = the length of the k−th link of the highway (in miles), j=1,...,K,

Q_k = the number of bus passengers per hour in the residential area k, k=1,...,K,

E_k = the number of auto passengers per hour in the residential area k, k=1,...,K,

L_k = the line−haul travel distance in miles, k=1,..,K,

ρ = the passenger load factor per auto vehicle,

A_{oh} = the auto operating cost per highway mile (in dollars),

A_{or} = the auto operating cost per local street mile (in dollars),

A_e = the social external cost per auto−mile (in dollars),

V_{ak} = the value of auto travel time for a representative commuter in the k−th residential area, k=1,...,K,

V_{bk} = the value of bus in−vehicle time for a commuter living in the k−th area ($/hour), k=1,...,K,

V_{wk} = the value of bus waiting time for commuters living in the k−th residential area (in dollars per hour),k=1,..,K,

V_{wwk} = the value of walking time for commuters living in the k−th residential area (in dollars per hour), k=1,...,K,

$C_k(w_k)$ = the construction cost of a capacity wk lane in the k−th link of the highway (in dollars per lane−mile per day).

Since buses have a limited passenger capacity, we add a constraint that no buses carry more than 50 passengers (the conventional bus capacity) at a time. By solving the minimization problem (Equation 7) with the bus capacity constraint, $h_k Q_k \leq 50$, we find the optimal highway capacity(w_k) and the optimal bus service levels(R_k, and h_k).

The optimal highway capacity will be expanded to the point where the marginal cost of an additional unit of highway capacity is equal to the marginal benefit of users and the agency brought about by the additional capacity investment. Similarly, the optimal bus service levels are determined by the point where the marginal cost of an additional bus service improvement is equal to the marginal value of commuters' cost savings (see Keeler and Small 1977 for details).

2) Optimal Prices for Transportation System

According to the first-best pricing rule, the marginal cost pricing structure is the pricing structure which maximizes the sum of consumers' and producers' surplus. For a given long-run optimal investment level determined by Equation 7, the optimal short-run congestion toll is simply the difference between the marginal cost (the optimal price) and average variable cost incurred directly by a commuter. The difference is composed of the incremental costs that an additional commuter imposes on other commuters in the system (Viton 1983 and Morrison 1986).

When there is an additional bus passenger to carry, the marginal cost associated with this passenger depends on the excess (carrying) capacity of the currently running buses. When the existing buses are full, the bus agency has to run an additional bus to carry the additional bus passenger. The additional bus increases the bus agency's operating costs and slows down all vehicles on the highway as well. Thus, the increased bus agency's operating cost is $B_h[\sum_{j=1}^{k} \frac{d_j}{\partial S_j/\partial f_k} \sum_{i=j}^{k} f_i] + TDC_k$, where Bh is the hourly bus operating cost (in dollars per bus-hour). This extra bus imposes congestion effect on both bus users and auto users. The cost of increased in-vehicle travel time of both auto and bus users caused by an additional bus for area k is[4] $\sum_{j=1}^{k} \frac{d_j}{\partial S_j/\partial f_k} (V_{bi}-V_{ai}) + \sum_{i=j}^{k} (Q_i V_{bi}+E_i V_{ai})]$, where V_{ak}, is the value of auto in-vehicle time, and V_{bk} is the value of bus in-vehicle time. On the other hand, running an additional bus to the k-th area will shorten the bus headway and bus waiting time in the k-th area. The value of reduced waiting time due to the reduced headway in the k-th area is $V_{wk}(Q_k R_k/2)(\partial h_k/\partial f_k)$, where V_{wk} is value of bus waiting time for commuters living in the k-th residential area (in dollars per hour). Therefore, the optimal bus toll per bus when buses are full is (8)

$$TDC_k + B_k \sum_{j=1}^{k} \frac{d_j}{\partial S_j / \partial f_k} \sum_{i=j}^{K} f_i + \sum_{j=1}^{k} \frac{d_j}{\partial S_j / \partial f_k} [(V_{bj} - V_{aj}) + \sum_{i=j}^{K} (Q_i V_{ai} + E_i V_{ai})] - V_{wk} \frac{Q_k R_k}{2} \frac{\partial h_k}{\partial f_k}.$$

4) Given a fixed peak hour travel demand (N_k), an additional bus passenger converted from auto users decreases the number of auto vehicle by $1/\rho$ and increase the number of bus vehicle by 1. Thus, traffic volume generated from the k-th area becomes $[(f_k+1)+(E_k-1)/\rho]$, where ρ is a passenger load factor. This traffic volume changes is implicitly explained in the speed function ($\partial S_j/\partial f_k$). Of course, a converted bus passenger from the auto user slightly influences on the users' cost of in-vehicle travel time on the system. This amount is explained by $\sum_{j=1}^{k} \frac{d_j}{S_j/f_k} (V_{bi}-V_{ai})$.

and this toll should be equally distributed among Q_k/f_k bus riders.

When buses are not full (there are empty seats in each bus), the marginal cost should be determined based on the marginal congestion effect caused by the currently operating buses (Morrison 1986, Keeler and Small 1977). The existing buses still impose congestion effects on bus users, auto users and the bus agency. Thus, the optimal bus toll per bus when bus is not full becomes[5)]

$$\sum_{j=1}^{k} \frac{d_j}{\partial S_j/\partial f_j} [(V_{bj}-V_{aj})+\sum_{i=j}^{K}(E_i V_{ai}+Q_i V_{bi}+f_i B_k)]. \quad (9)$$

Similarly, an additional auto from the k-th area not only slows down the traffic on the highway but also increases travel time and operating costs for auto and bus users. The optimal auto toll per auto departing from the k-th area may be shown to be[6)]

$$\sum_{j=1}^{k} \frac{d_j}{\partial S_i / \partial E_j}[(V_{aj}-V_{bj}) + \sum_{i=j}^{K} (E_i V_{ai} + Q_i V_{bi} + f_i B_k)] . \quad (10)$$

3) Equilibrium Computation Process

The equilibrium model should allow for the interaction between demand and supply. Viton's (1983) demand and supply adjustment mechanism, which ensures convergence to a local the optimum, (if any), is adopted. The optimal congestion tolls (fare) calculated from the previous supply model (Equation 8, 9 and 10) will be the equilibrium prices if no demand and supply adjustment process occurs at these prices. If the equilibrium is not achieved, the demand and supply adjustment process will be iterated until no demand and supply adjustment process occurs (see Figure 2). The iteration process is as follows:

Step 1 : The total passenger travel densities (N_k) for each area on the corridor are assumed. These will be based on the underlying population distribution. A base set of levels of modal service and initial highway widths are also assumed. The initial congestion tolls are assumed to be zero.

Step 2 : Estimate the initial passenger travel volume for each area by modes using the demand model in Equation 5 and 6. Denote by $_1E_k$ and $_1Q_k$, the initial auto and bus passenger travel volumes of area k, respectively.

Step 3 : Using the initial travel volume $_1E_k$ and $_1Q_k$, the optimal highway capacity and bus service levels are found using the supply model (i.e. minimizing total costs given by Equation 7). The optimal tolls for bus and auto under the optimized service charac

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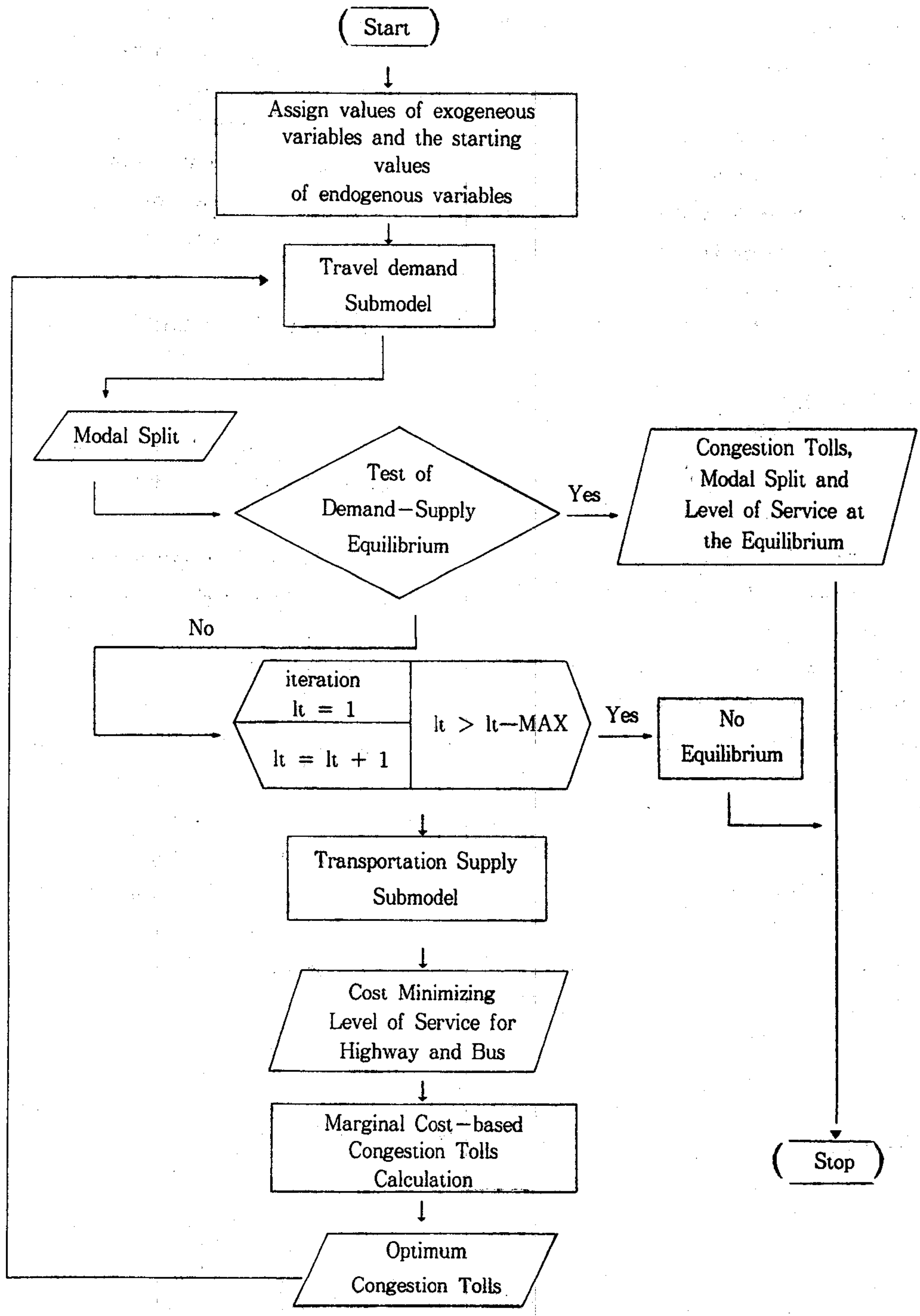
5) When the existing buses are not full, an additional bus passenger converted from auto users decreases only the number of auto vehicle by $1/\rho$. Thus, traffic volume generated from the k-th area becomes $[f_k + (E_k - 1)/\rho]$, where ρ is a passenger load factor. This traffic volume changes is implicitly explained in the speed function $(\partial S_j/\partial f_k)$.

6) Given a fixed peak hour travel demand (N_k), an additional auto user converted from bus passengers increases the number of auto vehicle by $1/\rho$. Thus, traffic volume generated from the k-th area becomes $[f_k + (E_k+1)/\rho]$, where ρ is a passenger load factor. This traffic volume changes is implicitly explained in the speed function$(\partial S_j/\partial f_k)$. Of course, a converted auto user from bus passengers slightly influences on the users' cost of in-vehicle travel time on the system. This amount is explained by $\sum_{j=1}^{k} \frac{d_j}{S_j/f_k}(V_{aj}-V_{bj})$.



< Figure 2> Transportation Demand-Supply Adjustment Process

teristics given by the supply model are determined by Equations 8, 9 and 10.

Step 4 : New mode choice probabilities by area are recalculated given the optimal toll and the service characteristics obtained by the step 3. Denote by $_2E_k$ and $_2Q_k$, the new auto and bus passenger travel volume of area k, respectively.

Step 5 : Test the following convergence criteria.

$|(_1E_k - {_2E_k})| \leq \varepsilon$, k=1,...,K and $|(_1Q_k - {_2Q_k})| \leq \varepsilon$, k=1,...,K,

where, ε is some predetermined criterion. If the convergence criteria are not satisfied, iterate from step 3, replacing the initial passenger volumes with $_2E_k$ and $_2Q_k$, otherwise stop the iteration.

This process ensures to reach at a demand and supply equilibrium. This demand−supply adjustment algorithm is not guaranteed to converge to a solution. Nor, if a solution is found, is it known to be globally optimal. (To establish this would require analysis of the convexity properties of the complicated objective function and the constraints, and it is beyond the scope of the this paper). To ensure the local solution found by the algorithm is unique for a certain range of decision variables, I have repeated the computation process with many different set of starting values[7]. Nonetheless, I believe that the local optima found by this algorithm are sufficiently interesting to be worth presenting, even in the absence of convergence or global results.

V. Empirical Parameters for the Model

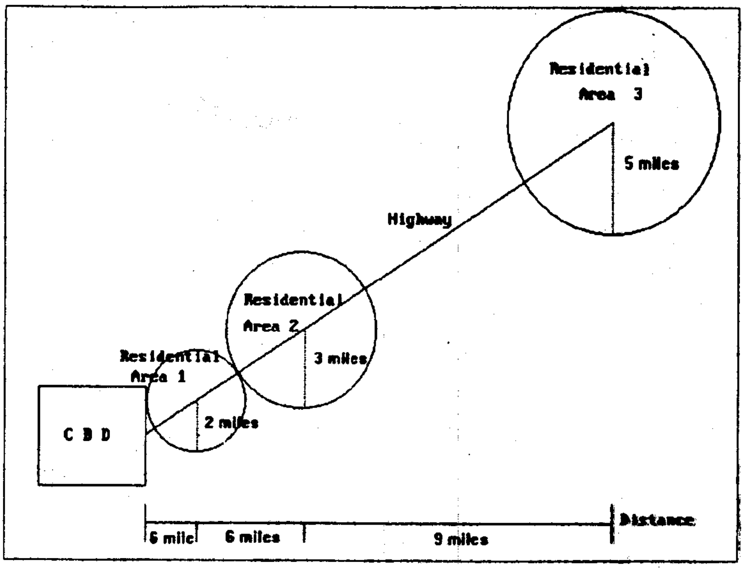
1. The Structure of An Urban Transportation Corridor

The transportation equilibrium model developed in the previous section is now applied to find spatial transportation equilibrium conditions (number of lanes, bus routes, bus headways and congestion tolls) for a simulated urban highway corridor. The corridor consists of one destination zone (CBD area) and three residential origins. The simulated corridor is designed to resemble the I−580 corridor in San Francisco Bay Area. The detail characteristics of the simulated corridor is shown in Table−1.

<Table 1> The Structure of The Simulated Corridor

	Area 1	Area 2	Area 3
Radius of the Residential area(r_k)	2 miles	3 miles	5 miles
Line−haul Length(L_k)	6 miles	12 miles	21 miles
Traffic density per lane(N_k)	4,000	2,000	1,000

7) This approach can be considered as a sort of a very simplified algorithm of solving the nonconvex bi−level programming problem (see Boyce and Kim 1987 for detail).



< Figure 3> The Structure of The Simulated Corridor

2. Speed and Flow Relationship on the Roadway

To explain the relationship between the traffic volume and travel speed on the highway, the U.S. Bureau of Public Road (BPR) link performance functions (equation 11) ,which is most widely used in the traffic engineering literature, is adopted.

$$T_k = T_k^0 [1 + \alpha (\frac{TV_k}{C_k})^\beta] .$$

where, T_k is total travel time on link k, T^0_k is free-flow travel time on link k, TV_k and C_k are traffic flow (volume) and link capacity, respectively, on link k, and α and β are empirical model parameters. The highway link capacity in this model is also assumed to be 2000 passenger cars per hour per lane (pcphpl) in order to compare the model's results with the results of previous studies. Since we are unable to find the α and β parameters that best fit the highway system in the San Francisco Bay Area, the typical values of 0.15 and 4.0 for α and β as suggested by BPR engineers are used. As suggested by Sheffi (1985, p.358), the free-flow traffic speed (T^0_k) is set to be 55 miles per hour. In the CBD area and residential areas, the attainable auto travel speed is assumed to be 25 miles per hour. Since buses are expected to have greater impact on highway congestion than autos, a 1.6 passenger-car equivalency rate of a bus is used to convert congestion impact of buses to passenger-car equivalents.

3. Highway Construction and Maintenance Costs

Highway construction and maintenance cost estimates are adopted form the work of Keeler and Small(1977). Since their esti mates are based on an auto-only highway, the original Keeler and Small estimates are adjusted to find the highway daily rental cost per lane-mile[8] (See Table 2).

< Table 2> Highway Rental costs per day per lane-mile

	Area 1	Area 2	Area 3
Daily Highway Rental Cost per lane-mile	$539.03	$180.13	$180.13

Source : 1. Keeler and Small (1977),p.10.
2. Viton (1983), p.89.

4. Cost Parameters Associated with Auto-mode

Keeler and Small (1975) adjusted The Federal Highway Administration (FHA) estimates[9] to reflect a higher prevailing wage in the San Francisco Bay Area and to distinguish freeway and arterial driving costs[10]. The average operating and maintenance cost for a compact car, for example, is $0.04 per vehicle mile. Crude estimates of accident costs are 3.79 cents per mile for freeways and 1.58 cents per mile for arterial streets (Keeler et al., 1975). The air pollution cost and the vehicle noise cost estimates are drawn from the work of Keeler et al. (1975) and Small (1977).

< Table 3 > Automobile cost Parameters (in 1972 price)

	Highways	Local Streets
Capital costs (cents per mile)	2.51	2.51
Operating costs (cents per mile)	4.17	4.17
External cost (cents per mile)	1.02	1.02
Total cost (cents per mile)	7.70	7.70

Sources : 1. Keeler, et al. (1975), p.52-91.
2. Viton (1985), p.90.

8) Keeler and Small estimates are multiplied by the factor 1.4 to get general purpose highway costs. Daily highway rental costs are derived based on the infinite life-time for highway construction, 35 years life-time for highway construction, and 255 business days per year.

9) Meyer, Cain and Pohl (1965), pp.213-214.

10) The adjustment is made based on engineering data that suggest that urban arterial driving costs 21% more than freeway driving in maintenance, 11% more in gasoline, and 300% more in tire wear.

5. Cost Parameters Associated with Bus－mode

Bus operating and maintenance costs are drawn from the work of Fisher and Viton (1974). They find that the average cost of a standard－size (50－passenger) bus is $9.28T + $.3007M, where T is bus operating time in hours and M is bus operating distance in miles. Since Fisher and Viton calculated the average total bus costs without distinguishing peak and non－peak hour bus operation, two modifications are made to their average costs to reflect peak hour bus operating costs.

First, since capital investments are likely to be made to meet the peak－intensive demand, a larger portion of the capital cost should be allocated to the peak. 50% of the capital cost is allocated to the morning peak hours.[11] This cost allocation (equivalent to freeing the off－peak of any capital costs) will ensure that we do not overestimate the efficiency of public transit (bus). For a bus whose purchase price is 42,000 dollars, the capital cost per major direction peak－hour operation is 9.63 dollars under the assumption of an industry standard 12 years vehicle life－time, 255 business days per year and a 6% interest rate.

Second, social external costs (air pollution and noise pollution) are not accounted for in Fisher and Viton's total cost. The air pollution and noise cost of buses are simply assumed to be twice those of autos. After adjusting for these two modifications, the bus operating time－based cost is 18.91 dollars per peak－hour bus, and bus mileage－based operating costs are 0.2999 dollars per mile on arterials and 0.30 dollars per mile on highways.

< Table 4 > Bus Cost Parameters (in 1972 Prices)

	Highways	Local Streets
Peak Hour Costs (dollars per hour)	18.91	18.91
Capital Costs (dollars per hour)	9.63	9.63
Labor Costs (dollars per hour)	9.28	9.28
Bus－mile cost (cents per mile)	29.99	30.01

Sources : 1. Fisher and Viton (1974), p.6.
2. Viton (1985), p.92.

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11) There is a strong argument against this approach because some bus equipments are used during off－peak periods as well (see Scherer 1976, Guldmann 1986 for the discussion of the cost allocation). This argument implies that some portion of bus capital costs should be allocated to off－peak periods. However, the off－peak use of bus equipment is not known until the optimal capital allocation for the peak is determined. The problem of allocating bus capital cost between peak and off－peak periods remains to be solved.

6. Travel Demand Models and Values of Travel Time

The empirical parameter of the demand model is adopted from the work of Jung (1991). He developed a spatially varying travel demand model specification from a sample of 810 commuters in the San Francisco Bay Area in 1972.[12] The parameter of the model along with descriptive statistics associated with the estimates are shown in Table 5.

< Table 5 > Empirical Travel Demand Model

Independent Variables	Spatially Varying Model	
	Estimates	t−value
Auto in−vehicle time (in tenth min.) (1)	−0.003269	−3.36
Auto in−vehicle time divided by travel distance (in miles)(1)	−0.014886	−1.73
Bus in−vehicle time (in tenth min.) (2)	−0.002166	−3.01
Bus in−vehicle time divided by income (in dollars) (2)	2.81293	2.64
Bus walk time (in tenth min.) In−CBD living commuters (2)	−0.002633	−3.49
Bus walk time (in tenth min.) Near−CBD or suburban living commuters (2)	−0.003758	−3.53
Bus Headways (in tenth min.) In−CBD and Near−CBD living commuters	−0.013566	−4.87
Bus Headways (in tenth min.) Suburb Living Commuters (2)	−0.115426	−1.78
Cost divide by wage (in cent) In−CBD living commuters (1,2)	−0.005161	−3.42
Cost divide by wage (in cent) Near−CBD living commuters (1,2)	−0.002180	−2.43
Cost divide by wage (in cent) Suburb living commuters (1,2)	−0.001669	−2.41
Bus alternative specific dummy (2)	−0.380465	−0.52
log likelihood function at $\beta = 0$	−561.45	
log likelihood function at β^*	−328.13	

Note : * Model applied: 1=auto, 2=bus.

Another important parameter in the equilibrium model to be supplied from the demand side is the value of travel time.[13] Values of times are computed from the demand model (Table 5) are shown in Table 6. Note that the value of time derived from the spatially varying demand model varies with residental location, travel distance and wage.

12) The original date set contains a sample of 991 workers' work trip behaviors before Bay Area Rapid Transit (BART) system opens for service in the San Francisco Bay Area in 1972. See McFadden and Associates (1977) and Talvitie (1976) for details on the data set.

13) See Small (1992), p.19 for details on the value of travel time.

<Table 6> Values of Travel Time Derived From The Travel Demand Model

		Area 1	Area 2	Area 3
Income (in dollars per year)		$14,920	$14,519	$21,280
Post-tax Wage (dollars per hour)		$4.26	$4.19	$6.59
Highway travel distance (miles)		6	12	21
Value of Time (dollars per hour)				
The Spatially Varying Demand Model	Auto in-vehicle time value	$4.75	$8.66	$13.86
	Bus in-vehicle time value	$1.63	$3.79	$8.03
	Bus walk time value	$2.17	$7.22	$14.84
	Bus waiting time value	$11.20	$26.07	$21.43

Note : Travel time values for area 1, 2, and 3 are adjusted to reflect a travel distance.

IV. Urban Transportation Pricing Structure

1. Three Transportation Pricing Policies

Since the demand and supply interaction is allowed in the transportation equilibrium model, the equilibrium conditions is expected to vary with transportation pricing policies. To observe the impact of transportation pricing policies on transportation equilibrium, I have developed three different spatial transportation pricing policy scenarios:

Policy 1: Marginal cost (MC) pricing rule is applied for both auto and bus,

Policy 2: MC pricing rule is applied for for bus and there is no auto congestion toll,

Policy 3: The break-even condition is imposed on bus agency and MC pricing rule is applied for auto.

1) The Pareto-Optimal Service Level When the optimal Pricing Policy is Implemented

Transportation equilibrium characteristics when the optimal investment and marginal costs pricing policies are implemented are simulated. The calculated equilibrium[14] is shown in Table 7. If correct pricing and investment policies are implemented, a bus mode market shares is about 90.96% in area 1. But, its market share drops to 69% in areas 2 and 59% in area 3. A high market share of the bus mode in area 1 is partially due to relatively lower travel time values of commuters in area 1 and and higher bus service levels in area 1. The low bus market share in area 3 is partly due to its residents' higher auto ownership as well as higher travel time values. The result implies that bus transportation is less competitive against auto in a suburban area where high income commuters live.

14) The equilibrium computation was done on a 33-mhz IBM 486-PC Compatible using the DBCONF multivariable nonlinear minimization routine in IMSL (Problem-Solving Software Systems 1987). Programs was compiled under version 5.0 of Microsoft FORTRAN.

Viton's (1983) uniform transportation equilibrium model predicted an 80% market share by the bus mode in a residential area with a 2,000 passenger hourly travel volume. This model predicts a 69% market share for area 2 which has similar physical characteristics to those in Viton's model. There are two reasons for the differences in these two models.

First, values of travel time drawn from work of Jung (1991) are significantly higher than those used in Viton's model. Keeler and Small (1977) show that a low value of travel time increases the optimal toll and leads to constructing fewer lanes, thus allowing highways to be congested. The high values of travel time used in this study causes us to build more lanes and to lower the congestion tolls. Thus, lower congestion tolls and a less congested highway are expected to contribute a higher market share by the auto mode.

Second, the peak-hour bus operating cost in this model is significantly higher than the average hourly bus operation cost used in Viton's model. Since bus operating costs in this model increase as the bus service distance increases, bus becomes less efficient transportation means as the service distance increases. As shown in Table 7, the bus service levels between areas 1 and 3 are significantly different.

< Table 7> Transportation Equilibrium Characteristics Under the Optimal Investment and Pricing Policies (The Base Model, Policy 1)

			Area 1	Area 2	Area 3
Modal Split at the Equilibrium		Auto	9.04%	31.27%	41.33%
		Bus	90.96%	68.73%	58.67%
Auto Congestion Tolls (in cents)		Per Trip	38.95	57.92	88.15
		Per vehicle-mile	6.49	4.83	4.20
Bus Fares (in cents)		Per Trip	2.78	8.44	16.84
		Per Vehicle-mile	0.46	0.70	0.80
Bus Service Level	Bus headway (min.)		3.17	5.18	13.88
	Number of bus routes		4	5	8
	Bus Walk-Access Time (min.)		8.17	9.33	10.10
	Bus Waiting Time (min.)		1.56	2.58	6.94
	Number of Passenger per Bus		50	23	17
Public Agency's Costs Per Day (in dollars)	Highway construction costs		2,181	639	395
	Bus Agency Costs	Capital costs	654	868	816
		Operationg Costs	1,416	1,963	1,877
Bus Agency's Losses per day (in dollars)			1,953	2,715	2,594
Subsidy per bus Passenger (in dollars)			0.54	1.97	4.42

The optimal auto congestion toll (per highway vehicle−mile) decreases as travel distance increases. On the other hand, the optimal bus fare (per passenger vehicle−mile) increases as travel distance increases because bus ridership decreases as the service distance increases (hence marginal costs must be divided amongst fewer passengers) and because peak−hour bus operating cost increases with service distance (see Table 7). Thus, the optimal spatial toll price structure should be constructed to give a travel distance discount for an auto and a distance premium for a bus.

Under the marginal cost pricing structure, the bus agency has to bear a huge deficit. Since the revenue generated from tolls is not sufficient enough to cover bus operating costs, tax monies must subsidize bus operation. Subsidies per bus passenger are $0.54, $1,97 and $4.42 in areas 1, 2 and 3, respectively, in order to provide bus service at the optimal levels. As a result, the marginal cost pricing policy benefits rich suburban bus commuters most because they pay less than 1% of their average costs.

2) The Impact of Auto congestion tolls on the Transportation Equilibrium

To observe the effects of auto congestion tolls on transportation equilibrium, I run the same model as the base model (shown in Table 7) without auto congestion tolls (see Table 8). Bus market shares, bus service level and bus fares are almost the same as the base model. However, highway construction and maintenance costs are increased by $217, $49 and $23 in areas 1, 2 and 3, respectively. Thus, the introduction of auto congestion tolls not only generates $869 of toll revenues per day but also results in a saving of $289 in highway construction and maintenance costs per day. The congestion toll revenue and highway construction cost saving combined amount to $590,580 per year (in 1972 prices) for this corridor (assuming 255 business days per year).

< Table 8 > Equilibrium Characteristics When Price Policy 2 (MC Pricing for Bus and No Auto Congestion Tolls) is implemented

		Area 1	Area 2	Area 3
Modal Split at the Equilibrium	Auto	11.70%	35.61%	45.00%
	Bus	88.03%	64.39%	55.00%
Highway Construction and Maintenance Costs($/day)		$2,397	$688	$417

3) The Equilibrium Conditions When the Break−even Condition is imposed on the Bus Agency

Under marginal cost pricing, the bus agency is losing about $3.7 million per year. There will be few local governments which is afford to bear such a huge deficit for a long time. In order to find the transportation equilibrium conditions when the break−even condition is imposed on the bus agency t(policy 3). The characteristics of the equilibrium are shown in Table 9. Compared to the equilibrium condition of policy 1, auto congestion tolls are almost the same as those in the policy 1. However, bus fares have increased dramatically. Bus fare (per mile) for area 1, 2 and 3 are 11.37 cents, 24.04 cents and 28.37 cents, respectively. These fares are 6 to 21 times higher than those under policy 1. Still, the optimal spatial toll structure (a distance discount for auto and a distance premium for bus) remains unchanged.

< Table 9> Equilibrium Characteristics When AC Pricing for Bus and MC Pricing for Auto are Applied (Policy 3)

		Area 1	Area 2	Area 3
Modal Split at the Equilibrium	Auto	52.03%	74.27%	72.62%
	Bus	47.97%	25.73%	27.38%
Auto Congestion Tolls (in cents)	Per Trip	36.72	55.21	84.80
	Per vehicle-mile	6.12	4.66	4.04
Bus Fares (in cents)	Per Trip	68.23	288.51	596.20
	Per Vehicle-mile	11.37	24.04	28.09

2. Welfare Analysis of various transportation pricing policies

To analyze welfare impacts of transportation pricing policies, the commuter's welfare change due to price changes is evaluated using the method developed by Small and Rosen (1981, equation 5.9). The formula to measure the consumer's welfare changes is[15)]

$$\Delta W = \frac{1}{\lambda}\left[\ln \sum_{1}^{2} \exp(V_i) \right]_{V_i^0}^{V_i^1},$$

where V_i is the observed systematic component of the utility of alternative i, λ is a marginal utility of income which converts the change of the expected utilities to monetary units, V^0_i is the utility function before price changes, and V^1_i is the utility function after price changes. The marginal utilities of income (by the stochastic version of Roy's Identity) and the expected utility under three different pricing policies are presented in Table 10. The sign of the compensating variations is adjusted so that positive compensating variations indicate a positive welfare change.

15) The term inside brackets is measured in abstract units of utility as known 'inclusive price', 'composite price', or 'expected utility'. the parameter λ is the marginal utility of income (obtained by Roy's Identity) and is expressed in 'units of cents per munite'.

<Table 10 > Expected Utilities under Four Transportation Pricing Policies

		Area 1	Area 2	Area 3
Marginal Utility of Income		0.0073	0.0031	0.0015
Expected Utility (Inclusive Prices)	Policy 1	−0.9109	−1.0522	−1.1847
	Policy 2	−0.8881	−1.0034	−1.1379
	Policy 3	−0.8861	−1.2191	−1.5267

The welfare effects of introducing auto congestion tolls (policy changes from policy 2 to policy 1) for both consumers and producers are shown in Table 11. All users are hurt by such a policy change (indicated by negative compensating variations). However, the higher−income commuters living in area 3 are hurt most because of a higher congestion toll for them (indicated by the highest negative compensating variation among the three groups). Travel time savings induced by auto congestion tolls are not sufficient enough to compensate for a high congestion toll for commuters in area 3. This implies that congestion pricing is not regressive as long as the bus service is provided at the optimal level. This contradicts Small's (1983b) earlier finding that congestion tolls are regressive. The reason is that bus service levels in our model, unlike his demand oriented model, are properly adjusted to the bus demand changes due to congestion pricing. Each individual's compensating variation (CV) is multiplied by the total number of commuters in each area to compute the total consumer's welfare changes. When congestion tolls and the producer's surplus such as bus agency's cost savings and highway construction costs savings are accounted for, the social net benefit from such policy changes is $148,979 per year for the simulated corridor (assuming 250 business days per year).

<Table 11> Welfare Changes When Auto Congestion Tolls are Implemented (in 1972 dollars)

	Area 1	Area 2	Area 3
Compensating Variation (CV) per passenger trip	−0.031	−0.157	−0.312
Aggregated CV	−124.98	−314.72	−312.24
Auto Toll Revenue Changes	140.87	362.54	364.68
Bus Agency's Cost Savings	−55.16	−111.89	−106.53
Highway Construction Cost Savings	216.42	48.77	22.52
Social Benefits Per Day	177.16	161.86	−46.91

Ⅶ. Concluding Remarks

There are strong interactive feedback between supply and demand for transportation: demand depends on service level and prices, while costs and service quality in turn depend crucially on highway congestion. Thus, Cost, demand, and pricing models should be integrated into an unified model, from which possible equilibrium for an entire urban transportation system could be described. Since, the demand and supply interactions take place within a geographical context, the spatial structure of transportation system and urban spatial characteristics are important factors in the unified model. Previously developed uniform transportation models (Viton 1983, 1986, Keeler and Small 1977) all have one important failing: the value of travel time is assumed to be constant over an urban space. In uniform transportation equilibrium models, the fact that the level of traffic congestion varies over an urban space was not properly accounted.

In this study, a new transportation equilibrium model which incorporated the spatial variability of travel time values and traffic congestion is developed. From the empirical study of the model using data drawn from the 1972 situation in the San Francisco Bay Area, several interesting findings are observed. First of all, at the equilibrium, bus market shares and the level of bus service vary significantly over an urban space. Second, the optimal bus fare structure not be discounted with respect to travel distance. Third, as long as the bus service levels are provided at optimal levels, auto congestion tolls are not regressive.

Above all, the most important finding concerns the importance of space itself. Although the equilibrium pattern depends on the specific population density distribution, income distribution and the physical layout of the transportation system over the urban space, there is a general conclusion to be drawn from the results of the simulation models. As long as there are taste variations over an urban space, the spatial transportation equilibrium conditions will also vary significantly over that space. This is in contrast with earlier studies in which the distribution of equilibrium characteristics was implicitly assumed to be spatially invariant.

The transportation equilibrium model presented in this paper is far from the perfect. There are many aspect remained to be solved in the future. It is very worthwhile to develop an algorithm which guarantees the convergence of the minimization problem in equation 7. To test the feasibility of an alternative transport system, we can extend the model by adding additional mode and mixed−modes such as subways, park−and−ride system and bus priority lanes (proposed by Small, 1983b). Potentially, a most important extension is to extend the urban space and network configuration itself to replicate the urban transportation environment more realistically.

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