Comm. Korean Math. Soc. 9 (1994), No. 3, pp. 649–658

ON THE BERWALD'S NEARLY KAEHLERIAN FINSLER MANIFOLD

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0. Introduction

The notion of the almost Hermitian Finsler manifold admitting an almost complex structure $f^i{}_j(x)$ was, for the first time, introduced by G. B. Rizza [8]. It is known that the almost Hermitian Finsler manifold (or a Rizza manifold) has been studied by Y. Ichijyo [2] and H. Hukui [1]. In those papers, the almost Hermitian Finsler manifold which the h-covariant derivative of the almost complex structure $f^i{}_j(x)$ with respect to the Cartan's Finsler connection vanishes was defined as the Kaehlerian Finsler manifold. The nearly Kaehlerian Finsler manifold was defined and studied by the former of authors [7]. The present paper is the continued study of above paper.

In the present paper, we define and study the Berwald's Kaehlerian Finsler manifold and the Berwald's nearly Kaehlerian Finsler manifold by the h-covariant derivatives of the almost complex structure $f^i{}_j(x)$ with respect to the Berwald's Finsler connection following the example of complex Riemannian geometry.

The terminology and notation are mainly referred to Matsumoto's monograph [4].

1. Preliminaries

We consider an n-dimensional Finsler manifold M with a fundamental function $L(x,y)$. The Finsler metric tensor $g_{ij}(x,y)$ is introduced by

$$g_{ij} = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j L^2(x,y), \tag{1.1}$$

Received April 2, 1994.

where $\dot{\partial}_i = \partial/\partial y^i$. Let $C\Gamma = (\Gamma_j{}^i{}_k, G^i{}_j, C_j{}^i{}_k)$ be the Cartan's Finsler connection. The connection coefficients $\Gamma_j{}^i{}_k$, $C_j{}^i{}_k$ and non-linear connection $G^i{}_j$ are constructed as follows:

$$
\begin{aligned}
\Gamma_j{}^i{}_k &= \frac{1}{2} g^{ir}(\delta_k g_{jr} + \delta_j g_{rk} - \delta_r g_{kj}),\\
C_j{}^i{}_k &= \frac{1}{2} g^{ir}(\dot{\partial}_k g_{jr} + \dot{\partial}_j g_{rk} - \dot{\partial}_r g_{kj}),\\
G^i{}_j &= \dot{\partial}_j G^i,
\end{aligned}
\tag{1.2}
$$

where $G^i = \gamma_j{}^i{}_k y^j y^k, \gamma_j{}^i{}_k = \frac{1}{2} g^{ir}(\partial_k g_{jr} + \partial_j g_{rk} - \partial_r g_{jk})$, $\delta_k = \partial_k - G^i{}_k \dot{\partial}_i$ and $\partial_i = \partial/\partial x^i$. For any Finsler tensor $K^i{}_j(x, y)$, the h-covariant and v-covariant derivatives with respect to the Cartan's Finsler connection $C\Gamma$ are defined as follows respectively

$$
K^i{}_{j|k} = \partial_k K^i{}_j - G^m{}_k \dot{\partial}_m K^i{}_j + \Gamma_m{}^i{}_k K^m{}_j - K^i{}_m \Gamma_j{}^m{}_k,
\tag{1.3}
$$

$$
K^i{}_j|_k = \dot{\partial}_k K^i{}_j + C_m{}^i{}_k K^m{}_j - K^i{}_m C_j{}^m{}_k.
\tag{1.4}
$$

Putting $G_j{}^i{}_k = \dot{\partial}_k G^i{}_j$, it is well known that $\Gamma_j{}^i{}_k$ and $G_j{}^i{}_k$ are the positively homogeneous functions of degree 0 with respect to y^i, and the relation

$$
G^i{}_j = \Gamma_k{}^i{}_j y^k
\tag{1.5}
$$

holds in [4].

As well as the Cartan's Finsler connection $C\Gamma = (\Gamma_j{}^i{}_k, G^i{}_j, C_j{}^i{}_k)$, we consider the Berwald's Finsler connection $B\Gamma = (G_j{}^i{}_k, G^i{}_j, 0)$. For any Finsler tensor $K^i{}_j(x, y)$, the h-covariant derivative with respect to the Berwald's Finsler connection $B\Gamma$ is defined as follows

$$
K^i{}_{j;k} = \partial_k K^i{}_j - y^m G_m{}^r{}_k \dot{\partial}_r K^i{}_j + K^m{}_j G_m{}^i{}_k - K^i{}_m G_j{}^m{}_k.
\tag{1.6}
$$

Let us assume that the Finsler manifold M admits an almost complex structure $f^i{}_j(x)$ and the fundamental function $L(x, y)$ satisfies the Rizza condition

$$
L(x, y\cos\theta + f(x)y\sin\theta) = L(x, y)
\tag{1.7}
$$

for any function θ.

Since $L(x, \lambda y) = \lambda L(x, y)$ for any positive number λ, the equation (1.7) can be expressed as

$$L(x, \tilde{c}y) = |\tilde{c}|L(x, y) \tag{1.8}$$

for non-zero complex number $\tilde{c} = a + bi$, where we put $\tilde{c}y^i = ay^i + bf^i{}_r(x)y^r$.

The Finsler manifold M which an almost complex structure $f^i{}_j(x)$ satisfying (1.7) or (1.8) is called an *almost Hermitian Finsler manifold* with an almost Hermitian Finsler structure $(f^i{}_j(x), g_{ij}(x, y))$.
In the present paper, we assume moreover the dimension of M is even because M admits an almost complex structure $f^i{}_j(x)$ [6]. Y. Ichijyo [2] showed that the Rizza condition (1.7) is equivalent to

$$g_{im}(x, y)f^m{}_j(x) + g_{jm}(x, y)f^m{}_i(x) + 2C_{ijm}(x, y)f^m{}_r(x)y^r = 0, \tag{1.9}$$

where $C_{ijm} = \dot{\partial}_m g_{ij}$.

M. Hukui [1] has proved that if $g_{ij}(x, y)$ and $f^i{}_j(x)$ satisfy the condition

$$g_{pq}(x, y)f^p{}_i(x)f^q{}_j(x) = g_{ij}(x, y), \tag{1.10}$$

then g_{ij} is a Riemannian metric, that is, (f, g) is an almost Hermitian structure.

On the other hand, in a Riemannian complex geometry, it is well know that in order that the almost complex structure $f^i{}_j(x)$ is integrable, it is necessary and sufficient that the Nijenhuis tensor

$$N^i{}_{jk} = (\partial_r f^i{}_j)f^r{}_k - (\partial_r f^i{}_k)f^r{}_j + f^i{}_r\partial_j f^r{}_k - f^i{}_r\partial_k f^r{}_j \tag{1.11}$$

vanishes. An almost Hermitian manifold with an almost Hermitian structure $(f^i{}_j(x), g_{ij}(x))$ is a Kaehlerian manifold if $\nabla_k f^i{}_j = 0$, where ∇_k is the covariant derivative with respect to the Levi-Civita connection $\left\{ {}_j{}^i{}_k \right\}$. The Hermitian manifold satisfying the following condition

$$\nabla_k f^i{}_j + \nabla_j f^i{}_k = 0 \tag{1.12}$$

is called a nearly Kaehlerian manifold. In a nearly Kaehlerian manifold, if the Nijenhuis tensor defined by (1.11) vanishes, then the nearly Kaehlerian manifold is a Kaehlerian manifold [10].

2. A Berwald's Kaehlerian Finsler manifold

In an almost Hermitian Finsler manifold; the h-covariant derivative of an almost complex structure $f^i{}_j(x)$ with respect to the Cartan's Finsler connection $C\Gamma = (\Gamma_j{}^i{}_k, G^i{}_j, C_j{}^i{}_k)$ is expressed as

$$f^i{}_{j|k} = \partial_k f^i{}_j + \Gamma_m{}^i{}_k f^m{}_j - f^i{}_m \Gamma_j{}^m{}_k. \tag{2.1}$$

An almost Hermitian Finsler manifold satisfying $f^i{}_{j|k} = 0$ is said to be a *Kaehlerian Finsler manifold.* In an almost Hermitian Finsler manifold, the h-covariant derivative of an almost complex structure $f^i{}_j(x)$ with respect to the Berwald's Finsler connection $B\Gamma = (G_j{}^i{}_k, G^i{}_j, 0)$ is expressed as

$$f^i{}_{j;k} = \partial_k f^i{}_j + G_m{}^i{}_k f^m{}_j - f^i{}_m G_j{}^m{}_k. \tag{2.2}$$

An almost Hermitian Finsler manifold satisfying $f^i{}_{j;k} = 0$ will be called a *Berwald's Kaehlerian Finsler manifold.* In a Kaehlerian Finsler manifold, we get from (2.1)

$$\partial_k f^i{}_j + \Gamma_m{}^i{}_k f^m{}_j - f^i{}_m \Gamma_j{}^m{}_k = 0. \tag{2.3}$$

Transvecting (2.3) with y^k and using (1.5), we have

$$y^m \partial_m f^i{}_j + G^i{}_m f^m{}_j - f^i{}_m G^m{}_j = 0. \tag{2.4}$$

Differentiating (2.4) partially with respect to y^k, we get

$$\partial_k f^i{}_j + G_m{}^i{}_k f^m{}_j - f^i{}_m G_j{}^m{}_k = f^i{}_{j;k} = 0.$$

Thus we have

THEOREM 2.1. *A Kaehlerian Finsler manifold is a Berwald's Kaehlerian Finsler manifold.*

REMARK. The inverse of this theorem is not satisfied.

We define the operators

$$(2.5)\qquad O^{ij}_{st} = \frac{1}{2}(\delta^i_s\delta^j_t - f^i{}_s f^j{}_t), {}^*O^{ij}_{st} = \frac{1}{2}(\delta^i_s\delta^j_t + f^i{}_s f^j{}_t)$$

for almost complex structure $f^i{}_j(x)$ respectively.

In the Berwald's Kaehlerian Finsler manifold, the tensors O^{ij}_{st} and ${}^*O^{ij}_{st}$ defined by (2.5) are h-covariant constant with respect to the Berwald's Finsler connection $B\Gamma = (G_j{}^i{}_k, G^i{}_j, 0)$.

If a Finsler tensor $K_i{}^h{}_j(x,y)$ satisfies $O^{ij}_{st}K_i{}^h{}_j = 0$, we say that $K_i{}^h{}_j$ is *hybrid* in i and j, and if ${}^*O^{ij}_{st}K_i{}^h{}_j = 0$, we say that it is *pure* in i and j.

Applying the Ricci identity of a Finsler tensor $K^i{}_j(x,y)$ for the Berwald's Finsler connection $B\Gamma = (G_j{}^i{}_k, G^i{}_j, 0)$, we have

$$(2.6)\qquad K^i{}_{j;h;k} - K^i{}_{j;k;h} = K^r{}_j H_r{}^i{}_{hk} - K^i{}_r H_j{}^r{}_{hk} - (\dot\partial_r K^i{}_j)R^r{}_{hk},$$

where $H_h{}^i{}_{jk}$ is the h-curvature tensor and $R^i{}_{jk}$ is the $(v)h$-torsion tensor and these are expressed as in [5]

$$(2.7)\qquad \begin{aligned} H_h{}^i{}_{jk} &= \delta_k G_h{}^i{}_j + G_r{}^i{}_k G_h{}^r{}_j - \delta_j G_h{}^i{}_k - G_r{}^i{}_j G_h{}^r{}_k,\\ R^i{}_{jk} &= \delta_k G^i{}_j - \delta_j G^i{}_k. \end{aligned}$$

In a Berwald's Kaehlerian Finsler manifold, the Ricci identity of an almost complex structure $f^i{}_j(x)$ are expressed as follows from (2.6) and $f^i{}_{j;k} = 0$,

$$(2.8)\qquad f^r{}_j H_r{}^i{}_{hk} - f^i{}_r H_j{}^r{}_{hk} = 0,$$

from which

$${}^*O^{ij}_{rl}(H_j{}^r{}_{hk}) = 0.$$

Hence we obtain

THEOREM 2.2. *In a Berwald's Kaehlerian Finsler manifold, the h-curvature tensor $H_j{}^r{}_{hk}$ is pure in r and j.*

Now, the Nijenhuis tensor $N^i{}_{jk}$ of the almost complex structure $f^i{}_j(x)$ defined by (1.11) is written as follows by substitution of (2.2)

$$\begin{aligned} N^i{}_{jk} &= (f^i{}_{j;r} - G_m{}^i{}_r f^m{}_j + f^i{}_m G_j{}^m{}_r) f^r{}_k \\ &\quad - (f^i{}_{k;r} - G_m{}^i{}_r f^m{}_k + f^i{}_m G_k{}^m{}_r) f^r{}_j \\ &\quad + f^i{}_r (f^r{}_{k;j} - G_m{}^r{}_j f^m{}_k + f^r{}_m G_k{}^m{}_j) \\ &\quad - f^i{}_r (f^r{}_{j;k} - G_m{}^r{}_k f^m{}_j + f^r{}_m G_j{}^m{}_k) \\ &= f^i{}_{j;r} f^r{}_k - f^i{}_{k;r} f^r{}_j + f^i{}_r f^r{}_{k;j} - f^i{}_r f^r{}_{j;k}. \end{aligned}$$

Hence we obtain

THEOREM 2.3. *In a Berwald's Kaehlerian Finsler manifold, the almost complex structure $f^i{}_j(x)$ is integrable.*

On the other hand, if an n-dimensional Finsler manifold $M(n > 3)$ satisfies $H_h{}^i{}_{jk} = K(g_{hj}\delta^i_k - g_{hk}\delta^i_j)$, then M is called a Finsler space of constant curvature [5]. In this case, (2.8) can be rewritten as

$$K f^r{}_j (g_{rh}\delta^i_k - g_{rk}\delta^i_h) - K f^i{}_r (g_{jh}\delta^r_k - g_{jk}\delta^r_h) = 0.$$

Now, we assume $K \neq 0$, then we have

$$f_{hj}\delta^i_k - f_{kj}\delta^i_h - f^i{}_k g_{jh} + f^i{}_h g_{jk} = 0.$$

Contracting this equation with respect to i and h, we get $(1-n)f_{kj} - f_{jk} = 0$. From (1.9), we find $(1-n)f_{kj} + f_{kj} + 2C_{kjm} f^m{}_r y^r = 0$. Since $n > 3$, we find $f_{ij} = \frac{2}{n-2} C_{ijm} f^m{}_r y^r$. Substituting this equation into (1.9) again, we have $C_{ijm} f^m{}_r y^r = 0$. Therefore we have $f_{ij} = 0$. This is a contradiction. Consequently we obtain $K = 0$. Thus we have

THEOREM 2.4. *Let M be an n-dimensional Berwald's Kaehlerian Finsler manifold. If M is a Finsler space of constant curvature and $n > 3$, then the h-curvature tensor of the Berwald's Finsler connection vanishes.*

3. A Berwald's nearly Kaehlerian Finsler Manifold

The almost Hermitian Finsler manifold satisfying

$$f^i{}_{j;k} + f^i{}_{k;j} = 0 \tag{3.1}$$

will be called a *Berwald's nearly Kaehlerian Finsler manifold.* In a Berwald's nearly Kaehlerian Finsler manifold, we have

$$\begin{aligned} 0 &= f^h{}_{s;t} + f^h{}_{t;s} \\ &= f^h{}_{s;t} + f^i{}_s f^j{}_t f^h{}_{i;j} = {}^*O^{ij}_{st} f^h{}_{i;j} \end{aligned} \tag{3.2}$$

because of $f^i{}_{r;k} f^r{}_j = -f^i{}_r f^r{}_{j;k}$.

Thus we have

THEOREM 3.1. *In a Berwald's nearly Kaehlerian Finsler manifold,* $f^i{}_{j;k}$ *is pure in* j *and* k.

Using (2.2), the Nijenhuis tensor $N^i{}_{jk}$ of the almost complex structure $f^i{}_j(x)$ defined by (1.11) is written as follows

$$\begin{aligned} N^i{}_{jk} &= f^i{}_{j;r} f^r{}_k - f^i{}_{k;r} f^r{}_j + f^i{}_r f^r{}_{k;j} - f^i{}_r f^r{}_{j;k} \\ &= 4 f^i{}_r f^r{}_{k;j} - 2 f^i{}_r (f^r{}_{k;j} + f^r{}_{j;k}) \\ &\quad - (f^i{}_{r;k} + f^i{}_{k;r}) f^r{}_j + (f^i{}_{j;r} + f^i{}_{r;j}) f^r{}_k. \end{aligned} \tag{3.3}$$

Therefore, in a Berwald's nearly Kaehlerian Finsler manifold we obtain

$$N^i{}_{jk} = 4 f^i{}_r f^r{}_{k;j}.$$

Thus we have

THEOREM 3.2. *Let M be a Berwald's nearly Kaehlerian Finsler manifold. In order that the Nijenhuis tensor* $N^i{}_{jk}$ *vanishes identically, it is necessary and sufficient that M is a Berwald's Kaehlerian Finsler manifold.*

In a Berwald's nearly Kaehlerian Finsler manifold, if $C_{kim}(x, y) f^m{}_j(x)$ is pure in i and j, then we have $C_{kim} f^m{}_j - C_{kmj} f^m{}_i = 0$, from which $C_{kim} f^m{}_j y^j = 0$ by virtue of $C_{kmj} y^j = 0$. Therefore, from (1.9) and

(1.10), g_{ij} is a Riemannian metric, that is, (f, g) is an almost Hermitian structure. Then we find $f_{ij} = -f_{ji}$ and $\Gamma_j{}^i{}_k = G_j{}^i{}_k = \left\{{}_j{}^i{}_k\right\}$ because of (1.2) and (1.5). Thus the h-covariant derivative with respect to the Berwald's Finsler connection coincides with the covariant derivative with respect to the Levi-Civita's connection. We get

$$\nabla_k f^i{}_j + \nabla_j f^i{}_k = f^i{}_{j;k} + f^i{}_{k;j}.$$

Hence we obtain

THEOREM 3.3. *In a Berwald's Finsler manifold M, if $C_{kim} f^m{}_j$ is pure in i and j, then M is a nearly Kaehlerian manifold.*

The functions $G^i(x, y)$ defined by (1.2) are (2)p-homogeneous in y. We put $G_i{}^h{}_{jk} = \dot{\partial}_i G_j{}^h{}_k$ and $G_{ij} = G_i{}^r{}_{jr}$. It notes that $G_i{}^h{}_{jk}$ and G_{ij} are symmetric in the indices i, j, k and i, j respectively.

By the Euler's theorem of homogeneous function in y^i, we have

$$\begin{aligned}
&G_i{}^h{}_{j0} = G_i{}^h{}_{0k} = G_0{}^h{}_{jk} = 0,\\
&G_i{}^h{}_0 (= G_0{}^h{}_i) = G^h{}_i,\\
&G_{0j} = G_{j0} = 0,\\
(3.4)\quad &G^h{}_0 = 2G^h,\\
&y^r \dot{\partial}_r G_i{}^h{}_{jk} (= y^r \dot{\partial}_i G_r{}^h{}_{jk} = y^r \dot{\partial}_j G_i{}^h{}_{rk} = y^r \dot{\partial}_k G_i{}^h{}_{jr}) = -G_i{}^h{}_{jk},\\
&y^r \dot{\partial}_r G_{ij} (= y^r \dot{\partial}_i G_{jr} = y^r \dot{\partial}_j G_{ir}) = -G_{ij},
\end{aligned}$$

where the index 0 denotes the contraction with element of support y.

If $G_j{}^i{}_k$ are functions of position alone, namely, $\dot{\partial}_h G_j{}^i{}_k = G_h{}^i{}_{jk} = 0$, then the Finsler manifold is said to be a Berwald space as in [9]. Moreover, the tensor field D with components

$$(3.5)\quad D_i{}^h{}_{jk} = G_i{}^h{}_{jk} - \frac{1}{(n+1)}(y^h \dot{\partial}_k G_{ij} + \delta_i^h G_{jk} + \delta_j^h G_{ki} + \delta_k^h G_{ij})$$

is known as Douglas tensor in [3].

THEOREM 3.4. *If a Berwald's nearly Kaehlerian Finsler manifold has a vanishing Douglas tensor, then it is a Berwald space.*

Proof. From the assumption, we have

$$\partial_k f^i{}_j + \partial_j f^i{}_k + G_m{}^i{}_k f^m{}_j + G_m{}^i{}_j f^m{}_k - 2f^i{}_m G_j{}^m{}_k = 0, \tag{3.6}$$

$$G_i{}^h{}_{jk} = \frac{1}{n+1}(y^h \dot{\partial}_k G_{ij} + \delta^h_i G_{jk} + \delta^h_j G_{ki} + \delta^h_k G_{ij}). \tag{3.7}$$

Differentiating (3.6) partially with respect to y^l, we find

$$G_l{}^i{}_{mk} f^m{}_j + G_l{}^i{}_{mj} f^m{}_k - 2f^i{}_m G_l{}^m{}_{jk} = 0. \tag{3.8}$$

Contracting (3.8) with i and l, we have

$$G_{mk} f^m{}_j + G_{mj} f^m{}_k - 2f^r{}_m G_r{}^m{}_{jk} = 0. \tag{3.9}$$

Transvecting (3.9) with y^j and using (3.4), we find

$$G_{mk} f^m{}_0 = 0. \tag{3.10}$$

Differentiating (3.10) with respect to y^r, we can get

$$f^m{}_0 \dot{\partial}_r G_{mk} + G_{mk} f^m{}_r = 0. \tag{3.11}$$

That is,

$$G_{mk} f^m{}_r = -f^m{}_0 \dot{\partial}_r G_{mk} = -f^m{}_0 \dot{\partial}_m G_{kr} = G_{mr} f^m{}_k. \tag{3.12}$$

On the other hand, substituting (3.7) into (3.8), we have

$$\begin{aligned}
&(y^i \dot{\partial}_k G_{lm} + \delta^i_l G_{mk} + \delta^i_m G_{kl} + \delta^i_k G_{lm}) f^m{}_j \\
&+ (y^i \dot{\partial}_j G_{lm} + \delta^i_l G_{mj} + \delta^i_m G_{jl} + \delta^i_j G_{lm}) f^m{}_k \\
&- 2f^i{}_m (y^m \dot{\partial}_k G_{lj} + \delta^m_l G_{jk} + \delta^m_j G_{kl} + \delta^m_k G_{lj}) = 0.
\end{aligned} \tag{3.13}$$

Contracting (3.13) with i and l, we have

$$(3.14)\quad \begin{aligned}(y^r\dot{\partial}_r G_{mk} + (n+2)G_{mk})f^m{}_j + (y^r\dot{\partial}_r G_{mj} + (n+2)G_{mj})f^m{}_k \\ - 2(f^r{}_0\dot{\partial}_r G_{kj} + f^r{}_j G_{kr} + f^r{}_k G_{rj}) = 0\end{aligned}$$

because of $\dot{\partial}_j G_{lm} = \dot{\partial}_l G_{mj} = \dot{\partial}_m G_{jl}$. From (3.12), the equation (3.14) is reduced to

$$(3.15)\qquad f^m{}_j y^r \dot{\partial}_r G_{mk} + f^m{}_k y^r \dot{\partial}_r G_{mj} + 2(n+1)G_{mk} f^m{}_j = 0.$$

By means of (3.4) and (3.12), the equation (3.15) turns out

$$2nG_{mk}f^m{}_j = 0,$$

that is, $G_{jk} = 0$. Substituting this equation into (3.7), we obtain $G_i{}^h{}_{jk} = 0$. Consequently, the manifold is a Berwald space. This completes the proof of the Theorem.

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