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ANALYTIC TORSION FOR HOLOMORPHIC VECTOR BUNDLES

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Let $E \to M$ be a hermitian holomorphic vector bundle over a compact (complex) hermitian manifold M of complex dimension n, and let

$$d''_p(E) : 0 \to A^{p,0}(E) \xrightarrow{d''} A^{p,1}(E) \to \cdots \to A^{p,n}(E) \to 0$$

be the Dolbeault complex. Then $A^{p,q}(E)$ become a prehilbert space so that the formal adjoint δ'' of d'' and the "Laplacian" $\Delta'' = \delta'' d'' + d'' \delta''$ are defined.

Now Ray-Singer's *analytic torsion* for the elliptic complex $d''_p(E)$ is given by

$$\tau_p(E) := \exp\left(\frac{1}{2}\sum_{q\geq 0}(-1)^q q \zeta'_{p,q}(0)\right),$$

where $\zeta_{p,q}$ denotes the zeta function [RS71, 73] associated to the positive semi-definite elliptic operator $\Delta''_{p,q} := \Delta''|_{A^{p,q}(E)}$.

Note that there is a commutative diagram

$$\begin{array}{llcccccc}
d''_p(E) : & 0 \to & A^{p,0}(E) & \xrightarrow{d''} & A^{p,1}(E) & \to \cdots \to & A^{p,n}(E) & \to 0 \\
 & & \downarrow{\scriptstyle h^*} & & \downarrow{\scriptstyle -h^*} & & \downarrow{\scriptstyle (-1)^n h^*} & \\
\delta''_{n-p}(E^*) : & 0 \to & A^{n-p,n}(E^*) & \xrightarrow{\delta''} & A^{n-p,n-1}(E^*) & \to \cdots \to & A^{n-p,0}(E^*) & \to 0
\end{array}$$

where the vertical arrow h^* is the (conjugate linear) Hodge star $\bar{*}$: $A^{p,q} \to A^{n-p,n-q}$ coupled with the "hermitian structure" $h : E \to E^*$. Thus two complexes $d''_p(E)$ and $\delta''_{n-p}(E^*)$ have the same analytic torsion.

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Since the bottom complex $\delta''_{n-p}(E^*)$ is the *adjoint* of

$$0 \to A^{n-p,0}(E^*) \xrightarrow{d''} A^{n-p,1}(E^*) \to \cdots \to A^{n-p,n}(E^*) \to 0,$$

the analytic torsion of $\delta''_{n-p}(E^*)$ is equal to $\tau_{n-p}(E^*)^{(-1)^{n+1}}$ and hence we get

$$\tau_p(E) = \tau_{n-p}(E^*)^{(-1)^{n+1}}.$$

Now we apply the above consideration to the trivial line bundle E to obtain the following theorem.

THEOREM. *Let M be a compact complex manifold of even dimension with the trivial canonical line bundle K_M. Then the analytic torsion of the Dolbeault complex*

$$0 \to A^{0,0} \xrightarrow{d''} A^{0,1} \to \cdots \to A^{0,n} \to 0$$

is identically equal to 1 for any hermitian metric on M.

References

[RS71] D. B. Ray and I. M. Singer, *R-torsion and the Laplacian on Riemannian Manifolds*, Adv. Math. **7** (1971), 145–210.

[RS73] ———, *Analytic torsion for complex manifolds*, Ann. of Math. **98** (1973), 154–177.

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