Comm. Korean Math. Soc. **9** (1994), No. 3, pp. 711–730

A CHARACTERIZATION OF GIBBS MEASURES ON $(\mathbf{R} \times W_{0,0})^{\mathbf{Z}^\nu}$ VIA STOCHASTIC CALCULUS

HYE YOUNG LIM[1], YONG MOON PARK[1] AND HYUN JAE YOO[2]

1. Introduction

We consider Gibbs measures on $(\mathbf{R} \times W_{0,0})^{\mathbf{Z}^\nu}$, $W_{0,0} = \{\omega \in C[0,1] : \omega(0) = \omega(1)\}$, which are associated to an interaction between particles in lattice boson systems (quantum unbounded spin systems). In [4], the Gibbs measures were introduced in the study of equilibrium states of interacting lattice boson systems and were characterized by means of the equilibrium conditions. In this paper we utilize the techniques of the stochastic calculus of variations and the infinite dimensional Itô integral to derive stochastic equations which we call the equilibrium equations. We show that under appropriate conditions the equilibrium conditions and the equilibrium equations are equivalent. The lattice boson systems with superstable and regular interactions, which we studied in [4], are typical examples.

There have been many studies on the infinite product spaces of paths related to the diffusion processes with infinite number of degrees of freedom and to the statistical mechanical systems (see [1], [6] and references therein). Recently Roelly and Zessin investigated Gibbs measures on $C[0,1]^{\mathbf{Z}^\nu}$, in fact on $W_0^{\mathbf{Z}^\nu}$ where $W_0 = \{\omega \in C[0,1] : \omega(0) = 0\}$, associated with a bounded interaction [6]. They derived the equilibrium equations by using the method of stochastic calculus of variations. In the case of bounded interactions they established the equivalence of the equilibrium conditions and the equilibrium equations. However there are two drawbacks in the work of Roelly and Zessin: First, the configuration space [4] associated to quantum boson systems is $\Omega = S^{\mathbf{Z}^\nu}$, $S = \{s \in C[0,1] : s(0) = s(1)\}$, instead of $W_0^{\mathbf{Z}^\nu}$ considered in [6]. Second, most of

Received April 27, 1994.

the interesting interactions in quantum boson systems belong to a class of unbounded interactions [3, 4, 7, 8].

The aim of this paper is to overcome the above drawbacks. Note that each path $s \in S$ is closed and so it is natural to impose periodic boundary conditions to any operation related to the paths s. We modify the (stochastic) derivative operator used in [6] to accommodate periodic boundary conditions. In order to deal with unbounded interactions which are superstable and regular, we use the probability estimates established in [4].

It would be interesting to study Dirichlet forms, Dirichlet operators and diffusion processes associated to the Gibbs measures considered in this paper. The questions of closability for Dirichlet forms and the problems of constructing the associated diffusion processes are under investigation.

The organization of this paper is as follows. In section 2, we give notations, definitions and necessary preliminaries, and then state our main results. In section 3, we produce all the proofs of the main results. We shall employ the methods used in [6] with suitable modifications.

2. Preliminaries and Main Results

We start with introducing the necessary notations. Let S be the Polish space $\mathbf{R} \times W_{0,0}$ with canonical filtration, where $W_{0,0} = \{\omega \in C[0,1] : \omega(0) = \omega(1) = 0\}$. That is, S has a σ-algebra generated by the products of Borel sets in $\mathbf{R}$ and cylinder sets in $W_{0,0}$ [9]. Let $\Omega \equiv S^{\mathbf{Z}^\nu} = (\mathbf{R} \times W_{0,0})^{\mathbf{Z}^\nu}$. For each subset $\Lambda \subset \mathbf{Z}^\nu$ we shall use the notation x_Λ and s_Λ for the elements of $\mathbf{R}^\Lambda$ and S^Λ, respectively. For each $i \in \mathbf{Z}^\nu$, let $p_i : \Omega \longrightarrow S$ be the canonical projection, $p_i(s) = s_i$, the value of the path s on the i-th site. For each $\Lambda \subset \mathbf{Z}^\nu$, we have a local σ-algebra $\mathcal{F}_\Lambda$, which is the minimal σ-algebra for which p_i, $i \in \Lambda$, are measurable. We simply write $\mathcal{F}$ for $\mathcal{F}_{\mathbf{Z}_\nu}$. By $\mathcal{P}(\Omega, \mathcal{F})$ we mean the set of all probability measures on Ω. Let λ be the reference measure on $\mathbf{R} \times W_{0,0}$ defined by

$$\lambda(ds) = dx \times P_{0,0}(d\omega), \quad s = (x, \omega) = x + \omega, \tag{2.1}$$

where dx is the Lebesgue measure on $\mathbf{R}$ and $P_{0,0}(d\omega)$ is the conditional Wiener measure on $W_{0,0}$ [9]. For any bounded $\Lambda \subset \mathbf{Z}^\nu$, put $\lambda^\Lambda(ds_\Lambda) = \prod_{i \in \Lambda} \lambda(ds_i)$.

We would like to also consider S as a topological space. We give an L^2-norm to S : For $s = (x, \omega) \in S$, let

$$\|s\| = \left(\int_0^1 |s(t)|^2 dt \right)^{1/2}, \tag{2.2}$$

where $s(t) = x + \omega(t)$. We say that a function $F : S \to \mathbf{C}$ is L^2-*continuous* if F is continuous in this topology.

REMARK 2.1. Notice that $\lambda(B) > 0$ for any nonempty open set in the L^2-topology. Since $\|s\| \le \|s\|_\infty$, any ball centered at the zero path in the L^2-norm contains the corresponding ball in L^∞-norm which has nonzero λ-measure by Lemma A.3 of [4]. Furthermore since every translations $\theta_s\lambda$ of λ defined by $\theta_s(\lambda)(\,\cdot\,) \equiv \lambda(\,\cdot - s), s \in S$, are absolutely continuous with respect to λ, we conclude that any nonempty open set has strictly positive λ-measure.

Let us introduce interactions and specifications.

DEFINITION 2.2. An interaction $\Phi = (\Phi_\Delta)_{\Delta \subset \mathbf{Z}^\nu}$ is a net of measurable functions Φ_Δ on $(\Omega, \mathcal{F})$ and satisfies the following conditions:
(a) Φ_Δ is $\mathcal{F}_\Delta$-measurable for all $\Delta \subset \mathbf{Z}^\nu$;
(b) Φ_Δ is invariant under translations of $\mathbf{Z}^\nu$;
(c) There is a (measurable) subset $\mathfrak{S}$ (depending on Φ) of Ω such that for all $\bar{s} \in \mathfrak{S}$ and finite $\Lambda \subset \mathbf{Z}^\nu$,

$$Z_\Lambda^\Phi(\bar{s}) \equiv \int_{S^\Lambda} \lambda^\Lambda(ds_\Lambda) \exp[-H_\Lambda^\Phi(s_\Lambda \bar{s}_{\Lambda^c})] < \infty, \tag{2.3}$$

where $s_\Lambda \bar{s}_{\Lambda^c} \in \Omega$ is a configuration which coincides with s_Λ on Λ and $\bar{s}_{\Lambda^c}$ on Λ^c, and for all $s \in \Omega$,

$$\begin{aligned} H_\Lambda^\Phi(s) &\equiv V(s_\Lambda) + W(s_\Lambda, s_{\Lambda^c}), \\ V(s_\Lambda) &= \sum_{\Delta \subset \Lambda} \Phi(s_\Delta), \\ W(s_\Lambda, s_{\Lambda^c}) &= \sum_{\substack{\Delta \cap \Lambda \neq \emptyset \\ \Delta \cap \Lambda^c \neq \emptyset}} \Phi_\Delta(s_\Delta). \end{aligned} \tag{2.4}$$

REMARK 2.3. (a) For physical systems [4] Φ_Δ is given by

$$\Phi_\Delta(s_\Delta) = \int_0^1 d\tau\, \widetilde{\Phi}_\Delta(s_\Delta(\tau)),$$

where for any $\Delta \subset \mathbf{Z}^\nu$, $\widetilde{\Phi}_\Delta$ is a Borel measurable function on $\mathbf{R}^\Delta$. Since we are concerning only on the Gibbs measures (not on the Gibbs states on a quasi-local algebra) it is sufficient, using Feynman-Kac formula, that Φ_Δ is defined as above.
(b) The introduction of the subset $\mathfrak{S}$ is necessary for general interactions which may not be bounded. Every Gibbs measures (if exist) for the interaction should be supported on $\mathfrak{S}$ [5].
(c) The formalism introduced so far is insensitive to the dimensionality of the range space of the paths $s \in S$. One can replace S by $\mathbf{R}^d \times \{\omega \in C([0,1], \mathbf{R}^d) : \omega(0) = \omega(1)\}$. Then all the results in this paper remain to hold.

Given an interaction Φ, the corresponding Gibbs specification $\gamma^\Phi = (\gamma_\Lambda^\Phi)_{\substack{\Lambda \subset \mathbf{Z}^\nu \\ \Lambda:\ \text{finite}}}$ with respect to $\mathfrak{S}$ is defined by [2,4,5]

$$\text{(2.5)} \qquad \gamma_\Lambda^\Phi(A|\overline{s}) = \begin{cases} Z_\Lambda^\Phi(\overline{s})^{-1} \int \lambda^\Lambda(ds_\Lambda) \exp[-V(s_\Lambda) - W(s_\Lambda, \overline{s}_{\Lambda^c})] & \\ \qquad \times 1_A(s_\Lambda \overline{s}_{\Lambda^c}) & \text{if } \overline{s} \in \mathfrak{S}, \\ 0 \qquad \text{if } \overline{s} \notin \mathfrak{S}, & \end{cases}$$

where $A \in \mathcal{F}$ and 1_A is the indicator function on A. It is easy to check that the Gibbs specification satisfies the consistent condition [2, 5]: For $\Delta \subset \Lambda$, $\overline{s} \in \mathfrak{S}$,

$$\begin{aligned} \gamma_\Lambda^\Phi \gamma_\Delta^\Phi(A|\overline{s}) &\equiv \int_{\mathfrak{S}} \gamma_\Lambda^\Phi(ds^*|\overline{s}) \gamma_\Delta^\Phi(A|s^*) \\ &= \gamma_\Lambda^\Phi(A|\overline{s}). \end{aligned}$$

For given interaction Φ the Gibbs measures on $(\Omega, \mathcal{F})$ are defined as follow:

DEFINITION 2.4. *A Gibbs measure μ for the potential Φ is a Borel probability measure on $(\Omega, \mathcal{F})$ satisfying the equilibrium conditions: For any bounded $\Lambda \subset \mathbf{Z}^\nu$*

$$\mu(A) = \int \mu(d\overline{s})\gamma_\Lambda^\Phi(A|\overline{s}), \qquad A \in \mathcal{F}.$$

We denote by $\mathcal{G}(\Phi)$ the family of all Gibbs measures for the interaction Φ.

Let us now define the derivative operators for the functions on S and Ω. From now on we interpret the closed interval $[0,1]$ as the torus T^1 of circumference 1 by matching the end points 0 and 1. Let $L^{1,2}(S, d\lambda) \equiv L^1(S, d\lambda) \bigcap L^2(S, d\lambda)$. By $W^{1;1,2}(S, d\lambda)$, we mean the set of functions in $L^{1,2}(S, d\lambda)$ such that $F \in W^{1;1,2}(S, d\lambda)$ is $L^{1,2}$-differentiable in the sense that for all $g \in L^2(T^1)$ the following limit exists in $L^{1,2}(S, d\lambda)$:

$$D_g F(s) \equiv \lim_{\varepsilon \to 0} \varepsilon^{-1}(F(s + \varepsilon\widetilde{g}) - F(s)), \quad s \in S, \tag{2.6}$$

where $\widetilde{g} \in S$ is defined by

$$\widetilde{g}(t) = \int_0^1 du \int_{t-u}^t d\tau\, g(\tau), \quad t \in [0,1] = T^1. \tag{2.7}$$

The above derivative operators are the modified versions of those defined in [1, 6]. See the remark stated below. In the integration in (2.7), we understand that $\int_{t-u}^t d\tau g(\tau) = \int_{1+t-u}^1 d\tau g(\tau) + \int_0^t d\tau g(\tau)$ if $t - u < 0$. As mentioned in [6], $D_g F$ is nothing but the Fréchet derivative of F.

REMARK 2.5. If one interchanges the order of integrations of u and τ in the definition of $\widetilde{g}(t)$, one has that

$$\widetilde{g}(t) = \int_0^t d\tau g(\tau) - t \int_0^1 d\tau g(\tau) + \int_0^1 d\tau\ \tau\, g(\tau).$$

The above definition is slightly different from that in [6], where $\widetilde{g}$ was given by $\widetilde{g}(t) = \int_0^t d\tau\, g(\tau)$. The reason for the definition of $\widetilde{g}$ as in (2.7) is to impose the periodic boundary conditions. Since we are dealing with a torus (not a line segment $[0,1]$) we have no preferable origin. Actually $\widetilde{g}$ in (2.7) is the mean for taking arbitrary point in T^1 as the origin.

Now let us consider functions on Ω. If there is no confusion involved, we shall use the same letters, e.g., s, etc, for elements of S or Ω. We say that a function F on Ω is Λ-*local* if $F(s) = F(s_\Lambda)$ for all $s \in \Omega$. Let $l^1(\mathbf{Z}^\nu;\ L^2(T^1))$ be the space of $L^2(T^1)$-valued functions which are defined on $\mathbf{Z}^\nu$ and absolutely summable. By $W^{1;1,2}_{\text{loc}}(\Omega)$, we mean the set of local functions in Ω which are $L^{1,2}$-differentiable in the sense that for any $g = (g_i)_{i\in\mathbf{Z}^\nu} \in l^1(\mathbf{Z}^\nu;\ L^2(T^1))$, the following limit exists:

$$
\begin{aligned}
(2.8) \qquad D_g F(s) &= \lim_{\varepsilon\to 0} \varepsilon^{-1}(F(s+\varepsilon\widetilde{g}) - F(s)) \\
&= \sum_{i\in\mathbf{Z}^\nu} D^i_g F(s),
\end{aligned}
$$

where $\widetilde{g} = (\widetilde{g}_i)_{i\in\mathbf{Z}^\nu}$ and $\widetilde{g}_i$, $i \in \mathbf{Z}^\nu$, are defined as in (2.7). Each $D^i_g F(s)$, interpreted as a function on S by $s_i \longrightarrow D^i_g F(s)$, is an element of $L^{1,2}(S, d\lambda)$, where s_i is the projection of s to the i-th site. Since F is a local function, only finite terms survive in the summation.

Before passing to the stochastic integration we define a space of functions on $\mathbf{R}$: By $W^{1;1,2}(\mathbf{R}, dx)$ we mean the set $\{F_0 \in L^1(\mathbf{R}, dx)\bigcap L^2(\mathbf{R}, dx) : F_0 \text{ is differentiable and } F_0' \in L^1(\mathbf{R}, dx)\bigcap L^2(\mathbf{R}, dx)\}$.

Now if the integration

$$
(2.9) \qquad \zeta_g(s) \equiv \int_0^1 g(t)ds(t), \quad s \in S,
$$

is meaningful (in the Itô sense) for $g \in L^2(T^1)$, we say that ζ_g is the stochastic integration with respect to the canonical process. And for $g = (g_i)_{i\in\mathbf{Z}^\nu} \in l^1(\mathbf{Z}^\nu;\ L^2(T^1))$, we put for $s \in \Omega$

$$
\begin{aligned}
(2.10) \qquad \zeta_g(s) &= \sum_{i\in\mathbf{Z}^\nu} \zeta_{g_i}(s_i) \\
&= \sum_{i\in\mathbf{Z}^\nu} \int_0^1 g_i(t)ds_i(t).
\end{aligned}
$$

We now state the main results of the paper.

PROPOSITION 2.6. *For any $F \in W^{1;1,2}(S, d\lambda)$ and $g \in L^2(T^1)$, the duality property*

$$E_\lambda(\zeta_g F) = E_\lambda(D_g F) \tag{2.11}$$

holds. Especially one has that for $F_0 \in W^{1;1,2}(\mathbf{R}, dx)$ and $g \in L^2(T^1)$

$$E_\lambda(F_0 e^{it\zeta_g}) = I(F_0)e^{-\frac{1}{2}t^2\sigma^2(g)}, \tag{2.12}$$

where $F_0(s) = F_0(s(0))$, $I(F_0) = \frac{1}{\sqrt{2\pi}} \int F_0(x)dx$ and $\sigma^2(g) = \int_0^1 g^2(t)dt - (\int_0^1 g(t)dt)^2$. Conversely if ρ is a measure on S which is absolutely continuous with respect to λ with a L^2-continuous and L^2-continuously differentiable (in the sense of (2.6)) Radon-Nikodym derivative and satisfies

$$E_\rho(\zeta_g F) = E_\rho(D_g F)$$

for all $F \in W^{1;1,2}(S, d\lambda)$ and $g \in L^2(T^1)$, then $\rho = c\lambda$ for some constant $c > 0$.

Using the above duality property between the derivative operator and the stochastic integration we can give a characterization for the Gibbs measures on Ω. In the following $H_i(\,\cdot\,)$ means $H^\Phi_{\{i\}}(\,\cdot\,)$. See the notation in (2.4).

PROPOSITION 2.7. *Let Φ be an interaction described as in Definition 2.2. Suppose that $H_i(\,\cdot\, \overline{s}_{\{i\}^c})$ is a differentiable function (in the sense of (2.6)) for all $\overline{s} \in \mathfrak{S}$. Let $\mu \in \mathcal{G}(\Phi)$ be such that μ_Λ, the restriction of μ to finite $\Lambda \subset \mathbf{Z}^\nu$, is absolutely continuous with respect to λ^Λ and such that $E_{\mu_{\{i\}}}(|\zeta_{g_i}|^2) \le c\,\|g_i\|^2_{L^2(T^1)}$ for some constant c for all $i \in \mathbf{Z}^\nu$ and $g_i \in L^2(T^1)$. Then the Gibbs measure μ satisfies the equilibrium equations:*

$$E_\mu(F\zeta_g) = E_\mu(D_g F) - E_\mu(F \sum_{i \in \mathbf{Z}^\nu} D^i_g H_i), \tag{2.13}$$

for any $F \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ and $g \in l^1(\mathbf{Z}^\nu;\, L^2(T^1))$.

We will show that for superstable and regular interactions the above conditions are satisfied (see Corollary 2.11).

On the other hand the relation (2.13) also characterizes the Gibbs measures for some interactions:

THEOREM 2.8. *Let Φ be an interaction given as in Definition 2.2. Suppose that $H_i(\cdot\,\overline{s}_{\{i\}^c})$ is L^2-continuous and L^2-continuously differentiable (in the sense of (2.6)) for all $i \in \mathbf{Z}^\nu$. Let $\mu \in \mathcal{P}(\Omega)$ be such that $\mu(\mathfrak{S}) = 1$ and for all $\overline{s} \in \mathfrak{S}$, the measures $E_\mu(\cdot\,|\mathcal{F}_{\Lambda^c})(\overline{s})$ on Ω^Λ are absolutely continuous with respect to λ^Λ with L^2-continuous and L^2-continuously differentiable Radon-Nikodym derivatives for all finite $\Lambda \subset \mathbf{Z}^\nu$. Furthermore suppose that for $F_1 \in W^{1;1,2}(S, d\lambda)$, $F_1(\cdot)e^{-H_i(\cdot\,\overline{s}_{\{i\}^c})}$ $\in W^{1;1,2}(S, d\lambda)$ for any $i \in \mathbf{Z}^\nu$ and $\overline{s} \in \mathfrak{S}$, and that for any $F \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ and $g \in l^1(\mathbf{Z}^\nu; L^2(T^1))$ each term in (2.14) below are well defined and the equality*

$$E_\mu(F\zeta_{g_i}) = E_\mu(D^i_g F) - E_\mu(F D^i_g H_i), \quad i \in \mathbf{Z}^\nu, \tag{2.14}$$

holds. Then $\mu \in \mathcal{G}(\Phi)$.

All the proofs are given in the next section. In the remainder of this section we check that the Gibbs measures constructed for a class of superstable and regular interactions [4] satisfy the assumptions in Proposition 2.7 (and Theorem 2.8), and so the relation (2.13) holds for any $\mu \in \mathcal{G}^\Phi(\Omega)$.

Let us introduce the superstable and regular interactions. Remember the definition of the norm in S given in (2.1). Briefly we write s^2 for $\|s\|^2$. Note that the reference measure λ has the property [4]: For all $\alpha > 0$

$$\int \lambda(ds)e^{-\alpha s^2} < \infty. \tag{2.15}$$

Let us now recall the definition of superstable and regular interactions [3, 4, 7, 8].

DEFINITION 2.9. Let $\Phi = (\Phi_\Delta)_{\Delta \subset \mathbf{Z}^\nu}$ be an interaction described as in Definition 2.2.
(a) The interaction is said to be *superstable* if there are $A > 0$ and $c \in \mathbf{R}$ such that for every $s_\Lambda \in S^\Lambda$,

$$V(s_\Lambda) = \sum_{\Delta \subset \Lambda} \Phi_\Delta(s_\Lambda) \geq \sum_{i \in \Lambda}(As_i^2 - c).$$

(b) The interaction is said to be *regular* if there exists a decreasing positive function Ψ on the natural integers such that

$$\Psi(r) \le K r^{-\nu-\varepsilon} \text{ for some } K \text{ and } \varepsilon > 0 \text{ with } \sum_{i \in \mathbf{Z}^\nu} \Psi(|i|) < A,$$

where for $i = (i_1, \ldots, i_\nu) \in \mathbf{Z}^\nu$, $|i| = \max_{1 \le l \le \nu} |i_l|$, and furthermore if $\Lambda_1 \bigcap \Lambda_2 = \emptyset$, then

$$|W(s_{\Lambda_1}, s_{\Lambda_2})| \le \sum_{i \in \Lambda_1} \sum_{j \in \Lambda_2} \Psi(|i-j|) \frac{1}{2}(s_i^2 + s_j^2),$$

where

$$W(s_{\Lambda_1}, s_{\Lambda_2}) = V(s_{\Lambda_1 \cup \Lambda_2}) - V(s_{\Lambda_1}) - V(s_{\Lambda_2}).$$

For systems with superstable and regular interactions we put

$$\text{(2.16)} \qquad \mathfrak{S} = \bigcup_{N \in \mathbf{N}} \mathfrak{S}_N,$$

$$\mathfrak{S}_N = \left\{ s \in \Omega : \forall l, \sum_{|i| \le l} s_i^2 \le N^2 (2l+1)^\nu \right\}.$$

We note that if Φ is superstable and regular, then by (2.15) the condition (c) of Definition 2.1 is satisfied. The following is a result obtained in [4]:

THEOREM 2.10 ([4], THEOREM 2.7). *Let Φ be a superstable and regular interaction. Then $\mathcal{G}(\Phi)$ is non-empty, convex, compact in the local convergence topology* [2], *and a Choquet simplex. Furthermore each $\mu \in \mathcal{G}(\Phi)$ is regular in the sense that there exist $A^* > 0$ and $\delta > 0$ such that if $F \in \mathcal{F}_\Lambda$, then the bound*

$$\text{(2.17)} \qquad E_\mu(F) \le \int \lambda^\Lambda(ds_\Lambda) 1_F(s_\Lambda) \exp[-A^* \sum_{i \in \Lambda} s_i^2 + \delta]$$

holds.

Using the above theorem we have the following result:

COROLLARY 2.11. *Let Φ be a superstable and regular interaction. Assume that $H_i(\cdot \bar{s}_{\{i\}^c})$ is differentiable for all $i \in \mathbf{Z}^\nu$ and $\bar{s} \in \mathfrak{S}$. Then the assumptions in Proposition 2.7 are satisfied. Thus the relation* (2.13) *holds for any $\mu \in \mathcal{G}^\Phi(\Omega)$.*

Before proving the above result it may be worth to give some examples of interactions satisfying the conditions stated in Corollary 2.11. Consider two body interactions of the following type:

$$\begin{aligned} \Phi_{\{i\}}(s_i) &= \int_0^1 d\tau\, P(s_i(\tau)), \\ \Phi_{\{i,j\}}(s_i, s_j) &= \int_0^1 d\tau\, f(|i-j|)s_i(\tau)s_j(\tau), \\ \Phi_\Delta(s_\Delta) &= 0 \quad \text{if } |\Delta| > 2, \end{aligned}$$

where $P(x)$ is a polynomial of degree $2n$ and f is a function on natural integers such that $|f(|i|)| \le \Psi(|i|)$. Then a direct calculation shows that

$$D_g^i H_i(s_i \bar{s}_{\{i\}^c}) = \int_0^1 d\tau\, \widetilde{g}_i(\tau) P'(s_i(\tau)) + \sum_{\substack{j \in \mathbf{Z}^\nu: \\ j \ne i}} f(|i-j|) \int_0^1 d\tau\, \widetilde{g}_i(\tau) s_j(\tau),$$

where $P'(x) = \frac{d}{dx}P(x)$. Thus the above interactions satisfy all of the assumptions in Corollary 2.11.

Proof of Corollary 2.11. Let $\mu \in \mathcal{G}(\Phi)$. We check that all the conditions of Proposition 2.7 are satisfied. First, we show that the restriction μ_Λ of μ to any finite $\Lambda \subset \mathbf{Z}^\nu$ is absolutely continuous with respect to λ^Λ. By the regularity property (2.17) of μ one has that for any $F \in \mathcal{F}_\Lambda$,

$$\begin{aligned} \mu_\Lambda(F) &= \mu(F) \\ &\le \int \lambda^\Lambda(ds_\Lambda) 1_F(s_\Lambda) \exp[-A^* \sum_{i \in \Lambda} s_i^2 + \delta]. \end{aligned}$$

Since $0 < \exp[-A^* \sum_{i\in\Lambda} s_i^2 + \delta]$, and by (2.15) $\int \lambda^\Lambda(ds_\Lambda)\exp[-A^* \sum_{i\in\Lambda} s_i^2 + \delta] \le (C_0)^{|\Lambda|} < \infty$ for some constant $C_0 > 0$, μ_Λ is absolutely continuous with respect to λ^Λ.

Now we show that for all $i \in \mathbf{Z}^\nu$ and $g_i \in L^2(T^1)$, we have $E_{\mu_{\{i\}}}(|\zeta_{g_i}|^2) \le c\,\|g_i\|^2_{L^2(T^1)}$. From the estimates in eqs. (A.13) – (A.15) of [4], one sees that there exist constants $c' > 0$ and $\delta' > 0$ such that

$$\int P_{x_0,x_0}(ds)\exp[-A^* s^2 + \delta] \le \exp[-c' x_0^2 + \delta'], \tag{2.18}$$

where $P_{x_0,x_0}(ds)$ is the conditional Wiener measure on $W_{x_0,x_0} = \{s \in C[0,1] : s(0) = s(1) = x_0\}$. Now using the regularity of μ and Schwarz inequality one obtains that for $i \in \mathbf{Z}^\nu$ and $g_i \in L^2(T^1)$,

$$\begin{aligned} &E_{\mu_{\{i\}}}(|\zeta_{g_i}|^2) \\ &\le \int dx_0 \int P_{x_0,x_0}(ds)|\zeta_{g_i}(s)|^2 \exp[-A^* s^2 + \delta] \\ &\le \int dx_0 \left(\int P_{x_0,x_0}(ds)|\zeta_{g_i}(s)|^4\right)^{1/2} \\ &\quad \left(\int P_{x_0,x_0}(ds)\exp[-2A^* s^2 + 2\delta]\right)^{1/2}. \end{aligned} \tag{2.19}$$

From (2.12) we know that

$$\int P_{x_0,x_0}(ds)\exp[it\zeta_{g_i}(s)] = \exp[-\frac{1}{2}t^2\sigma^2(g_i)].$$

Hence

$$\begin{aligned} &\int P_{x_0,x_0}(ds)|\zeta_{g_i}(s)|^4 \\ &= \frac{d^4}{dt^4}\left(\int P_{x_0,x_0}(ds)\exp[it\zeta_{g_i}(s)]\right)\Bigg|_{t=0} \\ &= 3\,(\sigma^2(g_i))^2 \\ &\le 3\,\|g_i\|^4_{L^2(T^1)}. \end{aligned} \tag{2.20}$$

We use (2.18) in the second integral of (2.19) and integrate out with respect to dx_0. From this and (2.20) we have $E_{\mu_{\{i\}}}(|\zeta_{g_i}|^2) \le c\,\|g_i\|^2_{L^2(T^1)}$. Now the relation (2.13) follows from Proposition 2.7.

3. Proofs of Main Results

In this section we provide all the proofs of the main results in the last section. Although the quantities we are dealing with are somewhat different from those in [6] the general strategy of the proofs is in parallel with those in [6]. In particular, for the proofs of Proposition 2.7 and Theorem 2.8 we shall adopt the methods similar to those used in [6]

Proof of Proposition 2.6. It is enough to show that the relation (2.11) holds for $F \in W^{1;1,2}(S, d\lambda)$ which depends only on finitely many points on the paths and for any step function $g \in L^2(T^1)$. So let $F(s) = F(s(t_1), s(t_2), \ldots, s(t_n))$, where $t_0 = 0 < t_1 < \cdots < t_n = 1\,(\equiv 0)$ and $g(t) = \sum_{i=1}^n a_i 1_{[t_{i-1}, t_i)}(t)$, $a_i \in \mathbf{R}$. We put $\tau_i \equiv t_i - t_{i-1}$, $1 = 1, 2, \ldots, n$, so $\sum_{i=1}^n \tau_i = 1$. Then by (2.6)

$$D_g F(s) = \sum_{i=1}^n \widetilde{g}(t_i) F_i'(s(t_1), s(t_2), \ldots, s(t_n)), \tag{3.1}$$

where

$$\widetilde{g}(t_i) = \Big(\sum_{k=i+1}^{i+n} \tau_k \sum_{l=k}^{i+n} a_l \tau_l - \frac{1}{2} \sum_{k=1}^{n} a_k \tau_k^2 \Big), \tag{3.2}$$

in which if the summation index, e.g., l goes over n, we mean $a_l = a_{l^*}$, where $l^* \in \{1, 2, \ldots, n\}$ and $l \equiv l^* \bmod n$, and F_i' means the usual derivative of F with respect to the i-th argument. Let us put

$$P_\tau(x, y) \equiv \frac{1}{\sqrt{2\pi\tau}} \exp[-\frac{1}{2\tau}|y - x|^2].$$

Using (3.1) and (3.2) and the definition of the reference measure λ we have

$$\begin{aligned} E_\lambda(D_g F) = \sum_{i=1}^n \widetilde{g}(t_i) \int dx_1 \cdots dx_n P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) \\ \times F_i'(x_1, \ldots, x_n), \end{aligned} \tag{3.3}$$

where $x_0 = x_n$. From the integration by parts, the right hand side of (3.3) becomes

$$(3.4)\quad \sum_{i=1}^{n} \widetilde{g}(t_i) \int dx_1 \cdots dx_n \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big] \times P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) F(x_1, \ldots, x_n).$$

We note that

(3.5)

$$\sum_{i=1}^{n} \widetilde{g}(t_i) \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big]$$

$$= \sum_{i=1}^{n} \Big(\sum_{k=i+1}^{i+n} \tau_k \sum_{l=k}^{i+n} a_l \tau_l - \frac{1}{2} \sum_{k=1}^{n} a_k \tau_k^2 \Big) \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big]$$

$$= \sum_{i=1}^{n} \Big(\sum_{k=i+1}^{i+n} \tau_k \sum_{l=k}^{i+n} a_l \tau_l \Big) \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big],$$

because the second summation in the second line sums to 0. In the summation (3.5) we interchange the order of the summation. Then (3.5) becomes

$$(3.6)\quad \sum_{k=1}^{n} \tau_k \sum_{i=1}^{n} \sum_{l=k}^{i(k)} a_l \tau_l \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big],$$

where

$$i(k) = \begin{cases} i & \text{if } k \le i, \\ i+n & \text{if } k > i. \end{cases}$$

Once again we interchange the order of summation in (3.6) and thereby (3.6) becomes

$$(3.7)\quad \sum_{k=1}^{n} \tau_k \sum_{l=k}^{k+n-1} a_l \tau_l \sum_{i=l}^{k+n-1} \Big[\frac{(x_i - x_{i-1})}{\tau_i} - \frac{(x_{i+1} - x_i)}{\tau_{i+1}}\Big]$$

$$= \sum_{k=1}^{n} \tau_k \sum_{l=k}^{k+n-1} a_l \tau_l \Big[\frac{(x_l - x_{l-1})}{\tau_l} - \frac{(x_k - x_{k-1})}{\tau_k}\Big]$$

$$= \sum_{l=1}^{n} a_l (x_l - x_{l-1}).$$

Substituting (3.7) into (3.4) we have

$$
\begin{aligned}
&E_\lambda(D_g F) \\
&= \int dx_1 \cdots dx_n P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) \, [\sum_{l=1}^{n} a_l (x_l - x_{l-1})] \\
&\qquad \times F(x_1, \ldots, x_n) \\
&= E_\lambda(\zeta_g F),
\end{aligned}
$$

which is the relation (2.11).

Let us now show the relation (2.12). For $F_0 \in W^{1;1,2}(\mathbf{R}, dx)$, we understand F_0 as being a function on S defined by $F_0(s) = F_0(s(0))$, $s \in S$. And let $g(t) = \sum_{i=1}^{n} a_i 1_{[t_{i-1}, t_i)}(t)$ be as above. We put

$$
\widehat{\rho}(F_0; t) \equiv E_\lambda(F_0 \, \exp[it\zeta_g]). \tag{3.8}
$$

Using (2.10), one obtains that

(3.9)

$$
\begin{aligned}
&\frac{d}{dt}\widehat{\rho}(F_0; t) \\
&= i \, E_\lambda(\zeta_g F_0 \, \exp[it\zeta_g]) \\
&= i \, E_\lambda(D_g(F_0 \, \exp[it\zeta_g])) \\
&= i \sum_{i=0}^{n-1} \widetilde{g}(t_i) \int dx_0 dx_1 \cdots dx_{n-1} P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) \\
&\qquad \times \frac{\partial}{\partial x_i} \left(F_0(x_0) \exp[it \sum_{j=1}^{n} a_j (x_j - x_{j-1})] \right) \\
&= i \, \widetilde{g}(0) \int dx_0 dx_1 \cdots dx_{n-1} P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) \\
&\qquad \times F_0'(x_0) \exp[it \sum_{j=1}^{n} a_j (x_j - x_{j-1})] \\
&\quad - t \sum_{i=0}^{n-1} \widetilde{g}(t_i)(a_i - a_{i+1}) \int dx_0 dx_1 \cdots
\end{aligned}
$$

$$dx_{n-1} P_{\tau_1}(x_0, x_1) \cdots P_{\tau_n}(x_{n-1}, x_n) \times F_0(x_0) \exp[it \sum_{j=1}^{n} a_j (x_j - x_{j-1})].$$

Note that the first integral in (3.9) is 0. In fact, after changing the variables $x_i - x_0 \to x_i$, $i = 1, 2, \ldots, n-1$, it becomes

$$\begin{aligned} &\int dx_0 F_0'(x_0) \int dx_1 \cdots dx_{n-1} P_{\tau_1}(0, x_1) \cdots P_{\tau_n}(x_{n-1}, 0) \\ &\qquad \times \exp[it \sum_{j=1}^{n} a_j (x_j - x_{j-1})], \qquad (x_0 = x_n = 0) \\ &\qquad = 0, \end{aligned}$$

because $F_0 \in W^{1;1,2}(\mathbf{R}, dx)$. In the second term of (3.9) one has

$$\begin{aligned} (3.10) \qquad & \sum_{i=0}^{n-1} \tilde{g}(t_i)(a_i - a_{i+1}) \\ &= \sum_{i=1}^{n} \tilde{g}(t_i)(a_i - a_{i+1}) \\ &= \sum_{i=1}^{n} (a_i - a_{i+1}) \Big(\sum_{k=i+1}^{i+n} \tau_k \sum_{l=k}^{i+n} a_l \tau_l - \frac{1}{2} \sum_{k=1}^{n} a_k \tau_k^2 \Big) \\ &= \sum_{i=1}^{n} (a_i - a_{i+1}) \sum_{k=i+1}^{i+n} \tau_k \sum_{l=k}^{i+n} a_l \tau_l . \end{aligned}$$

As before we interchange the order of the summation in (3.10). Then (3.10) becomes

$$\begin{aligned} (3.11) \qquad & \sum_{k=1}^{n} \tau_k \sum_{l=k}^{k+n-1} a_l \tau_l \sum_{i=l}^{k+n-1} (a_i - a_{i+1}) \\ &= \sum_{k=1}^{n} \tau_k \sum_{l=k}^{k+n-1} a_l \tau_l (a_l - a_k) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{l=1}^{n} a_l^2 \tau_l - \big(\sum_{l=1}^{n} a_l \tau_l\big)^2 \\
&= \int_0^1 g^2(t)dt - \big(\int_0^1 g(t)dt\big)^2 \\
&\equiv \sigma^2(g).
\end{aligned}
$$

Combining the above results, we obtain

$$\frac{d}{dt}\widehat{\rho}(F_0;t) = -t\sigma^2(g)\,\widehat{\rho}(F_0;t). \tag{3.12}$$

The above ordinary differential equation has a unique solution:

$$\widehat{\rho}(F_0;t) = I(F_0)\exp[-\frac{1}{2}t^2\sigma^2(g)], \tag{3.13}$$

where $I(F_0) = \frac{1}{\sqrt{2\pi}} \int dx_0\, F(x_0)$.

Finally suppose that a measure ρ on S, which is absolutely continuous with respect to λ with an L^2-continuous and L^2-continuously differentiable Radon-Nikodym derivative, say G, and satisfies $E_\rho(\zeta_g F) = E_\rho(D_g F)$ for any $F \in W^{1;1,2}(S, d\lambda)$ and $g \in L^2(T^1)$. Then we have

$$
\begin{aligned}
E_\rho(\zeta_g F) &= E_\lambda(\zeta_g FG) \\
&= E_\lambda(D_g(FG)) \\
&= E_\lambda(GD_gF) + E_\lambda(FD_gG).
\end{aligned} \tag{3.14}
$$

On the other hand

$$E_\rho(\zeta_g F) = E_\lambda(GD_gF). \tag{3.15}$$

From (3.14) and (3.15) $E_\lambda(FD_gG) = 0$ for all $F \in W^{1;1,2}(S, d\lambda)$ and $g \in L^2(T^1)$. Therefore for any $g \in L^2(T^1)$, $D_gG = 0$, $\lambda - a.s.$ But since D_gG is L^2-continuous, as noticed in Remark 2.1, we see that $D_gG(s) = 0$, $\forall s \in S$. Now the function $t \to G(t\widetilde{g})$ is an absolutely continuous function on $\mathbf{R}$, and hence

$$
\begin{aligned}
G(\widetilde{g}) &= G(0) + \int_0^1 \frac{d}{dt}G(t\widetilde{g})\,dt \\
&= G(0) + \int_0^1 D_gG(t\widetilde{g})\,dt \\
&= G(0).
\end{aligned}
$$

We notice that the set $\{\widetilde{g} : g \in L^2(T^1)\}$ is (L^2-) dense in S. Since G is also assumed to be continuous, $G(s) = G(0)$, $\forall s \in S$. This completes the proof of the proposition.

Proof of Proposition 2.7. Let Φ and $\mu \in \mathcal{G}(\Phi)$ be as in the statement of Proposition 2.7. Let $F \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ be Λ-local and $g = (g_i)_{i\in\mathbf{Z}^\nu} \in l^1(\mathbf{Z}^\nu;\, L^2(T^1))$ be given. We note from the hypothesis that

$$\begin{aligned} E_\mu(|F\zeta_{g_i}|) &\le E^{1/2}_{\mu_\Lambda}(|F|^2)\, E^{1/2}_{\mu_{\{i\}}}(|\zeta_{g_i}|^2) \\ &\le c\, E^{1/2}_{\mu_\Lambda}(|F|^2)\|g_i\|_{L^2(T^1)}. \end{aligned}$$

Since $g \in l^1(\mathbf{Z}^\nu;\, L^2(T^1))$,

$$|E_\mu(F\zeta_g)| = \sum_{i\in\mathbf{Z}^\nu} E_\mu(|F\zeta_{g_i}|) < \infty. \tag{3.16}$$

Let us simply write $\gamma_i(\,\cdot\,|\,\cdot\,)$ and $Z_i(\,\cdot\,)$ instead of $\gamma^\Phi_{\{i\}}(\,\cdot\,|\,\cdot\,)$ and $Z^\Phi_{\{i\}}(\,\cdot\,)$ respectively. From the equilibrium conditions of Gibbs measures it follows that for all $i \in \mathbf{Z}^\nu$

$$\begin{aligned} E_\mu(F\zeta_{g_i}) &= E_\mu(E_\mu(F\zeta_{g_i}|\mathcal{F}_{\{i\}^c})) \\ &= E_\mu(\gamma_i(F\zeta_{g_i}|\,\cdot\,)). \end{aligned}$$

For $\overline{s} \in \mathfrak{S}$, we use the above and the duality relation (2.11) to obtain

(3.17)

$$\begin{aligned} &\gamma_i(F\zeta_{g_i}|\overline{s}) \\ &= Z_i(\overline{s})^{-1}\int_S \lambda(ds)\, F(s\overline{s}_{\{i\}^c})\zeta_{g_i}(s)e^{-H_i(s\overline{s}_{\{i\}^c})} \\ &= Z_i(\overline{s})^{-1}E_\lambda\left(D_{g_i}(F(\,\cdot\,\overline{s}_{\{i\}^c})e^{-H_i(\,\cdot\,\overline{s}_{\{i\}^c})})\right) \\ &= Z_i(\overline{s})^{-1}E_\lambda\Big((D_{g_i}F(\,\cdot\,\overline{s}_{\{i\}^c}) - F(\,\cdot\,\overline{s}_{\{i\}^c})D_{g_i}H_i(\,\cdot\,\overline{s}_{\{i\}^c})) \\ &\qquad e^{-H_i(\,\cdot\,\overline{s}_{\{i\}^c})}\Big) \\ &= \gamma_i(D^i_gF - FD^i_gH_i). \end{aligned}$$

Using the equilibrium conditions once again we have

$$(2.14)\qquad E_\mu(F\zeta_{g_i}) = E_\mu(D_g^i F) - E_\mu(F D_g^i H_i), \quad \forall\, i \in \mathbf{Z}^\nu.$$

Since F is a Λ-local function, $D_g^i F = 0$ for $i \in \Lambda^c$. Therefore by (3.16),

$$(3.18)\qquad \sum_{i\in\mathbf{Z}^\nu} \left|E_\mu(F D_g^i H_i)\right| \le \sum_{i\in\mathbf{Z}^\nu} \left(\left|E_\mu(D_g^i F)\right| + \left|E_\mu(F\zeta_{g_i})\right|\right) < \infty.$$

This guarantees the relation (2.13).

Proof of Theorem 2.8. Consider an $F \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ of the form: $F(s) = \widetilde{F}(s_{\{i\}^c})F_i(s_i)$, where $\widetilde{F} \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ and $F_i \in W^{1;1,2}(S, d\lambda)$. Applying the relation (2.14) to this function we have

$$E_\mu(\widetilde{F}F_i\zeta_{g_i}) = E_\mu(\widetilde{F}D_g^i F_i) - E_\mu(\widetilde{F}F_i D_g^i H_i).$$

This also says that

$$(3.19)\qquad E_\mu\left(\widetilde{F}E_\mu(F_i\zeta_{g_i}|\mathcal{F}_{\{i\}^c})\right) = E_\mu\left(\widetilde{F}E_\mu(D_g^i F_i|\mathcal{F}_{\{i\}^c})\right) - E_\mu\left(\widetilde{F}E_\mu(F_i D_g^i H_i|\mathcal{F}_{\{i\}^c})\right).$$

Since the relation (3.19) holds for any $\widetilde{F} \in W^{1;1,2}_{\mathrm{loc}}(\Omega)$ ($\mathcal{F}_{\{i\}^c}$-measurable), we have

$$\begin{aligned} E_\mu(F_i\zeta_{g_i}|\mathcal{F}_{\{i\}^c}) &= E_\mu(D_g^i F_i|\mathcal{F}_{\{i\}^c}) - E_\mu(F_i D_g^i H_i|\mathcal{F}_{\{i\}^c}), \quad \mu-\text{a.s.} \\ (3.20)\qquad &= E_\mu(e^{H_i} D_g^i(F_i e^{-H_i})|\mathcal{F}_{\{i\}^c}), \quad \mu-\text{a.s.} \end{aligned}$$

for any $F_i \in W^{1;1,2}(S, d\lambda)$ and $g \in l^1(\mathbf{Z}^\nu;\, L^2(T^1))$.

Now let $\widetilde{\mu}_i^{\overline{s}}$ be the measure $\mu(\,\cdot\,|\mathcal{F}_{\{i\}^c})(\overline{s})$, $\overline{s} \in \mathfrak{S}$, on S. Then for μ-a.a. $\overline{s}$,

$$\mu(d(s_i\eta_{\{i\}^c})|\mathcal{F}_{\{i\}^c})(\overline{s}) = \widetilde{\mu}_i^{\overline{s}}(ds_i) \times \delta_{\overline{s}_{\{i\}^c}}(d\eta)$$

and so the relation (3.20) can be written as

$$E_{\widetilde{\mu}_i^{\overline{s}}}(F_i\zeta_{g_i}) = E_{\widetilde{\mu}_i^{\overline{s}}}\left(e^{H_i(\,\cdot\,\overline{s}_{\{i\}^c})} D_{g_i}(F_i e^{-H_i(\,\cdot\,\overline{s}_{\{i\}^c})})\right),$$

or

$$E_{\widehat{\mu}_i^{\overline{s}}}(F_i e^{-H_i(\cdot \overline{s}_{\{i\}^c})}\zeta_{g_i}) = E_{\widehat{\mu}_i^{\overline{s}}}(D_{g_i}(F_i e^{-H_i(\cdot \overline{s}_{\{i\}^c})})), \tag{3.21}$$

where we have put $\widehat{\mu}_i^{\overline{s}}(ds) = e^{H_i(s\overline{s}_{\{i\}^c})}\widetilde{\mu}_i^{\overline{s}}(ds)$. By the hypothesis given in the statement of Theorem 2.8, the measure $\widetilde{\mu}_i^{\overline{s}}$ is absolutely continuous with respect to λ with an L^2-continuous and L^2-continuously differentiable Radon-Nikodym derivative, and hence the measure $\widehat{\mu}_i^{\overline{s}}$ is also absolutely continuous with respect to λ with an L^2-continuous and L^2-continuously differentiable Radon-Nikodym derivative because for all $\overline{s} \in \mathfrak{S}$, $e^{-H_i(\cdot \overline{s}_{\{i\}^c})}$ is a strictly positive, L^2-continuous and L^2-continuously differentiable function on S. Therefore from (3.21), applying Proposition 2.6, we have $\widehat{\mu}_i^{\overline{s}} = c\lambda$ for some constant $c > 0$. That is, the probability measure $\widetilde{\mu}_i^{\overline{s}}$ should be as follows:

$$\widetilde{\mu}_i^{\overline{s}}(ds) = Z_i(\overline{s})^{-1} e^{-H_i(s\overline{s}_{\{i\}^c})}\lambda(ds), \ \forall i \in \mathbf{Z}^\nu, \ \overline{s} \in \mathfrak{S}.$$

This says that the conditional expectation of μ with respect to $\mathcal{F}_{\{i\}^c}$, is $\gamma_{\{i\}}^\Phi$. Since μ is supported on $\mathfrak{S}$, we may put $E_\mu(\cdot|\mathcal{F}_{\{i\}^c})(\overline{s}) = 0$ for all $\overline{s} \notin \mathfrak{S}$. Applying Theorem 1.33 of [2] we have $\mu \in \mathcal{G}(\Phi)$.

References

1. H. Föllmer, *Time reversal on Wiener space*, Lecture Notes in Math., vol. 1158, Springer-Verlag, 1986, pp 119–129.
2. H. O. Georgii, *Gibbs measures and phase transitions*, de Gruyter Stud. Math. vol. 9, Berlin, 1988.
3. J. L. Lebowitz and E. Presutti, *Statistical mechanics of systems of unbounded spins*, Commun. Math. Phys. **50** (1976), 195–218.
4. Y. M. Park and H. J. Yoo, *A characterization of Gibbs states of lattice boson systems*, J. Stat. Phys. (1994) (to appear).
5. C. Preston, *Random fields*, Lecture Notes in Math., vol. 534 Springer-Verlag, New York and Berlin 1976.
6. S. Roelly and H. Zessin, *A characterization of Gibbs measures on $C[0,1]^{\mathbf{Z}^\nu}$ by the stochastic calculus of variations*, Preprint, Bibos No. 488/91.
7. D. Ruelle, *Superstable interactions in classical statistical mechanics*, Comm. Math. Phys. **18** (1970), 127–159.
8. ———, *Probability estimates for continuous spin systems*, Comm. Math. Phys. **50** (1976), 189–194.

9. B. Simon, *Functional integration and quantum physics*, Academic Press, New York, 1979.

[1]Department of Mathematics and
Institute for Mathematical Sciences
Yonsei University
Seoul 120-749, Korea

[2]Department of Physics
Yonsei University
Seoul 120-749, Korea