Robust Decentralized Adaptive Controller for Trajectory Tracking Control of Uncertain Robotic Manipulators

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비중앙 집중식 강성 적응제어법을 통한 산업용 로봇 궤도추적제어

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Key Words : Feedforward compensation, variable structure (VS) - type robust control, decentralized(centralized) adaptive control, structured (unstructured) uncertainties, Lyapunov method, uniform ultimate boundedness(UUB), deterministic approach.

This paper presents a dynamic compensation methodology for robust trajectory tracking control of uncertain robot manipulators. To improve tracking performance of the system, a full model-based feedforward compensation with continuous VS - type robust control is developed in this paper(i.e., robust decentralized adaptive control scheme). Since possible bounds of uncertainties are unknown, the adaptive bounds of the robust control is used to directly estimate the uncertainty bounds(instead of estimating manipulator parameters as in centralized adaptive control). The global stability and robustness issues of the proposed control algorithm have been investigated extensively and rigorously via a Lyapunov method. The presented control algorithm guarantees that all system responses are uniformly ultimately bounded. Thus, it is shown that the control system is evaluated to be highly robust with respect to significant uncertainties.

Introduction

In recent years, there has been intensive research on the control system design for robotic manipulators in order to meet various demands of modern industries. Moreover, the practical applications of robots may be unlimited in the future. Therefore, the design of high-performance and reliable control algorithm is one of the active areas of current robotics research.

A class of nonlinear dynamical systems, including robot manipulators, contain various uncertainties in dynamic modeling and control processes. In general, the system uncertainties can arise in different forms. The uncertainties under consideration include structured(or parametric) uncertainties and unstructured uncertainties. However, a wide range of the current control schemes ignore the uncertainties associated with robot systems because of the complexity of their effects. Thus these algorithms sometimes cannot provide general solutions. To achieve satisfactory system performance and to extend the usage of control laws to a larger class of applications, the control strategies should account for possible uncertainties that cause deteriorating system performances.

More recently, numerous papers delaing with the control of uncertain dynamical systems, including robot manipulators, have been published. Since the strategy on the control of uncertain system is deterministic approach [1-10, 17], stochastic approach on the uncertainties is never assumed in this paper. One useful design method of controlling uncertain systems is a variable structure (VS) - type control scheme [1-9, 17]. Unfortunately, most control approaches generally require a priori knowledge of uncertainty bounds to implement controllers. Abdallah et al.[4] gives a recent survey of robust control of robot manipulators. An alternative approach to solving the problem of uncertain systems is an adaptive control method. So far a considerable amount of study has been done in the field of adaptive control to estimate unknown parameters [4, 12-17]. However, most current researches emphasize on the estimation of model-parameters of the system(i.e., centralized adaptive control). Ortega and Spong [15] presents an overall review of adaptive robot control. In this paper, instead of (on line) updating model-parameters of robot manipulators, the uncertainty bounds will be directly estimated to achieve satisfactory system performacne, i.e., decentralized adaptive control approach. Up to now, a few papers discuss either the stability problem under the effects of higher-order uncertainties [7, 9, 17] or the adaptive bounds on uncertainties [1-2, 6-7, 17]. Consequently, many advanced control strategies, such as robust controls and adaptive controls, suffer from some of the following drawbacks : (i) use discontinuous control laws, (ii) synthesize computationally inefficient algorithms, (iii) require a priori knowledge of the uncertainty bounds, and (iv) compensate relatively small uncertainties.

The main goal of this work is to develop a systematic design methodology for the advanced control system that overcomes some or all the defects found in earlier design methods. The proposed control algorithm consists of two parts : the nominal control, utilizing a model-based feedforward compensation plus proportional-derivatives (PD) control, is first introduced to stabilize the system without uncertainties ; then the roubust nonlinear control law is synthesized to cope with uncertainties in the system. To show that the proposed control scheme is robust enough to overcome significant uncertainties, the uncertainties assumed here are bounded by higher-order polynomials in the norms of system states with unknown gains. Since the worst possible bounds of uncertainties are unknown, th adaptive bounds of the robust controllers are presented to estimate the unknown bounds. With a feasible class of desired trajectories(smooth and uniformly bounded by constants), the proposed control laws guarantee that all signals of the closed-loop system are uniformly ultimately bounded in the presence of higher-order uncertainties.

The organization of this paper is as follows. Section 2 briefly describes preliminaries and problem formulation. In section 3, without possible knowledge of the bounding functions on uncertainties, the robust adaptive controllers are developed. Finally, the contributions and conclusions are given in Section 4.

Preliminaries and Problem Formulation

Henceforth, unless otherwise specified, the following notation and terminolgy will be used. $\boldsymbol{R}$ denotes the field of real numbers, $\boldsymbol{R}^+$ will denote the set of nonnegative real numbers, $\boldsymbol{R}^n$ is the usual n-dimensional vector space over $\boldsymbol{R}$, $\boldsymbol{R}^{n\times m}$ is the set of $n\times m$ matrices with entries in the set $\boldsymbol{R}$, and C^p is the set of p-times continuously differential function. The vecotr norm $\|x\|$ or $\|x(t)\|$ is the Euclidean one of vector $x\in\boldsymbol{R}^n$ at time t, i.e., $\|x\|=(x^T\ x)^{1/2}=[\sum_{i=1}^{n}|x_i|^2]^{1/2}$, and the matrix norm is the corresponding induced one of real matrix $A\in\boldsymbol{R}^{n\times n}$, i.e., $\|A\|=[\sigma_{\max}(A^T\ A)]^{1/2}$, where $\sigma_{\max}(\bullet)[\sigma_{\min}(\bullet)]$ denotes the maximum (minimum) eigenvalue of the designated matrix. $A>0$ $(A<0)$ denotes a positive (negative) definite matrix A.

The robot manipulator system under consideration is a set of n moving rigid bodies connected in a serial open chain mechanism with all revolute joints. By using the Lagrangian formulation(for the convenience of control system design), an n-DOF robot dynamics can be written as [3-5, 9-10, 13-17].

$$M(q;\Theta)\ddot{q}+C(q,\dot{q};\Theta)\dot{q}+G(q;\Theta)+T_u=T,\ t\geq 0 \tag{1}$$

where q, $\dot{q}$, and $\ddot{q}\in\boldsymbol{R}^n$ denote the joint position, velocity, and acceleration vectors, respectively ; $M(q;\Theta)\in\boldsymbol{R}^{n\times n}$ is an inertia matrix ; $C(q,\dot{q};\Theta)\in\boldsymbol{R}^{n\times n}$ is a matrix function comprising the centripetal and Coriolis terms; $G(q;\Theta)\in\boldsymbol{R}^n$ is the vector the gravity torques; $T_u(q,\dot{q})\in\boldsymbol{R}^n$ represents the unstructured uncertainties, arising from such as friction, actuator and sensor noises, unmodelled dynamics, and external disturbances ; $T\in\boldsymbol{R}^n$ is the vector function consisting of applied generalized forces/torques ; and $\Theta \in R^m$ is the vector of (bounded) system parameters (link masses, link lengths, moments of inertia, etc.). It is important to note that the system uncertainties are considered as a part of the system dynamics. For the sake of brevity, some arguments of the dynamics (1) are omitted when no confusion is likely to arise.

For the design of advanced control algorithms, we will exploit some physical properties of robot dynamics (1). These are summarized as follows[3-5, 9-10, 13-17] :

Property 1. $M(q)$ is a symmetric and positive-definite matrix, i.e., $M(q)=M(q)^T > 0$, $\forall_q$, and $M(q)^{-1}$ exists in a singularity-free region. Furthermore, $M(q)$ and $M(q)^{-1}$ are differentiable matrix functions(C^∞ in q) and uniformly bounded above and below by

$$\delta_l \le \| M(q) \| \le \delta_u \text{ and } \frac{1}{\delta_u} \le \| M(q)^{-1} \| \le \frac{1}{\delta_l}, \ \forall_q \in R^n$$

where δ_l and δ_u are some positive scalar constants that depend on the mass properties of the given manipulator.

Property 2. $x^T(\dot{M}-2C)x=0$, $\forall_x \in R^n$, that is, $(\dot{M}-2C)$ is a skew-symmetric matrix (Hamilton property). Then, C can be expressed as $C(q,\dot{q})=[\dot{q}^T\bar{C}_k(q)]_{k=1,\cdots,n}$, where Christoffel symbols $\bar{C}_k \in R^{n\times n}$ is symmetric and bounded matrix for all $q \in R^n$ and defined as

$$C_k(q)=1/2\left(\frac{\partial m_k}{\partial_q}+\frac{\partial m_k^T}{\partial_q}-\frac{\partial M}{\partial q_k}\right)$$

Here m_k denotes the k th column (or row) of M and q_k is the k th element of q.

Property 3. $C(q, y)z=C(q, z)y$, for any q, y, $z \in R^n$

Moreover, the following assumptions are made concerning the system dynamics (1) for the system formulation.

Assumption 1. The nature of uncertainties (T_u) strongly influences system performance.

Assumption 2. The system state vectors (q, $\dot{q}$) are measurable for all $t \ge 0$ but joint accelerations $\ddot{q}$ are not.

Assumption 3. The model-parameter vector, $\Theta=(\theta_1,\cdots,\theta_m)^T$, is not precisely known, and the variation of each parameter θ_i is within the prescribed range $\Psi_i := [\underline{\theta}_i, \overline{\theta}_i] \subset R$, where $\underline{\theta}_i$ and $\overline{\theta}_i$ are known (or unknown) positive constants (i.e., the parameter bounds). In addition, we have $\Psi := \Psi_1 \times \cdots \times \Psi_m$ and $\Theta \in \Psi \in R^m$, where the set Ψ is known (or unknown) and a non-empty bounded set(compact set).

Assumption 4. Without loss of any generality, the desired trajectory ($q_d \in C^2$) and its derivatives are all continuous and uniformly bounded functions of time

$$d_1=\sup_{t\in R^+} \| q_d \| < \infty, \quad d_2=\sup_{t\in R^+} \| \dot{q}_d \| < \infty,$$
$$\text{and } d_3=\sup_{t\in R^+} \| \ddot{q}_d \| < \infty,$$

where d_1, d_2, and d_3 are some positive constants.

Before proceeding, we introduce the following definitions for the desired system behavior.

Definition 1 : The uniform ultimate boundedness(UUB).

See Appendix A for a definition.

Definition 2 : Let B represent the closed ball in R^n of radius $\zeta > 0$ centered at $x=0$

$B_\zeta(x) := (x \in R^n : \| x \| \le \zeta)$.

Now, a number of tracking error vectors are introduced for further discussions. $e \in R^n$ is the vector of the position tracking error defined as $e = q - q_d$, $q_d \in R^n$. Then the "reference" tracking error $\dot{e}_r \in R^n$ are defined by $\dot{e}_r = \dot{q}_d - Ae$, where $A \in R^{n \times n}$ is a gain matrix chosen by the designer, $A = \mu E_n$, $\mu > 0$, and E_n is an $n \times n$ identity matrix. And define the sliding variable vector $e_s \in R^n$ as $e_s = \dot{q} - \dot{e}_r$.

Lemma 1 : If $\| e_s(t) \| \leq \gamma (< \infty)$ is satisfied for any $t \in [t_0, \infty)$ with a scalar constant γ and some $t_0 \in R^+$, then

$$\| e(t) \| \leq exp[-\mu(t-t_0)] \{ \| e(t_0) \| - \frac{\gamma}{\mu} \} + \frac{\gamma}{\mu} \text{and} \| \dot{e}(t) \| \leq \gamma + \mu \| e(t) \| .$$

Thus, the ultimate bounds are given by

$$\lim_{t \to} \| e(t) \| = \frac{\gamma}{\mu} \text{ and } \lim_{t \to} \| \dot{e}(t) \| = 2\gamma.$$

Proof : The proof of Lemma 1 is a straightforward (see Refs [9-10, 17] for the proof).

With the above preliminaries, this study addresses the following trajectory tracking problem.

Problem Statement : For the given system dynamics (1) with some or all robot parameters being unknown, derive a realizable control law by using a dynamic compensation methodology such that despite significant uncertainties in the system model, every signal in the resulting closed-loop system remains within the desired degree of accuracy in some sense after a finite interval of time, i.e., guaranteeing asymptotic stability and uniform ultimate boundedness of all the signals.

In order to guarantee the system behavior presented above, we will make some assumptions on the structure of the uncertainties. In the subsequent section, we will present a control algorithm requiring minimal on-line control computation while maintaining good tracking and robustness properties.

Controller Design

As mentioned in the problem statement, the controller design problem is to formulate a control input vector so that the actual system states track closely the desired states under uncertainties. In this study, the following form is used to compute the driving torques of the obotic manipulators [9-10, 17]

$$T = T^n + T^r, \ t \geq 0 \tag{2}$$

where

$T^n = M_0(q_d ; \Theta_0)\ddot{e}_r + C_0(q_d, \dot{q}_d ; \Theta_0)\dot{e}_r + G_0(q_d ; \Theta_0) - k_a e_s$; M_0, C_0, G_0, and Θ_0 denote the estimates (or "available" values) of the true values M, C, G, and Θ via modeling, respectively ; the feedback gain matrix k_a is chosen by the designer, i.e., $k_a = k_a E_n$, $k_a > 0$. The control algorithm thus consists of a nominal control vector, T^n, which is a model-based feedforward plus PD control law to stabilize the system in the absence of the uncertainties (based on the available model), and a robust control T^r, which is used to compensate for both the resulting errors of the nominal control (or compensation error) and unstructured uncertainties via adaptation scheme. The adaptive bound of robust control takes a continuous VS-type control law to avoid undesirable chattering, and its structure (nonlinear feedback term) will be specified later. This two-stage control scheme is intended to achieve better robustness and tracking performances to significant uncertainties. The torque computation in the model-based portion can be performed off-line since the desired trajectories($q_d, \dot{q}_d, \ddot{q}_d$) and the nominal (fixed) values of the system parameters (Θ_0) are known in advance, while many other methods rely heavily on the on-line computations(i.e., as in centralized adaptive control). It is worth noting that the terms M_0, C_0, G_0, and Θ_0 are not updated on-line in this study.

Remark 1 : In general, the true values (Θ) are not available, however, the possible ranges of parameter variations may be give. Then, the nominal value θ_{0i} may be selected as $\theta_{0i}=1/2(\underline{\theta}_i+\overline{\theta}_i)$, i.e., the mean value of admissible range of θ_i, or in any other manner by designer's convenience.

After substituting (2) into (1) and subtracting $M\ddot{e}_r+C\dot{e}_r+G$ on both sides of the resulting equation, the error dynamics can be expressed in the general form as

$$M(q;\Theta)\dot{e}_s=-C(q,\dot{q};\Theta)e_s-(T_u+\Delta R_r) -k_a e_s+T^r, \quad (3)$$

where

$$\Delta R_r := [M(q;\Theta)-M_0(q_d;\Theta_0)]\ddot{e}_r + [C(q,\dot{q};\Theta)-C_0(q_d,\dot{q}_d;\Theta_0)]\dot{e}_r + [G(q;\Theta)-G_0(q_d;\Theta_0)] \quad (4)$$

Here, ΔR_r represents the structured uncertainties which are arising from manipulator parameter variations ($\tilde{\Theta}=\Theta-\Theta_0$), unknown payload, compensation errors, and so on. It should be noted that ΔR_r may be zero for the complete model-following of system. Unfortunnately, in most real applications, the estimated parameters always differ from the actural ones, in other words, $\Delta R_r \neq 0$.

In this paper, robust control (T^r) requires possible structures on uncertainties. In what follows, we investigate the bounding properties of the uncertainties. To develop the upper bound of ΔR_r, the following assumption is introduced.

Assumption 5. there exist scalar constants ρ_{11}, ρ_{12}, ρ_{13}, and $\rho_{14}(\in R^+)$ such that

$$\text{(i)}\ \rho_{11} := \sup_{(\Theta,\Theta_0)\in\Psi}\ \sup_{(q,q_d)} \|M(q;\Theta)-M_0(q_d;\Theta_0)\|$$

$$\text{(ii)}\ \|C(q,\dot{q};\Theta)-C_0(q_d,\dot{q}_d;\Theta_0)\| \le \rho_{12}\|\dot{q}\| + \rho_{13}\|\dot{q}_d\|$$

in which

$$\rho_{12} := \sup_{\Theta\in\Psi}\ \sup_{q} \sum_{k=1}^{n} \|\overline{C}_k(q;\Theta)\|$$

$$\rho_{13} := \sup_{\Theta_0\in\Psi}\ \sup_{q_d} \sum_{k=1}^{n} \|\overline{C}_k(q_d;\Theta_0)\|$$

and

$$\text{(iii)}\ \rho_{14} := \sup_{(\Theta,\Theta_0)\in\Psi}\ \sup_{q,q_d} \|G(q;\Theta)-G_0(q_d;\Theta_0)\|$$

On defining the augmented state error vector $x_e\in R^{2n}$ as $x_e=(e^T,\ \dot{e}^T)^T$, the strengths of system uncertainties (ΔR_r and T_u) are given as follows.

Lemma 2 : There exist unknown constants a_0, a_1, and a_2 ($\in R^+$) such that

$$\|\Delta R_r\| \le a_0+a_1\|x_e\|+a_2\|x_e\|^2. \quad (5)$$

Proof : See the Appendix B for the proof of this lemma.

Remark 2 : Even if the (possible) estimated values of a_0, a_1, and a_2 may be obtained by Assumption 5, these values are generally unknown constants.

Assumption 6. The unstructured uncertainties (T_u) satisfy the following norm bounds of the system states

$$\| T_u \| \leq b_0 + b_1 \| x_e \| + \cdots + b_p \| x_e \|^p, \quad (6)$$

where $b_i(i=0, \cdots, p)$ are unknown constants, and p is the highest order of x_e in the uncertainties. Note that the uncertainties (T_u) assumed are bounded by higher-order polynomials in the Euclidean norms, while the modeling error(ΔR_r) is at most quadratically bounded in the norms of the system states. Here, the uncertainty bounds (a_i and b_i) are unknown rather than being assumed known, then the adaptive scheme can be constructed to estimate the unknown bounds on the basis of functional property of the uncertainties.

Remark 3 : Most current robust control laws are based on ***a priori*** knowledge of the uncertainty bounds. Unfortunately, these bounds may not be easily obtained nor feasible to draw for a variety of reasons. Thus for safety, one may choose some conservative bounds, but that choice requires excessively large control energy.

In the adaptive bound of the robust control (or decentralized adaptive control), the prerequisites on the uncertainty bounds can be relaxed. Now, to develop adaptation mechanism, define the "regressor-like" functions $R_r(x_e)$ and $R_u(x_e)$ as

$$R_r := [1, \| x_e \|, \| x_e \|^2] \text{ and}$$
$$R_u := [1, \| x_e \|, \cdots, \| x_e \|^p],$$

respectively, and the vectors of the unknown constants $a \in R^3$ and $b \in R^{p+1}$ can be defined as $a := [a_0, a_1, a_2]^T$ and $b := [b_0, b_1, \cdots, b_p]^T$, respectively. Then the bounding functions can be further expressed as

$$\sum_{j=0}^{2} a_j \| x_e \|^j : R_r a := \phi_r \text{ and}$$
$$\sum_{i=0}^{p} b_i \| x_e \|^i := R_u b =: \phi_u,$$

respectively, where ϕ_r and $\phi_u (\in R^+)$ are unknown scalar bounding functions. The above equations represent the linear parameterization of the unknown bounding functions.

Remark 4 : If all uncertainties are at most quadratically bounded, i.e., $p \leq 2$, in the above equations, then we may combine ϕ_r and ϕ_u to obtain one simple term by choosing more conservative bounds.

The estimated versions for the scalar bounding functions can be defined as

$$\hat{\phi}_r = R_r \hat{a} \text{ and } \hat{\phi}_u = R_u \hat{b},$$

where $\hat{a}$ and $\hat{b}$ are the estimated vectors of the unknown gains a and b, respectively. In this study, the circumflex ($\hat{\bullet}$) represents the estimated value of ($\bullet$) provided by the adaptation law. Define the error vectors of uncertainty bounds $\tilde{a} \in R^3$ and $\tilde{b} \in R^{p+1}$ as $\tilde{a} = a - \hat{a}$ and $\tilde{b} = b - \hat{b}$, respectively. Then, the unknown gains are estimated by the following update laws :

$$\dot{\hat{a}} = \Gamma_r (R_r^T \| e_s \| - \omega_r \hat{a}) \quad (7)$$
$$\text{and } \dot{\hat{b}} = \Gamma_u (R_u^T \| e_s \| - \omega_u \hat{b}), \quad (8)$$

where the adaptation gains $\Gamma_r \in R^{3\times3}$ and $\Gamma_u \in R^{(p+1)\times(p+1)}$ may be selected as diagonal matrices, i.e., $\Gamma_r = \lambda_r E$, $\lambda_r > 0$, and $\Gamma_u = \lambda_u E$, $\lambda_u > 0$, respectively. The "leakage" terms ω_r (>0) and ω_u(>0) in the adaptive laws belong to a class of σ-modification [12, 17] which are designed to improve robustness of adaptive schemes.

As mentioned previously, the robust control (T^r) is primarily intended to cope with the total uncertainties (ΔR_r and T_u) given in polynomial bounds and to ensure global stability of the closed-loop system. In order to accomplish the design objectives, the adaptive bound of robust control algorithm is given by

$$T^r = -\frac{(\phi_r)^2 e_s}{\| e_s \| (\phi_r) + \xi} - \frac{(\phi_u)^2 e_s}{\| e_s \| (\phi_u) + \xi} \quad (9)$$

with $\xi(t) = \alpha \exp(-\beta t)$, where α and β are non-negative constants which can be selected by a designer. The major feature of this algorithm is that very conservative bounds (and functional structures) on the uncertainties (ΔR_r and T_u) may be chosen to cope with significant uncertainties. That is, the proposed control algorithm is designed to tolerate a larger class of uncertainties than those considered in most other approaches. In control law (9), there are some alternatives of choosing the specific structures of $\xi(t)$, that is, $\xi=0$ ($\alpha=0$) (signum function), i.e., a purely discontinuous VS control law ; $\xi \neq 0$ ($\alpha > 0$, $\beta > 0$) (smooth function), i.e., continuous VS-type controller, $\xi \neq 0$ ($\alpha > 0$, $\beta = 0$), i.e., a saturation-type (or boundary layer) controller. The existence of ξ in T^r guarantees the continuity of control input even if $\| e_s \| \to 0$. In case of $\xi=0$, T^r becomes discontinuous on the surface $e_s=0$. The drawback of discontinuous-type control law is that it causes undesirable phenomena, such as chattering problem associated with requiring excessive control energy as well as exciting high-frequency unmodelled dynamics in practical implementations.

Now, the stability and the tracking properties of the closed-loop system (3) are summarized in the following theorem.

Theorem 1 : For all (bounded) desired trajectories and with unknown gains a_i and b_i on the uncertainty bounds, the solutions (e_s, a, b) of the closed-loop system (3), along with the adaptive laws (7) and (8), are uniformly ultimately bounded. That is, every solution starting in Ψ^c enters the residual set Ψ in a finite time and there after remains in Ψ (where Ψ^c denotes the complement of Ψ) :

$$\Psi = \{ (t, e_s, a, b) \in R^+ \times R^n \times R^3 \times R^{(p+1)} : V \leq V_r \},$$

where the ultimate bounds are given by

$$\xi \neq 0 \; (\alpha > 0, \; \beta = 0); \; V_r = \frac{2\alpha + \gamma}{\gamma_0}$$

and

$$\xi \neq 0 \; (\alpha > 0, \; \beta > 0); \; V_r = \begin{cases} \dfrac{\gamma}{\gamma_0}, & \gamma_0 \neq \beta \\ \gamma, & \gamma_0 = \beta \end{cases}$$

with

$$\gamma = \frac{\omega_r}{2} \| a \|^2 + \frac{\omega_u}{2} \| b \|^2 \text{ and}$$

$$\gamma_0 = \min \left\{ \frac{2k_a}{Q_{\max}}, \frac{\omega_r}{Q_{\max}}, \frac{\omega_u}{Q_{\max}} \right\}$$

Proof : To show the global stability of the closed-loop system, define the Lyapunov-like function (a C^1 function), $V : (t, e_s, a, b) | R^+ \times R^n \times R^3 \times R^{(p+1)} \to R^+$, such that

$$V = 1/2 \; \{ e_s^T M e_s + a^T \Gamma_r^{-1} a + b^T \Gamma_u^{-1} b \} \quad (10)$$

By Rayleigh's principle, an upper and lower bounds on V can be estimated as

$$1/2 \, Q_{\min} \{ \| e_s \|^2 + \| a \|^2 + \| b \|^2 \} \leq V$$

$$\leq 1/2Q_{\max}\ \{\|e_s\|^2+\|\tilde{a}\|^2+\|\tilde{b}\|^2\}\quad(11)$$

where

$$Q_{\min}=\min\{\sigma_{\min}(M),\ \sigma_{\min}(\Gamma_r^{-1}),\ \sigma_{\min}(\Gamma_u^{-1})\}$$

$$Q_{\max}=\max\{\sigma_{\max}(M),\ \sigma_{\max}(\Gamma_r^{-1}),\ \sigma_{\max}(\Gamma_u^{-1})\}$$

with $Q_{\min}>0$. Thus V is a positive-definite function. Evaluating its time derivative (10) along the closed-loop system (3) gives

$$\dot{V}=e_s^T M\dot{e}_s+1/2e_s^T\dot{M}e_s+\tilde{a}^T\Gamma_r^{-1}\dot{\tilde{a}}+\tilde{b}^T\Gamma_u^{-1}\dot{\tilde{b}}$$
$$=e_s^T\{-C(q,\dot{q})e_s-(T_u+\Delta R)-k_a e_s-\frac{(\hat{\phi}_r)^2e_s}{\|e_s\|(\hat{\phi}_r)+\xi}-\frac{(\hat{\phi}_u)^2e_s}{\|e_s\|(\hat{\phi}_u)+\xi}\}$$
$$+1/2e_s^T\dot{M}e_s-\tilde{a}^T(R_r^T\|e_s\|-\omega_r\hat{a})-\tilde{b}^T(R_u^T\|e_s\|-\omega_u\hat{b})\quad(12)$$

where the adaptation laws presented in (7) and (8) are chosen, with $\dot{\tilde{a}}=-\dot{\hat{a}}$ and $\dot{\tilde{b}}=-\dot{\hat{b}}$ (assuming that $\dot{a}=\dot{b}=0$). Applying Property 2 on (12) gives

$$\dot{V}\leq-e_s^Tk_ae_s+\|e_s\|R_r\,a+\|e_s\|R_ub-\frac{(\hat{\phi}_r)^2\|e_s\|^2}{\|e_s\|(\hat{\phi}_r)+\xi}-\frac{(\hat{\phi}_u)^2\|e_s\|^2}{\|e_s\|(\hat{\phi}_u)+\xi}$$
$$-\tilde{a}^TR_r^T\|e_s\|+\tilde{a}^T\omega_r\hat{a}-\tilde{b}^TR_u^T\|e_s\|+\tilde{b}^T\omega_u\hat{b}$$

Thus, it follows that

$$\dot{V}\leq-e_s^Tk_ae_s+\|e_s\|R_ra+\|e_s\|R_ub-(a^T-\tilde{a}^T)R_r^T\|e_s\|-(b^T-\tilde{b}^T)R_u^T\|e_s\|$$
$$-\frac{(\hat{\phi}_r)^2\|e_s\|^2}{\|e_s\|(\hat{\phi}_r)+\xi}-\frac{(\hat{\phi}_u)^2\|e_s\|^2}{\|e_s\|(\hat{\phi}_u)+\xi}$$
$$+\tilde{a}^T\omega_r(a-\tilde{a})+\tilde{b}^T\omega_u(b-\tilde{b})\leq-e_s^Tk_ae_s+\frac{\xi\hat{\phi}_r\|e_s\|}{\|e_s\|(\hat{\phi}_r)+\xi}+\frac{\xi\hat{\phi}_u\|e_s\|}{\|e_s\|(\hat{\phi}_r)+\xi}$$
$$-\omega_r\|\tilde{a}\|^2+\omega_r\|\tilde{a}\|\ \|a\|-\omega_u\|\tilde{b}\|^2+\omega_u\|\tilde{b}\|\ \|b\|\quad(13)$$

By using the following inequality

$$-\omega_r\|\tilde{a}\|^2+\omega_r\|\tilde{a}\|\ \|a\|-\omega_u\|\tilde{b}\|^2+\omega_u\|\tilde{b}\|\ \|b\|\leq-\frac{\omega_r}{2}\|\tilde{a}\|^2+\frac{\omega_r}{2}\|a\|^2-\frac{\omega_u}{2}\|\tilde{b}\|^2+\frac{\omega_u}{2}\|b\|^2$$

the differential inequality (13) can be upper bounded by

$$\dot{V}\leq-k_a\|e_s\|^2-\frac{\omega_r}{2}\|\tilde{a}\|^2-\frac{\omega_u}{2}\|\tilde{b}\|^2+2\xi+\frac{\omega_r}{2}\|a\|^2+\frac{\omega_u}{2}\|b\|^2\quad(14)$$

Letting

$$\gamma=\frac{\omega_r}{2}\|a\|^2+\frac{\omega_u}{2}\|b\|^2\ \text{and}$$

$$\gamma_0=\min\{\frac{2k_a}{Q_{\max}},\ \frac{\omega_r}{Q_{\max}},\ \frac{\omega_u}{Q_{\max}}\}$$

for convenience, where $\gamma\geq0$ and $\gamma_0>0$. Then it follows from (14) that

$$\dot{V}\leq-\gamma_0V+2\xi+\gamma\quad(15)$$

If $\xi\neq0$ ($\alpha>0,\ \beta=0$), then, for some $V_r=\frac{2\alpha+\gamma}{\gamma_0}\geq0$, one have $\dot{V}<0$ whenever $V>V_r$ (or $V\subset\Psi^c$). A detailed solution of (15) is given by

$$V\leq\exp(-\gamma_0t)[V_0-\frac{2\alpha+\gamma}{\gamma_0}]+\frac{2\alpha+\gamma}{\gamma_0},\ t\geq0\quad(16)$$

where $V_0=V(t=0,\ \bullet,\ \bullet,\ \bullet)$ and $0\leq\lim_{t\to\infty}V=\inf_t V=V_r=\frac{2\alpha+\gamma}{\gamma_0}\leq V_0<\infty$.

If $\xi\neq0$ ($\alpha>0,\ \beta>0$), then the uniform boundedness of V can be obtained by for all $t\geq0$.

$$V\begin{cases}\leq\exp(-\gamma_0t)[V_0-\frac{2\alpha}{\gamma_0-\beta}-\frac{\gamma}{\gamma_0}]+\exp(-\beta t)\\ \frac{2\alpha}{\gamma_0-\beta}+\frac{\gamma}{\gamma_0},\ \gamma_0\neq\beta\leq\exp(-\gamma_0t)[V_0-\gamma]\\ +2\alpha t\ \exp(-\gamma_0t)+\gamma,\ \gamma_0=\beta\end{cases}\quad(17)$$

where $0\leq\lim_{t\to\infty}V=\inf_t V=V_r=\frac{\gamma}{\gamma_0}\leq V_0<\infty$, $\gamma_0\neq\beta$ and $0\leq\lim_{t\to\infty}V=\inf_t V=V_r=\gamma\leq V_0<\infty$, $\gamma_0=\beta$.

The rate of convergence depends on the values of γ_0 and β, that is, V decreases exponentially until the solutions reach the target ball (or the residual set) Ψ. Moreover, from (16) and (17), the norm bounds of joint-space tracking errors can be estimated as follows :

In case of $\xi \neq 0$ ($\alpha > 0$, $\beta = 0$),

$$\| e_s \| \leq \sqrt{\frac{2}{Q_{\min}}[\exp(-\gamma_0 t)(V_0 - \frac{2\alpha+\gamma}{\gamma_0}) + \frac{2\alpha+\gamma}{\gamma_0}]^{1/2}}$$

and in case of $\xi \neq 0$ ($\alpha > 0$, $\beta > 0$),

$$\| e_s \| \leq \begin{cases} \sqrt{\frac{2}{Q_{\min}}}[\exp(-\gamma_0 t)(V_0 - \frac{2\alpha}{\gamma_0 - \beta} - \frac{\gamma}{\gamma_0}) + \exp(-\beta t)\frac{2\alpha}{\gamma_0 - \beta} + \frac{\gamma}{\gamma_0}]^{1/2}, & \gamma_0 \neq \beta \\ \sqrt{\frac{2}{Q_{\min}}}[\exp(-\gamma_0 t)(V_0 - \gamma) + 2\alpha t \exp(-\gamma_0 t) + \gamma]^{1/2}, & \gamma_0 = \beta \end{cases}$$

Finally, it can be easily shown that the tracking errors converge to the following compact set as $t \to \infty$:

If $\xi \neq 0$ ($\alpha > 0$, $\beta = 0$), then

$$B(e_s) = \left\{ e_s \in R^n : \| e_s \| \leq \sqrt{\frac{2(2\alpha+\gamma)}{Q_{\min}\gamma_0}} \right\}$$

And if $\xi \neq 0$ ($\alpha > 0$, $\beta > 0$), then

$$B(e_s) = \begin{cases} e_s \in R^n : \| e_s \| \leq \sqrt{\frac{2\gamma}{Q_{\min}\gamma_0}}, & \gamma_0 \neq \beta \\ e_s \in R^n : \| e_s \| \leq \sqrt{\frac{2\gamma}{Q_{\min}}}, & \gamma_0 = \beta \end{cases}$$

Consequently, uniform ultimate boundedness of tracking error e_s can be easily established by using Definitional 1 and 2, and the uniform ultimate boundednesses of other signals ($\tilde{a}$, $\tilde{b}$) are also guaranteed by similar manipulations.

Therefore, the control law (2) with the adaptive laws in Eqs. (7) and (8) renders the closed-loop system (3) uniformly ultimately bounded. In other words, all signals are finally attracted into the target ball (Ψ) in a finite time regardless of the uncertainties, and the UUB of system responses (e_s, a, b) are established with respect to V_r in this design. Moreover, the size of the tracking errors can be reduced by manipulating the design parameters before actuator saturation occurs.

Remark 5 : By using the result of Lemma 1, the ultimate boundednesses of the (position/velocity) tracking errors (e and $\dot{e}$) can be deduced from the boundedness of the sliding variable vector(e_s).

Remark 6 : For the specific value of $\xi = 0$ (or $\alpha = 0$) in (9), it is readily shown that the closedloop system is also uniformly ultimately bounded. Furthermore, if $\xi = 0$ and $\gamma = 0$, then the gloval exponential stability result can be obtained, i.e., $\lim \| e_s(t) \| = 0 \to \lim_{t \to \infty} \| e(t) \| = 0$ and $\lim_{t \to \infty} \| \dot{e}(t) \| = 0$.

Conclusions

In this paper, a dynamic compensation methodology for motion control of uncertain robot system has been presented. The proposed control scheme consists of two parts : the nominal control is used to stabilize the system in the absence of the uncertainties, and the robust control law is synthesized to cope with the system uncertainties. The outstanding features of the proposed control algorithms are summarized as follows : (i) Joint accelerations are not required in the control law. (ii) Joint torques in the model-based portion can be pre-calculated since the desired trajectories and the nominal values of dynamic parameters are known in advance(i.e., off-line feedforward compensation). (iii) To avoid centralized manner, the robust adaptation law is used to directly estimate the system uncertainty bounds instead of estimating all the system parameters. (iv) The robust nonlinear control does not require a priori knowledge of uncertainty bounds. Furthermore, the robust control law can cope with higher-order uncertainties. Finally, it is shown that the proposed control laws can guarantee the uniform ultimate boundedness of all signals under significant uncertainties.

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Appendix

Appendix A : Uniform ultimate boundedness(UUB).

Consider an uncertain dynamical model described by

$$\dot{x}(t)=f(t,\ x(t),\ u(t),\ w(t)),\ x(t_0)=x_0$$

where $x(t)\in R^n$ is the system state vector with $t\in R^+$; $u(t)\in R^m$ is the control input vector ; $w(t)\in R^r$ is the system uncertainties and its values lie within a prescribed compact set (closed and bounded) $\Pi\subset R^r$, $\forall_t\in R^1$ and $w\in\Pi$, i.e., $w(\bullet):R^+\rightarrow\Pi\subset R^r$. The uncertainty bounding set Π may be known or unknown. For a given initial condition $(t_0,\ x_0)\in R^+\times R^n$, there exists a state feedback control function $p(t,\ x(t)):R^+\times R^n\rightarrow R^m$ such that the corresponding closed-loop system is given by

$$\dot{x}(t)=\bar{f}(t,\ x,\ p(t,\ x),\ w).$$

Then the solution of the uncertain dynamical system $x(t)$ with $x_0=x(t_0)$ are uniformly ultimately bounded. That is, for any $\bar{\xi}(\in R^+)\geq\underline{\zeta}:(\in R^+)$ and $\zeta_0(\in R^+)$, there exists a finite time(a non-negative constant) $\bar{T}(\underline{\zeta},\ \zeta_0)\in[t_0,\ \infty)$, such that if $x(\bullet):[t_0,\ \infty)\rightarrow R^n$ is a solution of the system with $\|x(t_0)\|\leq\zeta_0$, then, $\|x(t)\|\leq\bar{\zeta},\ \forall_t>t_0+\bar{T}$.

Appendix B : Proof of Lemma 2

The modeling error ΔR_r can be estimated as follows :

Taking the norms on both sides of (4) gives

$$\|\Delta R_r\|\leq\|M-M_0\|\ \|\ddot{e}_r\|+\|C-C_0\|\ \|\dot{e}_r\|+\|G-G_0\|$$

By Assumptions 4 and 5, the following inequality can be obtained

$$\|\Delta R_r\|\leq\rho_{11}\|\ddot{e}_r\|+(\rho_{12}\|\dot{q}\|+\rho_{13}\|\dot{q}_d\|)\|\dot{e}_r\|+\rho_{14}\leq\rho_{11}(\|\ddot{q}_d\|+\mu\|\dot{e}\|)+[\rho_{12}(\|\dot{q}_d\|+\|\dot{e}\|)+\rho_{13}\|\dot{q}_d\|+\mu\|e\|)+\rho_{14}\leq[\rho_{11}d_3+(\rho_{12}+\rho_{13})d_2^2+\rho_{14}]+(\rho_{12}+\rho_{13})\mu d_2\|e\|+(\rho_{11}\mu+\rho_{12}d_2)\|\dot{e}\|+\mu\rho_{12}\|e\|\ \|\dot{e}\|$$

Then, it follows that

$$\|\Delta R_r\|\leq\rho_0+\rho_1\|e\|+\rho_2\|\dot{e}\|+\rho_3\|e\|\ \|\dot{e}\|$$

where

$$\rho_0=\rho_{11}d_3+(\rho_{12}+\rho_{13})d_2^2+\rho_{14},$$
$$\rho_1=(\rho_{12}+\rho_{13})\mu d_2,$$
$$\rho_2=\rho_{11}\mu+\rho_{12}d_2,\ \text{and}\ \rho_3=\mu\rho_{12}$$

Note that $\|e\|\leq\|x_e\|$ and $\|\dot{e}\|\leq\|x_e\|$, then

$$\|\Delta R_r\|\leq\rho_0+(\rho_1+\rho_2)\|x_e\|+\rho_3\|x_e\|^2$$

Finally, the upper bounds on ΔR_r can be expressed as

$$\|\Delta R_r\|\leq a_0+a_1\|x_e\|+a_2\|x_e\|^2,$$

where $a_0=\rho_0$, $a_1=\rho_1+\rho_2$, and $a_2=\rho_3$. This completes the proof.