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불규칙 작용힘들간의 Correlation이 평판의 진동레벨에 미치는 영향

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Effects of Source Correlation on Plates Driven by Multi-point Random Forces

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ABSTRACT

The problem of reducing the vibration level of elastic plates driven by multiple random point forces is analyzed in this study. First, the analytical solution for the vibration level of finite thin plates with four simply supported edges under the action of multiple random point force is derived. By assuming the plates to be lightly damped, an approximate solution for the vibration level of the plate is obtained. A numerical study is carried out to determine an optimal spacing distance between the multiple point forces in order to produce a relative minimum in the plate's vibration level. The optimal spacing distance is shown to depend on the given excitation band. The effects of wave cancellation in the near field of the multiple point forces are discussed by using the equivalence of certain stationary random responses and deterministic pulse responese.

Key Words : Multi-point random forces, Source correlation, Vibration level, Wave cancellation, Near field

1. Introduction

The forced vibration of machines and structural elements continues to be an important problem in engineering, noise control, and machine design. Widespread interest in the vibration control of structures is a relatively recent development stemming largely from desires to increase machine life and working efficiency. It is a topic of practical concern to machine designers and acoustical engineers.

In recent years, numerous researchers have been working in the field of forced structural vibration under the action of multiple point forces. Studies in this area have led to reductions in the structural vibration level by adjusting the number and location of the exciting forces and the joint statistical properties of the random forces. These results are encouraging to engineers interested in the design

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of quieter machines.

The present study concentrates on reducing the vibration level in structures under the action of multiple random point forces by adjusting the applied force positions. The effects on the vibration level of the correlation between input forces containing band-liminted frequency components is investigated in detail. The spacing between multiple point forces is shown to have an important effect on the vibration level. An optimal spacing between the applied forces is found that minimized the vibration.

In the following sections the problem is formulated in general, and formal solutions are obtained. The principal calculations, however, are performed using approximate procedures. Special consideration is given to stationary wide-band excitation of a square plate. The optimal spacing between input forces to produce a relative minimum in the vibration response is predicted and explained by the effect of wave cancellation in the near field of the applied forces.

2. Random vibration of a finite thin plate

Consider a simply supported rectangular homogeneous plate, whose sides are of lengths L_x, and L_y. This plate is acted upon by N transverse forces $f_I(t)$ located at the positions $x=a_I$ and $y=b_I$. The equation of motion for the transverse plate displacement $W(x, y, t)$ may be written as

$$D\nabla^4 W + C\frac{\partial W}{\partial t} + \rho h\frac{\partial^2 W}{\partial t^2} = \sum_{I=1}^{N}\delta(x-a_I)\delta(y-b_I)f_I(t) \qquad (1)$$

where ρh is the mass per unit area and C is the viscous damping per unit area. The operator ∇^4 is $(\delta^2/\delta x^2+\delta^2/\delta y^2)^2$. For a flat rectangular plate of thickness h, the bending modulus is $D=Eh^2/12(1-\upsilon^2)$ where E is Young's modulus and υ is Poisson's ratio.

The differential equation of motion for the undamped structure within the periphery of the plate is given by

$$D\nabla^4\psi_{jp} = \rho h\omega_{jp}^2\psi_{jp} \qquad (2)$$

where the natural modes ψ_{jp} for a rectangular plate

$$\psi_{jp} = 2\sin\frac{j\pi x}{L_x}\sin\frac{p\pi y}{L_y} \qquad (3)$$

satisfy the eigenvalue problem determined by the boundary conditions, and ω_{jp} is the natural frequency corresponding to $\psi_{jp}(x, y)$

$$\omega_{jp} = (D/\rho h)^{1/2}[(j\pi/L_x)^2+(p\pi/L_y)^2]. \qquad (4)$$

The modes also satisfy the orthogonality condition

$$\int_0^{L_x}\int_0^{L_y}\rho h\psi_{jp}\psi_{kq}dxdy = M\delta_{jk}\delta_{pq} \qquad (5)$$

where now $M=\rho hL_xL_y$ is the total mass of the plate. The asymptotic average frequency spacing between successive natural frequencies is[3]

$$\Delta\omega=(D/\rho h)^{1/2}\frac{4\pi}{L_xL_y} \qquad (6)$$

The unit complex frequency response function for the two-dimensional structure can be represented by the modal sum

$$H(x,y,\omega;a_I,b_I) = \sum_{j,p=1}^{\infty}\sum_{I=1}^{N}\psi_{jp}(x,y)\psi_{jp}(a_I,b_I)H_{jp}(\omega) \qquad (7)$$

where $H_{jp}(\omega)$ is the unit complex frequency respnse function[3] for the j-th and p-th modes :

$$H_{jp}(\omega) = \frac{1}{M(\omega_{jp}^2-\omega^2+i\beta\omega)} \qquad (8)$$

and the modal bandwidth β is

$$\beta = \frac{C}{\rho h} . \tag{9}$$

We consider the mean-square displacement at the location (x, y), which may be written as

$$E[W^2(x,y)] = \sum_{j,p=1}^{\infty} \sum_{k,q=1}^{\infty} \psi_{jp}(x,y) \psi_{kq}(x,y) I_{jpkq} \tag{10}$$

where I_{jpkq} represents the integral

$$I_{jpkq} = \int_{-\infty}^{\infty} H_{jp}(\omega) H_{kq}^*(\omega) \Phi_{jpkq}(\omega) d\omega \tag{11}$$

and the superscript "*" means the complex conjugate. $\Phi_{jpkq}(\omega)$ is the modal excitation cross -spectral density function

$$\Phi_{jpkq} = \sum_{I=1}^{N} \sum_{J=1}^{N} \psi_{jp}(x_1,y_1) \psi_{kq}(x_2,y_2) S_f(a_I,b_I,a_J,b_J,\omega). \tag{12}$$

The cross-spectral density function of the applied forces is given by

$$S_f(a_I,b_I,a_J,b_J,\omega) = \delta(x_1-a_I)\delta(y_1-b_I) \delta(x_2-a_J)\delta(y_2-b_J) S_{IJ}(\omega) \tag{13}$$

where N is the number of the applied forces and $S_{IJ}(\omega)$ is the power spectral density between the I-th and J-th forces.

The summation of the mean-square displacement may be thought as an index of the vibration level of the entire plated and can be obtained by integrating the mean-square displacement over the entire surface

$$W = \int_0^{L_x} \int_0^{L_y} E[W^2(x,y)] dxdy$$

$$= \int_0^{L_x} \int_0^{L_y} \sum_{j,p=1}^{\infty} \sum_{k,q=1}^{\infty} \sum_{I,J=1}^{N} \psi_{jp}(x_1,y_1) \psi_{kq}(x_2,y_2) \int_{-\infty}^{\infty} H_{jp}(\omega) \cdot H_{kq}^*(\omega) \psi_{jp}(a_1,b_1) \psi_{kq}(a_2,b_2) S_{IJ}(\omega) d\omega dxdy. \tag{14}$$

3. Approximate Solutions based on modal sum

In the following, some approximate solutions for the vibration level of the plate are proposed in order to simplify the calculation. Two assumptions are made which are particularly popular and useful in random vibration analysis.

The first assumption is that the damping of the plates is light enough and the resonance frequencies are sufficiently separated, so that the 'off-diagonal coupling' terms may be neglected[1]. This implies omitting all terms in the fourfold modal summation except those for which j=k and p=q.

If the plate is square, the mode degeneracy is a symmetrically occurring phenomenon. In this case, every mode $\Psi_{jp}(j \neq p)$ has a mate Ψ_{pj} with the identical natural frequency. The important contributions to the dynamic response come not only from the terms representing modal auto-correlations but also from the terms representing modal cross-correlations arising from those modes with j=q and p=k.

The second assumption is the "white noise" approximation, which implies that the auto- and cross-spectral density functions $S_{IJ}(\omega)$ are slowly varying with respect to frequency relative to the rapidly varying mode response functions $H_{jp}(\omega)$, especially near the frequencies $\omega = \omega_{jp}$. The modal sum (10) for the mean-square displacement than becomes

$$E[W^2(x,y)] = \sum_{j,p=m}^{n} \sum_{I,J=1}^{N} \psi_{jp}^2(x,y) \psi_{jp}(a_I,b_I) \psi_{jp}(a_J,b_J) \cdot \int_{-\infty}^{\infty} |H_{jp}(\omega)|^2 S_{IJ}(\omega) d\omega \tag{15}$$

where the summation over the mode number j and p is confined whthin the range where thd condition of $\omega_1 < |\omega| < \omega_2$ is satisfied, and ω_1 and ω_2 represent lower and upper cut-off frequncies of the excitation band, and m and n represent the lower and the upper cut-off frequencies of the excitation band, and m and n represent the lower and upper resonant mode numbers within the given band.

In the case of band-limited white noise spectra, the spectral density for these forces may be written as

$$S_{II} = S_{JJ} = \begin{cases} S_0, & \omega_1 < |\omega| < \omega_2 \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

$$S_{IJ} = S_{JI} = \begin{cases} rS_0 & \omega_1 < |\omega| < \omega_2 \\ 0, & \text{otherwise} \end{cases}$$

where r is the excitation correlation coefficient ($-1 \le r \le 1$). The integral in (15) has the value $W_1^2 = nS_0/M^2\beta$ which is independent of ω_{jp}[1]. The mean-square displacement in the rectangular plate is thus approximated by the modal sum

$$E[W^2(x,y)] = W_1^2 \sum_{j,p=m}^{n} \sum_{I,J=1}^{N} \frac{16}{\omega_{jp}^2} \sin^2\frac{j\pi x}{L_x} \sin^2\frac{p\pi y}{L_y} \cdot \sin\frac{j\pi a_I}{L_x} \sin\frac{p\pi b_I}{L_y} \sin\frac{j\pi a_J}{L_x} \sin\frac{p\pi b_J}{L_y}. \tag{17}$$

For the case of a simply supported square plate ($L_x = L_y = L$), every mode Ψ_{jp} for $j \neq p$ has a mate Ψ_{pj} with the identical natural frequency. The plate response, including both modal cross-correlation terms (j=q, p=k) and auto-correlation terms (j=k, p=q), may be written as :

$$E[W^2(x,y)]/W_1^2 = A(x,y) + B(x,y) \tag{18}$$

with

$$A(x,y) = \sum_{j,p=m}^{n} \sum_{I,J=1}^{N} \frac{16}{\omega_{jp}^2} \sin^2\frac{j\pi x}{L} \sin^2\frac{p\pi y}{L} \cdot \sin\frac{j\pi a_I}{L} \sin\frac{p\pi b_I}{L} \sin\frac{j\pi a_J}{L} \sin\frac{p\pi b_J}{L} \tag{18-a}$$

$$B(x,y) = \sum_{j,p=m}^{n} \sum_{I,J=1}^{N} \frac{16}{\omega_{jp}^2} \sin\frac{j\pi x}{L} \sin\frac{p\pi y}{L} \sin\frac{p\pi x}{L} \cdot \sin\frac{j\pi y}{L} \sin\frac{j\pi a_I}{L} \sin\frac{p\pi b_I}{L} \sin\frac{p\pi a_J}{L} \sin\frac{j\pi b_J}{L} \tag{18-b}$$

where A(x, y) is due to by the modal auto-correlations and B(x, y) is due to by the modal cross-correlations. The integration of B(x, y) over the entire plate vanishes due to the orthogonality of the modes. Therefore the vibration level of the entire plate become[4]

$$\begin{aligned} W &= \int_0^L \int_0^L E[W^2(x,y)]dxdy \\ &= W_1^2 \int_0^L \int_0^L A(x,y)dxdy \\ &= 4\pi L^2 S_0/M^2\beta \sum_{j,p=m}^{n} \sum_{I,J=1}^{N} \sin\frac{j\pi a_I}{L} \sin\frac{p\pi b_I}{L} \sin\frac{j\pi a_J}{L} \sin\frac{p\pi b_J}{L} \,/\, \omega_{jp}^2. \end{aligned} \tag{19}$$

Equation (19) may be written as

$$W = W_a + W_c \tag{20}$$

where the term W_a is produced by the source auto-correlation,

$$W_a = 4\pi L^2 S_0/M^2\beta \sum_{j,p=m}^{n} \sum_{I=J=1}^{N} \sin^2\frac{j\pi a_I}{L} \sin^2\frac{p\pi b_I}{L} \,/\, \omega_{jp}^2. \tag{20-a}$$

and the term W_c is produced by the source cross-correlation. Thus,

$$W_c = 4\pi L^2 S_0 / M^2 \beta \sum_{j,p=m}^{n} \sum_{I \neq J}^{N} \sin \frac{j\pi a_I}{L}$$

$$\sin \frac{p\pi b_I}{L} \sin \frac{j\pi a_J}{L} \sin \frac{p\pi b_J}{L} / \omega_{jp}^2. \quad (20\text{-b})$$

where N is the number of applied forces.

4. Numerical results and discussion

In the present study attempts were made to investigate the mean-square displacement distribution of a plate and to determine an optimal spacing distance between input forces to produce a relative minimum in the vibration level under the excitation of band limited white noise point forces. When all forces have identical spectra, the mean-square response distribution depends on the cross-correlations between the forces.

In the present paper we consider structures excited by two or more point forces whose time histories are sample functions of wideband random processed with known cross-correlations. As has been proviously shown[(2)], the mean-square response of such structures is generally quite uniform, except in certain regions. These details of the response depend upon the number of forces and their cross-correlations.

The mean-square displacement distribrtions were calculated for different values of the correlation coefficient r : (1) $r=1$, (2) $r=0$, (3) $r=-1$. These different cases were calculated in order to study the associated mean-square displacement distributions.

In the numerical analysis a square aluminum plate with dimensions of 60 inch × 60 inch and thickness of 0.25 inch was chosen. Two and four point forces were chosen and the excitation was from "white noise" using octave bands centered at 500 Hz.

The numerical result were used to construct two dimensional contour maps to show the mean-square response distributions of the plate. The data in each case has been normalized so that it appears in the range form 0 to +1. The scaled data is identified through integral contour labels, where label 1 corresponds to the largest positive value (maximum) and label 3 corresponds to a zero value (minimum). As usual, the density of the contour lines is indicative of the local vibration level gradient.

The effects of multiple forces may be introduced most simply by considering the case of two point forces. Figures 1, 2 and 3 examine thd effects of correlation between two point forces when the excitation is an octave band centered at 500 Hz. The locations of the applied point forces are indicated using dark dots. Figure 1, in the extreme case of complete correlation ($r=+1$), shows that the mean-square displacement is enforced in the neighborhood of the drive points and their symmetric image points. There is some additional local enhancement midway between the drive points and at the middle point of the plate. Figure 2, for a pair of uncorrelated driving forces ($r=0$), demonstrates a flattening of the mean-

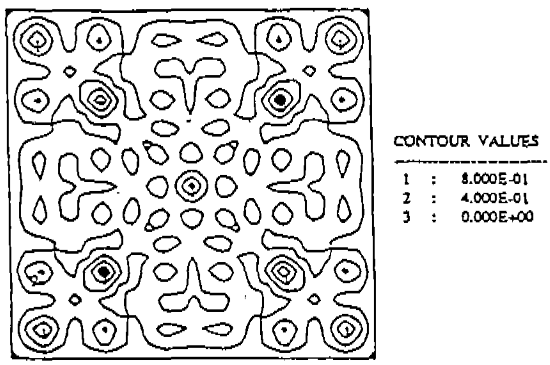

Fig. 1 Mean-square displacement produced by two point forces with correlation coefficeient $r=+1$ (octave band at 500 Hz)

square response along the diagonal located between the two point forces and the enforced mean-square displacement distribution in the neighborhood of the two drive points. Figure 3, for the case r=-1 between forces, shows that the diagonal located between the two point forces seves as a reference line of zero mean-square displacement. That is, one half of the plate is seen to be the mirror image of the other half.

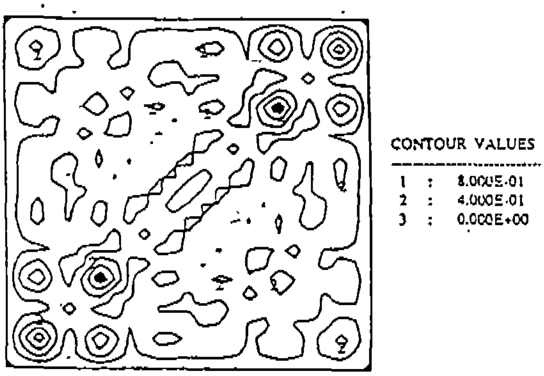

Fig. 2 Mean-square displacement produced by two point forces with correlation coefficient r= 0 (octave band at 500 Hz)

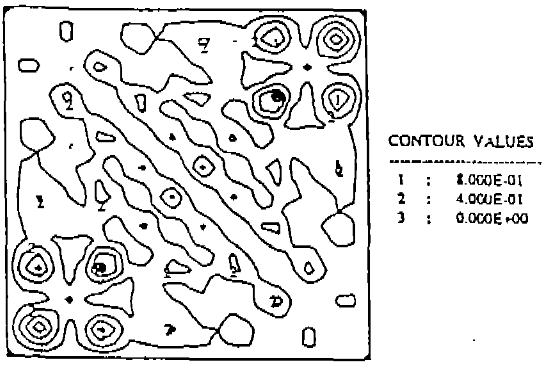

Fig. 3 Mean-square displacement produced by two point forces with correlation coefficient r= -1 (ocatave band at 500 Hz)

Figure 4 shows that the variation of the correlation coefficient (-1≤r≤1) may change the total mean-square vibration level W of the plate by adding or subtracting the vibration level due to the source cross-correlation, W_c. When the driving forces are uncorrelated (r=0) the vibration leval of the plate W reduces to a superposition of the distributions which result when each force acts alone. The vibration level of the plate due to the source auto-correlation has a constant value, the difference between W and W_c, and is independent on the correlation coefficient. In the extreme cases (r= ±1) the vibration level of the plate W is increased or decreased linearly 19% compared to the case when r is equal to zero.

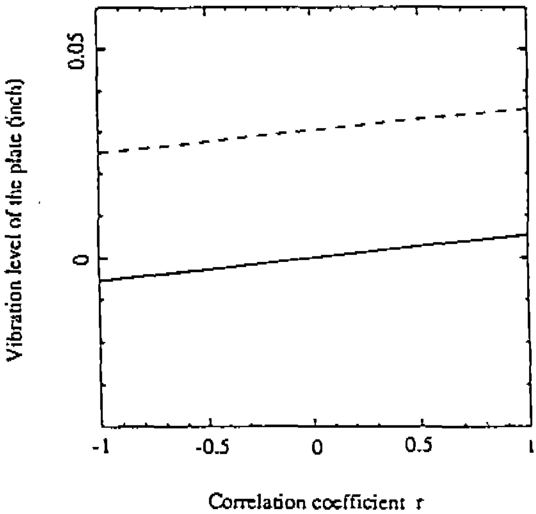

Fig. 4 Vibration level (mean-square displacement) of the plate (···) and vibration level of the plate due to the source cross-correlation (—) versus the correlation coefficient

It is also of interest to examine optimal arrangements of point forces that produce minima in the total plate response. A particualr case was investigated in which four point forces were located on the main diagonals of the plate, at a radial spacing R[5]. The total plate response was calculated for different r values and the results are shown in figure 5. This figure shows that an optimal force spacing exists to produce a relative minimum in the vibration level. By adjusting the spatial distance R between the applied forces the magnitude of the vibration level of the plate shows a dramatic reduction, relative to its maximum value when the point forces are located at the center of the plate. The reason for this behavior will be discussed in the following section by considering the effects of wave cancellation in the near-field of the applied forces. This discussion will take place in the time domain, making use of the equivalence between certain stationary random responses and deterministic responses[1].

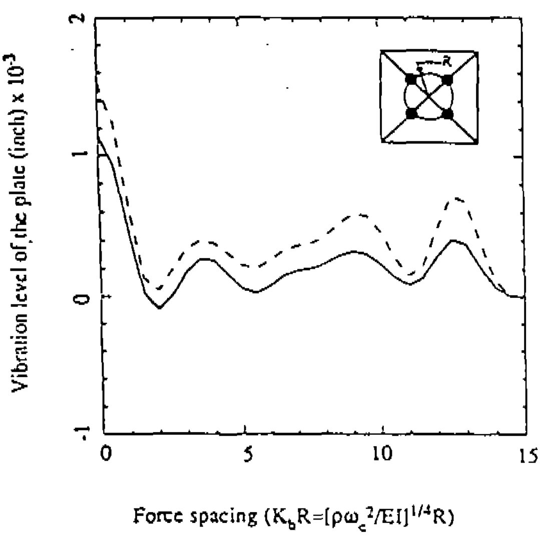

Fig. 5 Overall vibration level of the plate (…) and the vibation level of the plated due to the source cross-correlation (−) versus the spacing distance of four positively correlated ($r=+1$) random point forces.

5. Deterministic pulse response in time domain

In the previous sections, solutions were developed for the mean-square displacement of a structure undergoing stationary vibration due to a distributed stationary random excitation $f(x, y, t)$. This excitation was described by its space-time cross spectral density function $S_f(x_1, y_1, x_2, y_2, \omega)$. As pointed out by Crandall[1], "for certain special forms of these functions there exist 'equivalent' deterministic force response problems from which the mean-square response in the random case can be inferred."

This equivalence may be exploited to simplify the numerical calculations. In the present study, the equivalence relation is used to estimate the stationary response of a structure using a modal sum calculation of a deterministic force response.

The special form required of the space-time cross-spectral density function $S_f(x_1, y_1, x_2, y_2, \omega)$ is that it must factor in the following way[1]

$$S_f(x_1,y_1,x_2,y_2,\omega)=r(x_1)r(y_1)r(x_2)r(y_2)S(\omega) \qquad (21)$$

where $r(x)$ and $r(y)$ and real functions and $S(\omega)$ is the mean-square spectral density.

Using the spectral form (21) the mean-square displacement (10) can be described as

$$E[W^2(x,y)]=\int_{-\infty}^{\infty} S(\omega)d\omega \int_0^{L_x}\int_0^{L_y}\int_0^{L_x}\int_0^{L_y} r(\xi_1)r(\eta_1)r(\xi_2)r(\eta_2)\cdot H(x_1,y_1,\omega;\xi_1,\eta_1) H^*(x_2,y_2,\omega;\xi_2,\eta_2)d\xi_1 d\eta_1 d\xi_2 d\eta_2. \qquad (22)$$

Following Crandall[1], it is possible to show that a deterministic $f(x, y, t)$ exists such that the time integral of $W^2(x, y, t)$ is proportional to Equation (22).

Assume the exciatation to be of the form

$$f(x,y,t)=r(x)r(y)f(t) \qquad (23)$$

where $f(t)$ is to be determined. The displacement response to (23) is

$$W(x,y,t)=\int_0^{L_x}\int_0^{L_y} r(\xi)r(\eta)d\xi d\eta$$

$$\int_{-\infty}^{\infty} f(t-\tau)h(x,y,\tau;\xi,\eta)d\tau. \quad (24)$$

The Fourier transform of the displacement response (24) is

$$W(x,y,\omega)=F(\omega)\int_0^{L_x}\int_0^{L_y} r(\xi)r(\eta) H(x,y,\omega;\xi,\eta)d\xi d\eta. \quad (25)$$

The integral of the square of (25), after interchanging the order of integration, can be written in the frequency domain through use of Parseval's identity[1]

$$\int_{-\infty}^{\infty} W^2(x,y,t)dt = 1/2\pi \int_{-\infty}^{\infty} W(x,y,\omega) W^*(x,y,\omega)d\omega$$
$$= 1/2\pi \int_{-\infty}^{\infty} [F(\omega)F^*(\omega)]d\omega \int_0^{L_x}\int_0^{L_y}\int_0^{L_x}\int_0^{L_y} r(\xi_1)r(\eta_1)r(\xi_2)r(\eta_2)\cdot H(x_1,y_1,\omega;\xi_1,\eta_1)H^*(x_2,y_2,\omega;\xi_2,\eta_2)d\xi_1 d\eta_1 d\xi_2 d\eta_2. \quad (26)$$

Comparing (22) and (26) we see that the equivalence relationship is

$$F(\omega)F^*(\omega)/2\pi T_c=S(\omega) \quad (27)$$

where T_c is a constant of proportionality having the dimension of time and $F^*(\omega)$ is the complex conjugate of $F(\omega)$. In the frequency domain the selection of a deterministic excitation $F(\omega)$ that is equivalent to the given random spectrum $S(\omega)$ is made according to (27). We consider the case where the force process is band-limited white noise, i.e., where

$$S(\omega)=\begin{cases} S_0, & \omega_1<|\omega|<\omega_2 \\ 0, & \text{otherwise} \end{cases} \quad (28)$$

A solution to (27) is provided by

$$F(\omega)=\begin{cases} \sqrt{2\pi S_0 T_c}\exp(-i\omega t') & \omega_1<|\omega|<\omega_2 \\ 0, & \text{otherwise.} \end{cases} \quad (29)$$

Consider several "equivalent" deterministic forces applied to a simply supported square plate. The equation of motion for the transverse displacement W(x, y, t) is given by (1). The transverse displacement W(x, y, t) is defined as

$$W(x,y,t)=\sum_{j,p=1}^{\infty} \eta_{jp}(t)\sin\frac{j\pi x}{L}\sin\frac{p\pi y}{L}. \quad (30)$$

Substituting (3) into (1), the equation of motion (1) becomes

$$\ddot{\eta}_{jp}(t)+\beta\dot{\eta}_{jp}(t)+\omega_{jp}^2\eta_{jp}(t) = \frac{1}{\rho h} f_{jp}(t) \quad (31)$$

where $f_{jp}(t)=4/L^2\sum_{I=1}^{N} f_I(t)\sin(j\pi a_I/L)\sin(p\pi b_I/L)$

Initial conditions are

$$\eta_{jp}(0)=0, \qquad \dot{\eta}_{jp}(0)=0. \quad (32)$$

Applying the Duhamel's integral[6] to (31) we obtain

$$\eta_{jp}(t)=\frac{\exp(-\beta t/2)}{\omega_d}\int_{-\infty}^{\infty}\exp(\beta t'/2) \frac{f_{jp}(t')}{\rho h}\sin\omega_d(t-t')dt' \quad (33)$$

where ω_d is the frequnecy of damped oscillation. Crandall[1] obtained the equivalent deterministic force $f_I(t')$, the Fourier transform of (29), where the force process is band-limited white noise, i.e.,

$$f_I(t')=\sqrt{\frac{2S_0T_c}{\pi}}\left[\frac{\sin\omega_2 t'}{t'} - \frac{\sin\omega_1 t'}{t'}\right]. \quad (34)$$

Substituting (33) into (30), the transverse displacement of the plate is found

$$W(x,y,t)=\sqrt{\frac{2S_0T_c}{\pi}\frac{4}{\rho hL^2}}\sum_{j,p=m}^{n}\sum_{I=1}^{N}\sin\frac{j\pi x}{L}\sin\frac{p\pi y}{L}\sin\frac{j\pi a_I}{L}\sin\frac{p\pi b_I}{L}\cdot\frac{\exp(-\beta t/2)}{\omega_d}\int_{-\infty}^{\infty}\exp(-\beta t'/2)\left[\frac{\sin\omega_2 t'}{t'}-\frac{\sin\omega_1 t'}{t'}\right]\sin\omega_d(t-t')dt'. \quad (35)$$

Equation (35) will be used to calculate the wave propagation in the plate.

Figures 6, 7 and 8 show the wave propagation in the plate for the octave band centered at 500 Hz to demonstrate the effect of wave cancellation in the near field of the applied forces. Those Figures (6, 7 and 8) plot the numerical results for the three cases corresponding to changing force positions. The positions of applied forces along the diagonals are as follows : (a) located at center (R=0 inch), (b) located at corner (R=25 inch), (c) located around the center of the plate (R=4 inch). Four sample times (0.0005, 0.001, 0.0015 and 0.002 second) were chosen in each figure to compare the plate deflection by wave propagation as time increases.

Figure 6 shows wave propagation for the concentrated forces at the center of the plate. At time t=0.0005 second, in Figure 6(a), the plate is seen to be at rest except near the positions of the applied forces. As time goes by, waves propagates toward the edges of the plate and entire plate vibrates. Figure 7 shows a similar behavior, although for this case, the force positions were located at the corners. Waves propagate from the corners to the center of the plate. The whole plate eventually vibrates as in Figure 6. Figure 8 shows a reduction in the vibration level due to the effects of wave cancellation in the near fields of the applied forces, relative to the provious two cases.

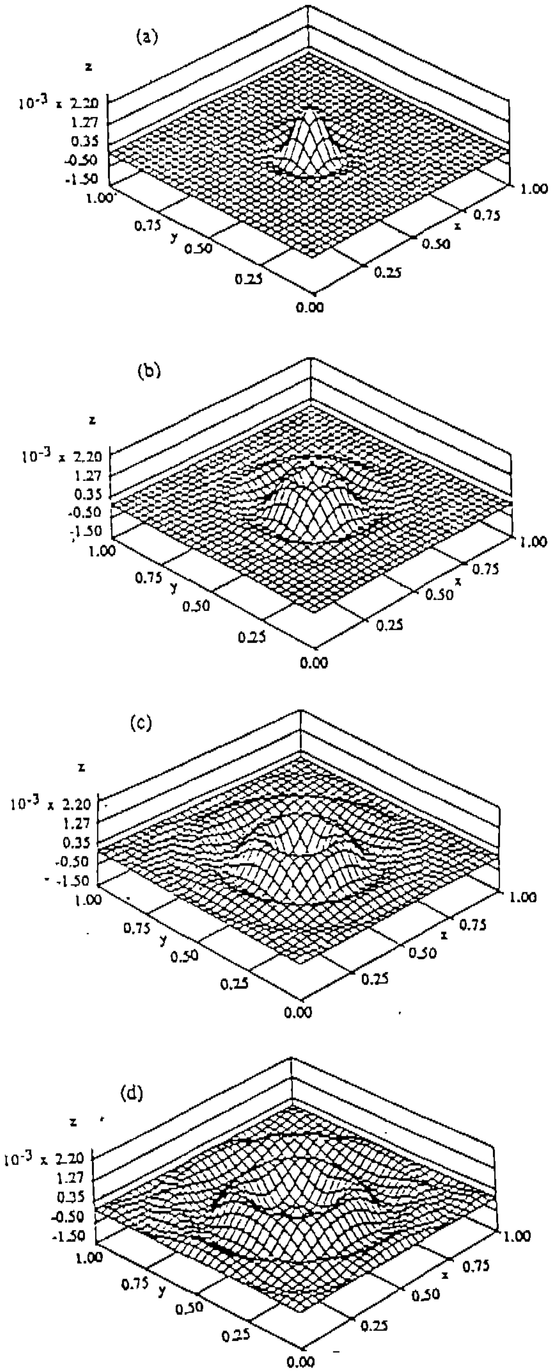

Fig. 6 Wave propagation for the force spacing R=0 inch resulting form deterministic point forces
(a) t=0.0005 second (b) t=0.001 second
(c) t=0.0015 second (d) t=0.002 second

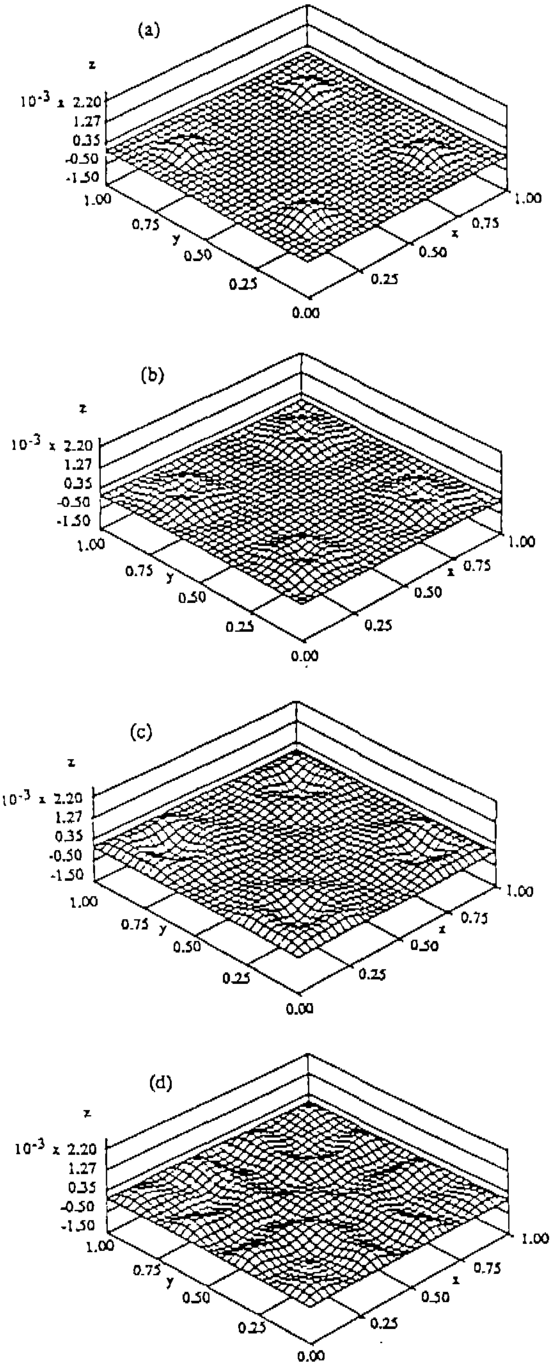

Fig. 7 Wave propagation for the force spacing R=25 inch resulting form four deterministic point forces
(a) t=0.0005 second (b) t=0.001 second
(c) t=0.0015 second (d) t=0.002 second

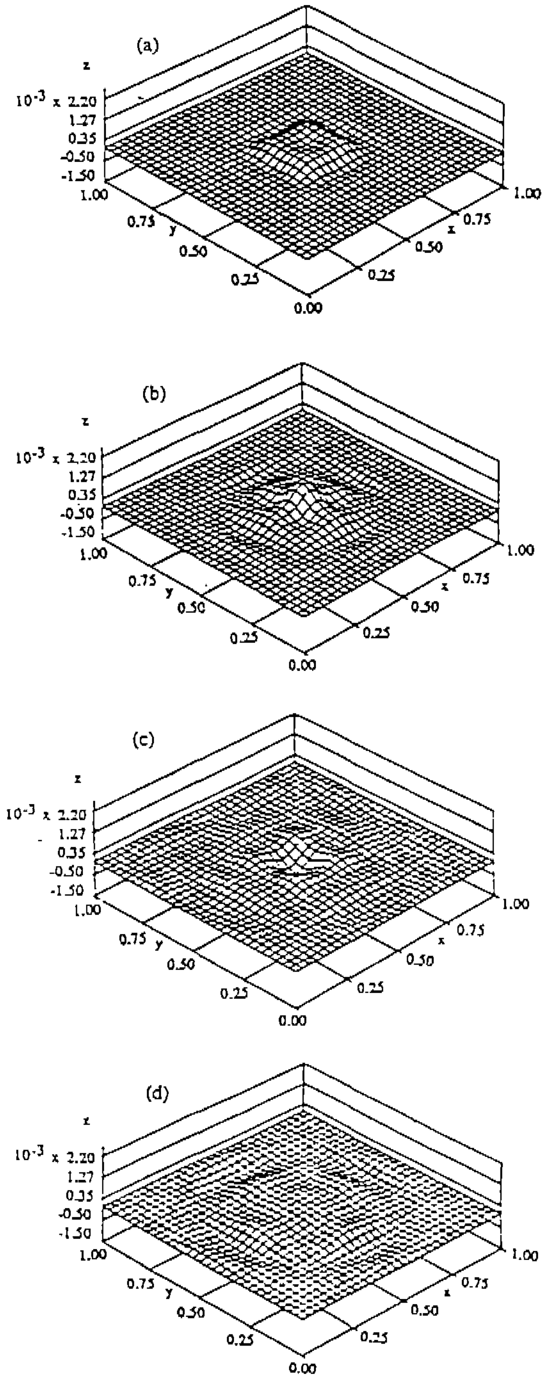

Fig. 8 Wave propagation for the force spacing R=4 inch resulting from four deterministic point forces
(a) t=0.0005 second (b) t=0.001 second
(c) t=0.0015 second (d) t=0.002 second

6. conclusion

It has been found in this study that the spatial distributions of multiple correlated point forces have significant influences on the vibration level of plates. In the case of a plate excited by band-limited white noise, the spatial distance between the applied forces causes changes in the vibration level of the plate. The specific case investigated consisted of a finite square plate, and four point forces. For a given excitation band, there was found an optimal spacing distance of the applied forces which leads to a relative minimum in the vibration level of the plate. The physical mechanism responsible was found to be the wave cancellation in the near fields of the applied forces.

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