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Comparisons of Acceptance Sampling Plans for the Exponential Lifetime Distribution

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ABSTRACT

Reliability acceptance sampling is concerned with whether to accept or reject a collection of items based upon the information obtained from life testing. Although various reliability acceptance sampling plans have been developed, little is known about their relative performances. This paper compares reliability acceptance sampling plans under Type II censoring, Hybrid censoring, and Time-Truncated Type II censoring assuming that the lifetimes of items in a lot follow an exponential distribution. The three plans are compared in terms of the power, the expected number of failures, and the expected time required to reach a decision. Computational experiments are conducted and the results are tabulated to provide guidelines for selecting an appropriate plan for a given situation.

KEYWORDS: Type II censoring, Hybrid censoring, Time-Truncated Type II censoring.

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1. INTRODUCTION

This article is concerned with comparing relative performances of typical reliability acceptance sampling plans(ASP) for the exponential lifetime distribution. First, assume that the lifetime X' of items in a lot follows an exponential distribution, the probability density function of which is given by

$$f(x') = (1/\theta')e^{-x'/\theta'}; \quad x' > 0, \quad \theta' > 0.$$

Then, based upon the data from a life test we want to test the following hypothesis on the mean lifetime θ'.

$$\begin{aligned} H_0 &: \theta' = \theta'_0 \\ H_1 &: \theta' = \theta'_1 (< \theta'_0). \end{aligned} \tag{1.1}$$

Two kinds of risks are involved in the above hypothesis test. One is the producer risk α which is the probability that a lot with mean lifetime $\theta' = \theta'_0$ is rejected, and the other is the consumer risk β which is the probability that a lot with $\theta' = \theta'_1$ is accepted. An ASP must be designed such that these risk requirements are met for given θ'_0, θ'_1, and other constraints.

Various ASP's have been developed under the assumption of exponentially distributed lifetimes([1]-[6]). In this article, we consider three such ASP's, namely, ASP's under Type II censoring, Hybrid censoring, and Time-Truncated Type II censoring(Hereafter, they will be denoted by ASP(II), ASP(H), and ASP(T), respectively). These three plans are easy to understand and implement, and therefore, most extensively discussed in the literature. However, little is known about their relative performances in terms of the power, the expected number of failed units, the completion time to reach a decision, *etc.* Thus it is difficult for reliablity engineers to choose an appropriate ASP for the given situation.

ASP(II) was developed by Epstein and Sobel(1953). The corresponding lifetest is terminated as soon as the first $r(1 \le r \le n)$ failures are observed where n is the number of units on test. Then, the maximum likelihood estimate

$\hat{\theta}'$ of θ' is calculated based upon the lifetime data, and H_0 in (1.1) is acccepted if $\hat{\theta}' > C'$ where critical value C' is a constant determined to satisfy the risk requirements, and rejected otherwise. Although ASP(II) has an advantage that the number of failures(r) can be predetermined, its completion time is random, and therefore, cannot be controlled. To avoid this difficulty, Epstein (1954) also develped ASP(H) where H_0 in (1.1) is rejected if r_h failures are observed before a prespecified time τ_h, and accepted otherwise. In his original plan, τ_h is assumed to be given, r_h is set equal to r of the corresponding ASP(II), and the sample size n_h is determined to approximately satisfy the risk requirements. However, this is an approximate way of constructing an ASP(H), and therfore, exact values of r_h and n_h given τ_h are first determined in this article before comparisons are made with other types of plans.

ASP(T) is similar to ASP(H) in that it also has a prespecified time limit(τ_t) and a prespecified number of failures (r_t). However, in the former the maximum likelihood estimate of θ' is calculated if r_t failures are observed before τ_t. Then, H_0 is either accepted or rejected in a similar manner as in ASP(II). This type of ASP was first proposed by Fertig and Mann(1980) for the Weibull lifetime distribution. In their original plan, the sample size n_t, r_t, and critical value C'_t were determined assuming pure Type II censoring, and then, τ_t is determined to make the consumer risk approximately equal to that of the corresponding ASP(II). In this paper, we modify their original plan in such a way that r_t and C'_t are respectively set equal to r and C' of the corresponding ASP(II), τ_t is set equal to τ_h, and n_t is determined to make the consumer risk approximately equal to that of the corresponding ASP(II). This modification is made to compare ASP(T) with ASP(II) or ASP(H) on an equal basis.

The purpose of this article is to compare performances of the above three ASP's in terms of the power, the expected number of failed units, and the expected time required until a decision is reached. To compare these plans on an equal basis, exact ASP(H) are developed and the original ASP(T) is modified. Computational experiments are conducted for various combinations of parameter values to assess the relative performances of the three plans, and guidelines are provided for selecting an appropriate plan for a given situation.

2. PRELIMINARIES

2.1. Notation

In this article the following notation and acronyms are used.

θ	Mean lifetime of an exponential distribution.
θ_0, θ_1	Specified mean lifetimes under H_0 and H_1, respectively.
α, β	Producer and consumer risks, respectively.
X	Random variable denoting the lifetime of a test unit.
$X_{i,m}$	The i-th failure time given m items on test.
ASP	Acceptance sampling plan. ASP(II) : ASP under Type II censoring. ASP(H) : ASP under Hybrid censoring. ASP(T) : ASP under Time-Truncated Type II censoring.
n, n_h, n_t	Sample sizes for ASP(II), ASP(H), and ASP(T), respectively.
r, r_h, r_t	Prespecified numbers of failures for ASP(II), ASP(H), and ASP(T), respectively.
τ_h, τ_t	Prespecified censoring times for ASP(H) and ASP(T), respectively.
C, C_t	Critical values for ASP(II) and ASP(T), respectively.
ϵ	Constant for the assurance of β error in ASP(T).
R	Random variable which denotes the number of failures until a decision is reached.

T	Random variable denoting the completion time to reach a decision.
$E_\theta(R)$	Expected value of R when the true mean lifetime is θ.
$E_\theta(T)$	Expected value of T when the true mean lifetime is θ.
OC	Operating characteristic.
$L(\theta)$	OC function of θ.
$\chi^2_{p,k}$	The p-th quantile of a chi-square distribution with k degrees of freedom.
$B(k; m, p)$	Binomial probability of k failures with parameters m and p.
pdf	Probability density function.

For each parameter related to time (e.g., θ), a prime is used to indicate the value in the original scale.

2.2. Assumptions

The following assumptions are made in this article.

(1) The lifetime distribution of a test unit follows an exponential distribution.

(2) The lifetimes of test units are statistically independent.

(3) The items in lots are produced by a stable manufacturing process.

(4) Failed items are not replaced by new items.

2.3. Standardization of Parameters and Risk Requirements

Consider the following transformation of the original lifetime X'.

$$X = X'/\theta'_0,$$

where θ'_0 is defined in (1.1). Then, the *pdf* of X is given by

$$f(x) = (1/\theta)e^{-x/\theta}, \quad x > 0,\ \theta > 0, \tag{2.1}$$

where $\theta = \theta'/\theta'_0$. Under the above transformation, hypotheses in (1.1) are changed to

$$H_0 \;:\; \theta = \theta_0 (= 1)$$

$$H_1 \;:\; \theta = \theta_1 (= \theta'_1/\theta'_0 < 1).$$

In addition, all the other time-related parameters are assumed to be standardized with respect to θ'_0, and will be used without a prime in the remaining of this article.

Let $L(\theta) = \Pr(\text{Accept } H_0 \mid \theta)$. Then, the risk requirements can be represented by

$$L(\theta_0) = L(1) = 1 - \alpha, \tag{2.2}$$

$$L(\theta_1) = \beta. \tag{2.3}$$

For a certain type of ASP, the above requirements may not be exactly satisfied due to discreteness of some parameters. In such cases, either one or both of the above equalities needs to be relaxed as $L(1) \geq 1 - \alpha$ and/or $L(\theta_1) \leq \beta$.

3. RELIABILITY ACCEPTANCE SAMPLING PLANS

3.1. ASP under Type II Censoring

Assume that n test units are drawn at random from distribution (2.1) and placed on life test at time 0. Under Type II censoring, experimentation is discontinued as soon as the r-th item fails ($1 \leq r \leq n$, prespecified). Then the maximum likelihood estimator of θ is given by(Epstein and Sobel, 1953)

$$\hat{\theta}_{r,n} = \left\{X_{1,n} + X_{2,n} + \cdots + X_{r,n} + (n-r)X_{r,n}\right\}/r. \tag{3.1}$$

It was also shown by Epstein and Sobel(1953) that $2r\hat{\theta}_{r,n}/\theta$ follows a chi-square distribution with $2r$ degrees of freedom, and that the risk requirements,

$L(\theta_0) = 1 - \alpha$ and $L(\theta_1) \leq \beta$, are satisfied if we define the acceptance region for H_0 as $\hat{\theta}_{r,n} \geq C$ where $C = \theta_0 \chi^2_{\alpha,2r}/(2r) = \chi^2_{\alpha,2r}/(2r)$, and r as the smallest integer that satisfies $\chi^2_{\alpha,2r}/\chi^2_{1-\beta,2r} \geq \theta_1/\theta_0$.

The OC function is given by

$$\begin{aligned} L(\theta) &= \Pr(\hat{\theta}_{r,n} \geq C|\theta) \\ &= \Pr(\chi^2_{2r} \geq \frac{2rC}{\theta}|\theta), \end{aligned} \tag{3.2}$$

the expected number of failures to reach a decision by r, and the expected completion time to reach a decision by(Epstein and Sobel, 1953)

$$E_\theta(T) = E_\theta(X_{r,n}) = \theta \sum_{j=1}^{r} \frac{1}{n-j+1}.$$

3.2. ASP under Hybrid Censoring

A Hybrid-censored life test with n_h items placed on test is terminated at $\min(X_{r_h,n_h}, \tau_h)$, where τ_h is the censoring time beyond which the experiment will not be run and r_h is a prespecified number of failures.

An ASP(H) can be constructed by determining n_h and r_h given τ_h. The (n_h, r_h) plan under Hybrid censoring was constructed by Epstein(1954). However, his plans are approximate in that r_h is determined as if the life test were conducted under Type II censoring and the sample size is determined by $n_h = r_h/(1 - e^{-\tau_h/C})$ where $C = \chi^2_{\alpha,2r_h}/(2r_h)$. In this article, exact plans are developed based upon the following.

Since the probability of an item failing before τ_h is given by $p_h = 1 - e^{-\tau_h/\theta}$,

$$\begin{aligned} \Pr(R = k|\theta) &= B(k; n_h, p_h), \quad k = 0, 1, \cdots, r_h - 1 \\ \Pr(R = r_h|\theta) &= 1 - \sum_{k=0}^{r_h-1} B(k; n_h, p_h). \end{aligned}$$

Therefore,

$$L(\theta) = \Pr(\text{Accept } H_0|\theta)$$

$$= \Pr(R < r_h|\theta) = \sum_{k=0}^{r_h-1} B(k; n_h, p_h).$$

We want to find (n_h, r_h) for which (2.2) and (2.3) hold. However, under Hybrid censoring, it is not generally possible to obtain a plan which statisfies (2.2) and (2.3) exactly. Therefore, the smallest n_h and the corresponding r_h are determined such that $L(1) \geq 1 - \alpha$ and $L(\theta_1) \leq \beta$. Exact plans under Hybrid censoring are given in Table 3.1 for the following combinations of parameters.

$$\begin{aligned} (\alpha,\ \beta) &= (0.01,\ 0.01),\ (0.01,\ 0.05),\ (0.05,\ 0.01),\ (0.05,\ 0.05) \\ &\quad (0.10,\ 0.10),\ (0.10,\ 0.25),\ (0.25,\ 0.10),\ (0.25,\ 0.25) \\ \tau_h &= 1/3,\ 1/5,\ 1/10,\ 1/20 \\ \theta_1 &= 2/3,\ 1/2,\ 1/3,\ 1/5,\ 1/10. \end{aligned} \tag{3.3}$$

The expected number of failures to reach a decision is given by

$$\begin{aligned} E_\theta(R) &= \sum_{k=0}^{r_h} k \Pr(R = k|\theta) \\ &= r_h\Big[1 - \sum_{k=0}^{r_h-1} B(k; n_h, p_h)\Big] + \sum_{k=0}^{r_h-1} kB(k; n_h, p_h), \end{aligned}$$

and the expected completion time to reach a decision by (Epstein, 1954)

$$E_\theta(T) = \sum_{k=1}^{r_h} \Pr(R = k|\theta) E_\theta(X_{k,n_h}),$$

where

$$E_\theta(X_{k,n_h}) = \theta \sum_{j=1}^{k} \frac{1}{n_h - j + 1}.$$

We now illustrate with an example how to utilize Table 3.1. Assume that we want to test the following hypothesis on the mean lifetime θ' with $\alpha = \beta = 0.05$.

$$H_0 \;:\; \theta'_0 = 1000 \; hrs$$
$$H_1 \;:\; \theta'_1 = 500 \; hrs.$$

Furthermore, suppose that the amount of testing time(τ'_h) available is 100 hrs. In terms of standardized parameters, we have $\theta_1 = \theta'_1/\theta'_0 = 1/2$ and $\tau_h = \tau'_h/\theta'_0 = 1/10$. Then, from Table 3.1, we obtain (n_h, r_h) values as (175, 24). The corresponding life test procedures and decision rules are as follows.

(1) Place $n_h = 175$ items on test at time 0.

(2) If 24 items fail before $\tau'_h = 100$ hrs, then reject H_0. Otherwise (i.e., if less than 24 items fail until τ'_h), terminate the test at τ'_h and accept H_0.

3.3. ASP under Time-Truncated Type II Censoring

The life test procedures and decision rules for ASP(T) are as follows.

(1) randomly select n_t items from a lot and place them on test at time 0.

(2) If the number of failures until a predetermined time τ_t is less than a predetermined number of failures r_t, then accept the lot.

(3) If r_t failures occur before τ_t, stop testing at the r_t-th failure, and compute the maximum likelihood estimate $\hat{\theta}_{r_t,n_t}$ of θ as shown in (3.1). If $\hat{\theta}_{r_t,n_t} > C_t$ where C_t is a constant, then accept the lot. Otherwise, reject it.

Let X_{r_t,n_t} be the r_t-th failure time in a sample of size n_t. Then (Fertig and Mann,1980),

$$\begin{aligned} L(\theta) &= \Pr(\text{Accept } H_0|\theta) \\ &= \Pr(R < r_t|\theta) + \Pr(R \geq r_t|\theta)\Pr(\hat{\theta}_{r_t,n_t} \geq C_t|R \geq r_t, \theta) \end{aligned}$$

$$= \Pr(R < r_t|\theta) + \Pr(\hat{\theta}_{r_t,n_t} \geq C_t|\theta)$$
$$- \Pr(R < r_t|\theta)\Pr(\hat{\theta}_{r_t,n_t} \geq C_t|R < r_t, \theta).$$

Or, equivalently

$$L(\theta) = \Pr(\hat{\theta}_{r_t,n_t} \geq C_t|\theta) + \Pr(R < r_t|\theta)\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta). \quad (3.4)$$

In ASP(T), r_t and C_t values are set equal to those for ASP(II). However, these r_t and C_t values do not satisfy the risk requirements in general because of the second term in (3.4). With the proper choice of n_t, however, the differences can be made negligible. To see this, note that if the process is operating at $\theta = 1$, then the probability of acceptance becomes

$$\Pr(\text{Accept } H_0|1)$$
$$= \Pr(\hat{\theta}_{r_t,n_t} \geq C_t|1) + \Pr(R < r_t|1)\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta = 1)$$
$$= (1 - \alpha) + \Pr(R < r_t|1)\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta = 1). \quad (3.5)$$

This probability is strictly greater than $1 - \alpha$, which implies that the producer risk criterion is automatically satisfied if one accepts or rejects the lot based on the assupmtion of Type II censoring. The consumer risk is given by

$$\Pr(\text{Accept } H_0|\theta_1) = \beta' + \Pr(R < r_t|\theta_1)\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta_1)$$
$$< \beta' + \Pr(R < r_t|\theta_1), \quad (3.6)$$

where $\beta' = \Pr(\hat{\theta}_{r_t,n_t} \geq C|\theta_1)$ is the actual consumer risk of the corresponding ASP(II) with failure number r_t. From (3.6), we note that the consumer risk of an ASP(T) is strictly greater than that of the corresponding ASP(II).

We wish to require that the producer and consumer risks of an ASP(T) be as close to those of the corresponding ASP(II) as possible. First, consider the consumer risk. The term $\Pr(R < r_t|\theta_1)$ in (3.6) can be made arbitrarily small with the proper choice of n_t. To show this, note that

$$\Pr(R < r_t|\theta) = \sum_{i=0}^{r_t - 1} B(i; n_t, p_t),$$

where $p_t = 1 - e^{-\tau_t/\theta}$. Then, for a given τ_t we can find the smallest value of n_t that satisfies

$$\Pr(R < r_t|\theta_1) < \epsilon, \tag{3.7}$$

where ϵ is a small constant. We cannot take an arbitrarily small ϵ since that results in a large n_t, and therefore, the cost for testing may be increased. As a compromise, ϵ is taken to be 0.001. With $\epsilon = 0.001$, we calculated n_t values for the combinations of α, β, τ_t, and θ_1 in (3.3), and evaluated $\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta = 1)$ in (3.5) for each combination by Monte Carlo simulation. The results indicate that $\Pr(\hat{\theta}_{r_t,n_t} < C_t|R < r_t, \theta = 1)$ is almost negligible for all combinations considered, implying that the producer risk can be also made very close to α with $\epsilon = 0.001$. In summary, the risk requirements of an ASP(T) can be made very close to those of the corresponding ASP(II) by taking n_t appropriately.

Table 3.2 illustrates n_t, r_t and C_t values for ASP(T) with $\epsilon = 0.001$. Parameter values covered are the same as those for ASP(H).

The expected number of failures to reach a decision is given by

$$\begin{aligned} E_\theta(R) &= \sum_{k=0}^{r_t} k \Pr(R = k|\theta) \\ &= r_t\Big[1 - \sum_{k=0}^{r_t-1} B(k; n_t, p_t)\Big] + \sum_{k=0}^{r_t-1} kB(k; n_t, p_t), \end{aligned}$$

and the expected completion time to reach a decision by

$$E_\theta(T) = \sum_{k=1}^{r_t} \Pr(R = k|\theta) E_\theta(X_{k,n_t}),$$

where

$$E_\theta(X_{k,n_t}) = \theta \sum_{j=1}^{k} \frac{1}{n_t - j + 1}.$$

4. COMPARISONS OF ACCEPTANCE SAMPLING PLANS

4.1. Comparison of ASP(II) and ASP(T)

As discussed in Sections 3.1 and 3.3, constructions of ASP(II) and ASP(T) require specifications of α, β, and discrimination ratio $\theta_1 (= \theta'_1/\theta'_0)$. ASP(T) additionally requires specifications of censoring time τ_t and ϵ.

Sample size n_t of an ASP(T) is chosen to make the consumer risk approximately equal to that of the corresponding ASP(II). To compare the two ASP's on an equal basis, the sample size of ASP(II) is set equal to n_t.

The power of an ASP(II) at a θ can be calculated based on (3.2). Although, we were unable to analytically determine the power of an ASP(T), the difference in powers of the two plans at a θ is believed to be negligible since the consumer and the producer risks of the two were made approximately the same as discussed in Section 3.3.

ASP(II) and ASP(T) are compared in terms of the expected number of failures and expected completion time to reach a decision. Table 4.1 shows part of the results for the cases where $\alpha = \beta = 0.05$, $\theta_1 = 2/3$ or $1/10$, and $\tau_t = 1/3$ or $1/20$. Table 4.1 and the computational results for other combinations of parameter values indicate that $E_\theta(R)$ and $E_\theta(T)$ for ASP(T) are smaller than those for ASP(II) at all values of θ considered. Furthermore, as the discrimination ratio θ_1 and τ_t decrease the superiority of ASP(T) becomes more apparent. This may be explained as follows. As the discrimination ratio decreases a smaller sample size is required, and for a smaller τ_t the possibility of termination by censoring increases for an ASP(T).

4.2. Comparison of ASP(II) and ASP(H)

To compare the two plans on an equal basis, the sample size of an ASP(II) is set equal to that of the corresponding ASP(H). Then, the two plans are compared with respect to $E_\theta(R)$, $E_\theta(T)$ and power. Table 4.2 shows the results for the cases where $\alpha = \beta = 0.05$, $\theta_1 = 2/3$ or $1/10$, and $\tau_h = 1/3$ or $1/20$.

In general, the power of an ASP(II) is better than that of the corresponding ASP(H) at each value of θ considered.

When θ is close to 1, ASP(H) yields smaller $E_\theta(R)$ and $E_\theta(T)$ than the corresponding ASP(II). On the other hand, the trend is reversed when θ is close to θ_1. The reason for these phenomena may be explained as follows. For the parameter values considered, the number of failures(r_h) for an ASP(H) in most cases is larger than the number of failures(r) for the corresponding ASP(II), although this could not be analytically proved. As θ approaches to θ_1, both plans tend to be completed earlier. Especially, termination by time censoring in ASP(H) tends to be less frequently realized. Since r_h is in general greater than r, ASP(H) tends to require larger $E_\theta(R)$ and $E_\theta(T)$ in this case.

As the discrimination ratio decreases and/or τ_h decreases, the range of θ over which ASP(H) yields smaller $E_\theta(R)$ and $E_\theta(T)$ is widened. This is because as the discrimination ratio decreases a smaller sample size is required, and as τ_h decreases the possibility of termination by time censoring increases for an ASP(H).

4.3. Comparision of ASP(T) and ASP(H)

Since the exact power of ASP(T) could not be calculated, we were unable to compare the two plans in terms of the power. However, an indirect comparision could be made since the power of an ASP(T) is believed to be approximately equal to the corresponding ASP(II).

We compared the two plans in terms of $E_\theta(R)$ and $E_\theta(T)$. Table 4.3 shows the results for the cases where $\alpha = \beta = 0.05$, $\theta_1 = 2/3$ or $1/10$, and $\tau_h = \tau_t = 1/3$ or $1/20$.

For the parameter values considered, an ASP(T) yields a smaller $E_\theta(T)$ than the corresponding ASP(H). The number of failures r_t for an ASP(T) is set equal to r for the corresponding ASP(II) , and as discussed in Section 4.2, $r_t = r < r_h$ for the most cases considered. In addition, the sample size n_t for an ASP(T) is in general greater than the sample size n_h for the corresponding ASP(H). Since censoring times τ_h and τ_t are made the same to compare the two plans on an equal basis, ASP(T) tends to be terminated earlier than ASP(H).

For $E_\theta(R)$, we observe similar phenomena as in comparing ASP(II) and ASP(H). That is, when the true mean lifetime θ is close to 1, an ASP(H) yields smaller $E_\theta(R)$ than the corresponding ASP(T). However, when θ is close to θ_1 the reverse is true. This may be explained as follows. As θ approaches to 1, the possibility of time truncation increases for both plans. Since the sample size n_h for an ASP(H) is in general smaller than the sample size n_t for the corresponding ASP(T), the observed number of failures before time truncation for an ASP(H) tends to be smaller than those for the corresponding ASP(T). As θ approaches to θ_1, the possibility of time truncation tends to be reduced for both plans. Since $r_t < r_h$ in general, $E_\theta(R)$ for an ASP(T) tends to be smaller than $E_\theta(R)$ for the corresponding ASP(H). One exception to the above arises when θ_1 is small and τ_t(or τ_h) is large, and therefore, n_t becomes equal to n_h. In such cases, $E_\theta(R)$ for ASP(H) is larger for all θ.

Finally, as the discrimination ratio decreases the range of θ over which an ASP(H) yields a smaller $E_\theta(R)$ is widened. This is because as the discrimination ratio decreases r_h becomes close to r_t while $n_h < n_t$ in general, and therefore, the observed number of failures for an ASP(H) tends to be smaller for a fairly small θ. Again, exceptions arise when $n_t = n_h$.

5. CONCLUSION

To provide guidelines for selecting an appropriate acceptance sampling plan when the lifetime is exponetially distributed, this article compares relative performances of those plans that are under Type II censoring, Hybrid censoring, and Time-Truncated Type II censoring in terms of the power, the expected number of failures, and the expected time required to reach a decision.

To compare three ASP's, computational experiments were conducted and the results tabulated. Based upon the computational results, we recommend the following.

When testing time is of primary concern, ASP(T) is recommended because, as seen in computational results, it requires less time on the average than the other plans for all cases considered.

When it is desired to reduce the number of failed items, the test plan to be recommended may vary depending on the discrimination ratio and the true mean lifetime. If the discrimination ratio is small($\theta_1 < 1/2$, say), ASP(H) is recommended. If the discrimination ratio is large ($\theta_1 > 1/2$, say), ASP(H) is preferred when the true mean lifetime θ is close to 1, while ASP(T) is preferred when θ is close to θ_1.

In terms of the power, ASP(II) is better than ASP(H) and is believed to be slightly better than ASP(T). Therefore, when there is no strict limit on testing time and the number of tests items(although this situation may not arise frequently in practice), ASP(II) may be considered.

An important area of furture research may include extending the present study to the case of Weibull or other lifetime distributions.

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Table 3.1. Exact ASP's under Hybrid Censoring.

$\theta_1 = \theta_1'/\theta_0'$	$\tau_h = \tau_h'/\theta_0'$ 1/3		1/5		1/10		1/20	
$\alpha = 0.01$ $\beta = 0.01$								
2/3	136[a]	402[b]	134	611	133	1143	133	2218
1/2	47	122	47	186	46	340	46	656
1/3	20	43	19	61	19	112	19	214
1/5	10	17	10	24	10	43	9	74
1/10	6	8	6	10	5	14	5	26
$\alpha = 0.01$ $\beta = 0.05$								
2/3	102	293	100	442	99	823	99	1596
1/2	36	89	36	135	35	244	35	470
1/3	15	30	16	47	15	81	15	154
1/5	9	14	8	18	8	31	8	57
1/10	5	6	5	8	5	12	4	18
$\alpha = 0.05$ $\beta = 0.01$								
2/3	97	295	96	451	95	843	95	1636
1/2	34	92	33	137	33	256	33	495
1/3	14	32	14	48	13	83	13	159
1/5	7	13	7	18	7	33	7	62
1/10	5	7	4	7	4	12	4	22
$\alpha = 0.05$ $\beta = 0.05$								
2/3	70	206	68	309	67	574	67	1114
1/2	25	64	24	94	24	175	24	338
1/3	10	21	11	34	10	58	10	110
1/5	5	8	6	14	5	21	5	39
1/10	4	5	3	5	3	8	3	14

a : number of failures (r_h), b : sample size (n_h)

Table 3.1 - continued -

$\theta_1 = \theta_1'/\theta_0'$	$\tau_h = \tau_h'/\theta_0'$							
	1/3		1/5		1/10		1/20	
$\alpha = 0.10$ $\beta = 0.10$								
2/3	43	126	42	190	41	350	41	679
1/2	15	38	15	58	15	108	15	209
1/3	7	14	7	21	6	34	6	65
1/5	4	6	4	9	3	12	3	23
1/10	2	2	2	3	2	5	2	9
$\alpha = 0.10$ $\beta = 0.25$								
2/3	26	72	26	111	26	208	25	388
1/2	10	23	10	35	10	65	10	124
1/3	5	9	5	13	5	24	4	36
1/5	3	4	3	6	3	9	3	17
1/10	2	2	2	3	2	4	2	6
$\alpha = 0.25$ $\beta = 0.10$								
2/3	25	77	24	114	23	207	23	403
1/2	9	24	9	37	8	63	8	122
1/3	4	9	4	13	4	24	4	47
1/5	2	3	2	5	2	9	2	17
1/10	2	2	2	3	2	5	1	5
$\alpha = 0.25$ $\beta = 0.25$								
2/3	13	37	12	53	12	100	12	194
1/2	5	12	5	18	5	34	5	65
1/3	3	6	3	8	2	10	2	19
1/5	2	3	2	4	2	6	2	12
1/10	2	2	1	1	1	2	1	3

Table 3.2. ASP's under Time-Trucated Type II Censoring.

$\theta_1 = \theta_1'/\theta_0'$	$\tau_t = \tau_t'/\theta_0'$											
	1/3			1/5			1/10			1/20		
	$\alpha = 0.01$ $\beta = 0.01$											
2/3	133[a]	414[b]	.8094[c]	133	641	.8094	133	1211	.8094	133	2354	.8094
1/2	46	130	.6891	46	199	.6891	46	373	.6891	46	722	.6891
1/3	19	46	.5441	19	69	.5441	19	128	.5441	19	245	.5441
1/5	9	18	.3891	9	26	.3891	9	47	.3891	9	89	.3891
1/10	5	8	.2555	5	10	.2555	5	18	.2555	5	31	.2555
	$\alpha = 0.01$ $\beta = 0.05$											
2/3	99	318	.7811	99	494	.7811	99	935	.7811	99	1818	.7811
1/2	35	103	.6489	35	159	.6489	35	299	.6489	35	579	.6489
1/3	15	38	.4981	15	58	.4981	15	107	.4981	15	207	.4981
1/5	8	16	.3629	8	24	.3629	8	43	.3629	8	83	3629
1/10	4	6	.2053	4	9	.2053	4	15	.2053	4	28	.2053
	$\alpha = 0.05$ $\beta = 0.01$											
2/3	95	307	.8374	95	476	.8374	95	902	.8374	95	1755	.8374
1/2	33	98	.7319	33	151	.7319	33	285	.7319	33	553	.7319
1/3	13	34	.5914	13	52	.5914	13	97	.5914	13	187	.5914
1/5	7	15	.4692	7	22	.4692	7	40	.4692	7	76	.4692
1/10	4	6	.3415	4	9	.3415	4	15	.3415	4	28	.3415
	$\alpha = 0.05$ $\beta = 0.05$											
2/3	67	226	.8079	67	352	.8079	67	669	.8079	67	1303	.8079
1/2	23	74	.6834	23	114	.6834	23	215	.6834	23	419	.6834
1/3	10	28	.5425	10	43	.5425	10	81	.5425	10	156	.5425
1/5	5	12	.3938	5	18	.3938	5	32	.3938	5	62	.3938
1/10	3	5	.2724	3	7	.2724	3	13	.2724	3	24	.2724

a = number of failures(r_t), b = sample size(n_t)
c = critical value(C_t)

Table 3.2 - continued -

$\theta_1 =$ θ_1'/θ_0'	$\tau_t = \tau_t'/\theta_0'$											
	1/3			1/5			1/10			1/20		
$\alpha = 0.10$ $\beta = 0.10$												
2/3	41	149	.8058	41	234	.8058	41	445	.8058	41	870	.8058
1/2	15	53	.6866	15	82	.6866	15	157	.6866	15	306	.6866
1/3	6	20	.5253	6	31	.5253	6	58	.5253	6	113	.5253
1/5	3	8	.3674	3	13	.3674	3	24	.3674	3	46	.3674
1/10	2	4	.2659	2	6	.2659	2	10	.2659	2	19	.2659
$\alpha = 0.10$ $\beta = 0.25$												
2/3	25	100	.7538	25	157	.7538	25	302	.7538	25	590	.7538
1/2	9	37	.6036	9	58	.6036	9	110	.6036	9	216	.6036
1/3	4	15	.4362	4	24	.4362	4	46	.4362	4	89	.4362
1/5	3	8	.3674	3	13	.3674	3	24	.3674	3	46	3674
1/10	2	4	.2659	2	6	.2659	2	10	.2659	2	19	.2659
$\alpha = 0.25$ $\beta = 0.10$												
2/3	23	94	.8526	23	148	.8526	23	283	.8526	23	554	.8526
1/2	8	34	.7445	8	53	.7445	8	102	.7445	8	200	.7445
1/3	4	15	.6338	4	24	.6338	4	46	.6338	4	89	.6338
1/5	2	7	.4806	2	10	.4806	2	19	.4806	2	38	.4806
1/10	1	3	.2877	1	4	.2877	1	7	.2877	1	14	.2877
$\alpha = 0.25$ $\beta = 0.25$												
2/3	12	58	.7932	12	92	.7932	12	177	.7932	12	347	.7932
1/2	5	25	.6737	5	40	.6737	5	77	.6737	5	150	.6737
1/3	2	10	.4806	2	16	.4806	2	32	.4806	2	63	.4806
1/5	1	5	.2877	1	7	.2877	1	14	.2877	1	28	.2877
1/10	1	3	.2877	1	4	.2877	1	7	.2877	1	14	.2877

Table 4.1. Comparisons of ASP(II) and ASP(T) with Respect to Expected Number of Failures and Expected Completion Time ($\alpha = \beta = 0.05$).

(a) $\theta_1 = 2/3$ ($r = r_t = 67$, $C = .8079$).

	$\tau_t = 1/3$ ($n = n_t = 226$)				$\tau_t = 1/20$ ($n = n_t = 1303$)			
	$E_\theta(R)$		$E_\theta(T)$		$E_\theta(R)$		$E_\theta(T)$	
θ	(II)	(T)	(II)	(T)	(II)	(T)	(II)	(T)
1.0000	67	62.57	.3507	.3237	67	61.85	.0528	.0486
.9444	67	64.37	.3312	.3160	67	63.91	.0498	.0475
.8889	67	65.69	.3117	.3046	67	65.46	.0469	.0458
.8333	67	66.49	.2923	.2897	67	66.40	.0440	.0436
.7778	67	66.86	.2728	.2721	67	66.83	.0410	.0409
.7222	67	66.98	.2533	.2532	67	66.97	.0381	.0381
.6667	67	67.00	.2338	.2338	67	67.00	.0352	.0352

(b) $\theta_1 = 1/10$ ($r = r_t = 3$, $C = .2724$).

	$\tau_t = 1/3$ ($n = n_t = 5$)				$\tau_t = 1/20$ ($n = n_t = 24$)			
	$E_\theta(R)$		$E_\theta(T)$		$E_\theta(R)$		$E_\theta(T)$	
θ	(II)	(T)	(II)	(T)	(II)	(T)	(II)	(T)
1.0000	3	1.39	.7833	.3189	3	1.14	.1306	.0484
.8500	3	1.58	.6658	.3126	3	1.31	.1110	.0476
.7000	3	1.81	.5483	.3022	3	1.55	.0914	.0464
.5500	3	2.12	.4308	.2840	3	1.86	.0718	.0441
.4000	3	2.49	.3133	.2500	3	2.29	.0522	.0396
.2500	3	2.86	.1958	.1847	3	2.78	.0326	.0301
.1000	3	3.00	.0783	.0783	3	3.00	.0131	.0131

Table 4.2. Comparisons of ASP(II) and ASP(H) with respect to Power, Expected Number of Failures, and Expected Completion Time ($\alpha = \beta = 0.05$).

(a) $\theta_1 = 2/3$, $\tau_h = 1/3$ ($n = n_h = 206$, $r = 67$, $r_h = 70$, $C = .8079$).

θ	$1 - Power$ (II)	$1 - Power$ (H)	$E_\theta(R)$ (II)	$E_\theta(R)$ (H)	$E_\theta(T)$ (II)	$E_\theta(T)$ (H)
1.0000	.9480	.9552	67	58.29	.3922	.3326
.9444	.8848	.8945	67	60.97	.3704	.3313
.8889	.7683	.7786	67	63.66	.3487	.3283
.8333	.5876	.5960	67	66.13	.3269	.3221
.7778	.3653	.3707	67	68.09	.3051	.3114
.7222	.1650	.1685	67	69.30	.2833	.2954
.6667	.0464	.0486	67	69.84	.2615	.2752

(b) $\theta_1 = 2/3$, $\tau_h = 1/20$ ($n = n_h = 1114$, $r = 67$, $r_h = 67$, $C = .8079$).

θ	$1 - Power$ (II)	$1 - Power$ (H)	$E_\theta(R)$ (II)	$E_\theta(R)$ (H)	$E_\theta(T)$ (II)	$E_\theta(T)$ (H)
1.0000	.9480	.9513	67	54.15	.0612	.0499
.9444	.8848	.8884	67	56.98	.0578	.0497
.8889	.7683	.7712	67	59.84	.0544	.0492
.8333	.5876	.5894	67	62.50	.0510	.0483
.7778	.3653	.3674	67	64.66	.0476	.0466
.7222	.1650	.1685	67	66.07	.0442	.0442
.6667	.0464	.0496	67	66.75	.0408	.0412

(c) $\theta_1 = 1/10$, $\tau_h = 1/3$ ($n = n_h = 5$, $r = 3$, $r_h = 4$, $C = .2724$).

θ	$1 - Power$ (II)	(H)	$E_\theta(R)$ (II)	(H)	$E_\theta(T)$ (II)	(H)
1.0000	.9501	.9750	3	1.42	.7833	.3314
.8500	.9266	.9590	3	1.62	.6658	.3301
.7000	.8865	.9282	3	1.89	.5483	.3274
.5500	.8124	.8642	3	2.25	.4308	.3213
.4000	.6650	.7202	3	2.77	.3133	.3059
.2500	.3657	.3959	3	3.47	.1958	.2602
.1000	.0120	.0118	3	3.99	.0783	.1277

(d) $\theta_1 = 1/10$ $\tau_h = 1/20$ ($n = n_h = 14$, $r = r_h = 3$, $C = .2724$).

θ	$1 - Power$ (II)	(H)	$E_\theta(R)$ (II)	(H)	$E_\theta(T)$ (II)	(H)
1.0000	.9501	.9718	3	.68	.2317	.0496
.8500	.9266	.9578	3	.79	.1969	.0494
.7000	.8865	.9327	3	.95	.1622	.0490
.5500	.8124	.8841	3	1.18	.1274	.0483
.4000	.6650	.7781	3	1.55	.0927	.0465
.2500	.3657	.5206	3	2.17	.0579	.0412
.1000	.0120	.0441	3	2.95	.0232	.0227

Table 4.3. Comparisons of ASP(T) and ASP(H) with respect to Expected Number of Failures and Expected Completion Time ($\alpha = \beta = 0.05$).

(a) $\theta_1 = 2/3$ ($C = .8079$).

	$\tau_h = \tau_t = 1/3$ ($n_t = 226$, $n_h = 206$) ($r_t = 67$, $r_h = 70$)				$\tau_h = \tau_t = 1/20$ ($n_t = 1303$, $n_h = 1114$) ($r_t = 67$, $r_h = 67$)			
	$E_\theta(R)$		$E_\theta(T)$		$E_\theta(R)$		$E_\theta(T)$	
θ	(T)	(H)	(T)	(H)	(T)	(H)	(T)	(H)
1.0000	62.57	58.29	.3237	.3326	61.85	54.15	.0486	.0499
.9444	64.37	60.97	.3160	.3313	63.91	56.98	.0475	.0497
.8889	65.69	63.66	.3046	.3283	65.46	59.84	.0458	.0492
.8333	66.49	66.13	.2897	.3221	66.40	62.50	.0436	.0483
.7778	66.86	68.09	.2721	.3114	66.83	64.66	.0409	.0466
.7222	66.98	69.30	.2532	.2954	66.97	66.07	.0381	.0442
.6667	67.00	69.84	.2338	.2752	67.00	66.75	.0352	.0412

(b) $\theta_1 = 1/10$ ($C = .2724$).

	$\tau_h = \tau_t = 1/3$ ($n_t = 5$, $n_h = 5$) ($r_t = 3$, $r_h = 4$)				$\tau_h = \tau_t = 1/20$ $n_t = 24$, $n_h = 14$) ($r_t = 3$, $r_h = 3$)			
	$E_\theta(R)$		$E_\theta(T)$		$E_\theta(R)$		$E_\theta(T)$	
θ	(T)	(H)	(T)	(H)	(T)	(H)	(T)	(H)
1.0000	1.39	1.42	.3189	.3314	1.14	.68	.0484	.0496
.8500	1.58	1.62	.3126	.3301	1.31	.79	.0476	.0494
.7000	1.81	1.89	.3022	.3274	1.55	.95	.0464	.0490
.5500	2.12	2.25	.2840	.3213	1.86	1.18	.0441	.0483
.4000	2.49	2.77	.2500	.3059	2.29	1.55	.0396	.0465
.2500	2.86	3.47	.1847	.2602	2.78	2.17	.0301	.0412
.1000	3.00	3.99	.0783	.1277	3.00	2.95	.0131	.0227