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Software Reliability Model with Multiple Change-Points

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Abstract

In this paper, we can see that software reliability model has been improved by considering multiple change-points. The condition for the existence of maximum likelihood estimate of the initial error content of a program is given and the maximum likelihood estimations of multiple change-points are derived. We assess the performance of our multiple change-points model on numerical application.

1. Introduction

Many software reliability models are used to evaluate the reliability of complex computer programs. One of the best known of these models was originally proposed by Jelinski and Moranda(JM)(1972). The JM model described the reliability growth during debugging of a computer program. Some drawbacks of this model have been pointed out by Forman and Singpurwalla(1977), Littlewood(1980), Spreij(1985), Joe and Reid(1985) and others. Recently, Zhao(19913) has shown that software reliability models can be improved by introducing the idea of the change-point. In particular, Zhao(1993) discussed reliability modelling with only one change-point.

This paper extends the modelling work in Zhao(1993) to software reliability models with multiple change-points. The multiple change-points problem in software reliability can occur whenever the running environment, testing strategy and the resource allocation during the program error (i.e. program bug) detection process are changed many times. Also, the increasing knowledge of the program, the testing facilities and other random factors can be the causes of the change-points.

We present a condition for the maximum likelihood estimate (MLE) of the initial error content of a program to be finite and suggest that this condition to be tested priori to using our multiple change-points model. Finally, we assess the performance of our model on application to the set of data reported by Musa(1979).

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2. Multiple change-points model

Let $F_1, F_2, \ldots, F_k, F_{k+1}$ be different lifetime distributions with density functions $f_1, f_2, \ldots, f_k, f_{k+1}$ with parameters $\theta_1, \theta_2, \ldots, \theta_k, \theta_{k+1}$, respectively. Also, let $X_1, \cdots, X_{\tau_1}, X_{\tau_1+1}, \cdots, X_{\tau_2}, \cdots, X_{\tau_{k-1}+1}, \cdots, X_{\tau_k}, X_{\tau_k+1}, \cdots, X_n$ be the times between successive failures of the program. The parameters $\tau_1, \tau_2, \ldots, \tau_k$ are change-points which are considered unknown. We give some assumptions to develope our papers.

Assumption 1. When debugging of a computer program starts, the program contains N errors.

Assumption 2. The original N errors have the same distribution F_1. After τ_1 failures are observed, the remaining $(N-\tau_1)$ errors have the distribution F_2. Successively, after $\tau_k-\tau_{k-1}$ failures are observed, the remaining $(N-\tau_k)$ errors have the distribution F_{k+1}.

Assumption 3. The sequences $\{X_1, \cdots, X_{\tau_1}\}, \{X_{\tau_1+1}, \cdots, X_{\tau_2}\}, \cdots, \{X_{\tau_{k-1}+1}, \cdots, X_{\tau_k}\}$, $\{X_{\tau_k+1}, \cdots, X_n\}$ are statistically independent.

We consider the MLEs of the change-points $\tau_1, \tau_2, \cdots, \tau_k$ and other parameters such as the initial error content N and $\theta_1, \theta_2, \ldots, \theta_k, \theta_{k+1}$, of distribution $F_1, F_2, \ldots, F_k, F_{k+1}$, assuming multiple change-points model based on Assumption 1, 2 and 3.

Let $T_1, T_2, \cdots, T_n$ denote the times at which software errors are detected ; then the likelihood function is

$$\begin{aligned} &L(\tau_1, \ldots, \tau_k, N, \theta_1, \ldots, \theta_k, \theta_{k+1} \mid T_1, \ldots, T_n) \\ &= \prod_{i=1}^{n}(N-i+1) \prod_{i=1}^{\tau_1} f_1(T_i|\theta_1)(1-F_1(T_{\tau_1}|\theta_1))^{N-\tau_1} \\ &\times \prod_{i=\tau_1+1}^{\tau_2} f_2(T_i|\theta_2)(1-F_2(T_{\tau_2}|\theta_2))^{N-\tau_2} \\ &\quad \cdots\cdots\cdots \\ &\times \prod_{i=\tau_{k-1}+1}^{\tau_k} f_k(T_i|\theta_k)(1-F_k(T_{\tau_k}|\theta_k))^{N-\tau_k} \\ &\times \prod_{i=\tau_k+1}^{n} f_{k+1}(T_i|\theta_{k+1})(1-F_{k+1}(T_n|\theta_{k+1}))^{N-n}. \end{aligned} \tag{2.1}$$

We focus on k change-points problem in the JM model for describing software failures. As in Jelinski and Moranda(1972), we assume that all errors in a program have the same size and each removal of the detected errors reduces the failure intensity by the same amount. We also assume that $F_1, F_2, \ldots, F_k, F_{k+1}$ are exponential distribution functions with scale parameters $\lambda_1, \lambda_2, \ldots, \lambda_k, \lambda_{k+1}$, respectively. Then we can compute the log likelihood function of the likelihood(2.1) in term of the $X_1, \cdots, X_n$ as follows

$$\begin{aligned}
\ell &\equiv \log L(\tau_1,\cdots,\tau_k, N, \lambda_1, \ldots,\lambda_k,\lambda_{k+1}|X_1,\cdots,X_n) \\
&= \sum_{i=1}^{n} \log(N-i+1) + \tau_1 \log\lambda_1 - \lambda_1 \sum_{i=1}^{\tau_1}(N-i+1)X_i \\
&\quad + (\tau_2-\tau_1)\log\lambda_2 - \lambda_2 \sum_{i=\tau_1+1}^{\tau_2}(N-i+1)X_i + \cdots \\
&\quad + (\tau_k-\tau_{k-1})\log\lambda_k - \lambda_k \sum_{i=\tau_{k-1}+1}^{\tau_k}(N-i+1)X_i \\
&\quad + (n-\tau_k)\log\lambda_{k+1} - \lambda_{k+1} \sum_{i=\tau_k+1}^{n}(N-i+1)X_i \ .
\end{aligned}$$

To derive MLEs for N and $\lambda_1, \lambda_2, \ldots, \lambda_k, \lambda_{k+1}$ we solve the equations $\partial \ell / \partial N = 0$, $\partial \ell / \partial \lambda_i = 0$, $i = 1, \ldots, k+1$. Then, the partial derivatives lead to :

$$\begin{aligned}
\hat{\lambda}_1 &= \frac{\tau_1}{\sum_{i=1}^{\tau_1}(\hat{N}-i+1)X_i}, \quad \hat{\lambda}_2 = \frac{\tau_2-\tau_1}{\sum_{i=\tau_1+1}^{\tau_2}(\hat{N}-i+1)X_i}, \\
&\vdots \\
\hat{\lambda}_k &= \frac{\tau_k-\tau_{k-1}}{\sum_{i=\tau_{k-1}+1}^{\tau_k}(\hat{N}-i+1)X_i}, \quad \hat{\lambda}_{k+1} = \frac{n-\tau_k}{\sum_{i=\tau_k+1}^{n}(\hat{N}-i+1)X_i},
\end{aligned} \tag{2.2}$$

$$\sum_{i=1}^{n} \frac{1}{\hat{N}-i+1} = \hat{\lambda}_1 \sum_{i=1}^{\tau_1} X_i + \hat{\lambda}_2 \sum_{i=\tau_1+1}^{\tau_2} X_i + \cdots + \hat{\lambda}_k \sum_{i=\tau_{k-1}+1}^{\tau_k} X_i + \hat{\lambda}_{k+1} \sum_{i=\tau_k+1}^{n} X_i.$$

The MLEs of the change-points $\tau_1, \tau_2, \ldots, \tau_k$ are the values $\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_k$ which simultaneously satisfy (2.2) for possible values of $\tau_1, \tau_2, \ldots, \tau_k$. Unfortunately, some data sets produce the MLE at infinite values of N and the MLEs of $\tau_1, \tau_2, \ldots, \tau_k$ may not exist. We suggest the following condition for the finite MLE of N

$$
\begin{aligned}
\frac{\sum_{i=1}^{\tau_1}(i-1)X_i}{\sum_{i=1}^{\tau_1}(i-1)} &> \frac{\sum_{i=1}^{\tau_1}X_i}{\tau_1}, \\
\frac{\sum_{i=\tau_1+1}^{\tau_2}(i-1)X_i}{\sum_{i=\tau_1+1}^{\tau_2}(i-1)} &> \frac{\sum_{i=\tau_1+1}^{\tau_2}X_i}{\tau_2-\tau_1}, \\
&\vdots \\
\frac{\sum_{i=\tau_{k-1}+1}^{\tau_k}(i-1)X_i}{\sum_{i=\tau_{k-1}+1}^{\tau_k}(i-1)} &> \frac{\sum_{i=\tau_{k-1}+1}^{\tau_k}X_i}{\tau_k-\tau_{k-1}}, \\
\frac{\sum_{i=\tau_k+1}^{n}(i-1)X_i}{\sum_{i=\tau_k+1}^{n}(i-1)} &> \frac{\sum_{i=\tau_k+1}^{n}X_i}{n-\tau_k}.
\end{aligned}
\tag{2.3}
$$

In Littlewood and Verrall(1981), it is proved that the MLE of N in the JM model is finite if and only if

$$
\frac{\sum_{i=1}^{n}(i-1)X_i}{\sum_{i=1}^{n}(i-1)} > \frac{\sum_{i=1}^{n}X_i}{n}.
$$

The condition given by (2.3) is easily obtained by regarding our multiple change-points model as a extension of the JM model.

The condition (2.3) is equivalent to :

$$
\begin{aligned}
\frac{\sum_{i=1}^{\tau_1}(i-1)X_i}{\tau_1} - \left[\frac{\sum_{i=1}^{\tau_1}X_i}{\tau_1}\right]\left[\frac{\sum_{i=1}^{\tau_1}(i-1)}{\tau_1}\right] &> 0, \\
\frac{\sum_{i=\tau_1+1}^{\tau_2}(i-1)X_i}{\tau_2-\tau_1} - \left[\frac{\sum_{i=\tau_1+1}^{\tau_2}X_i}{\tau_2-\tau_1}\right]\left[\frac{\sum_{i=\tau_1+1}^{\tau_2}(i-1)}{\tau_2-\tau_1}\right] &> 0, \\
&\vdots
\end{aligned}
\tag{2.4}
$$

$$\frac{\sum_{i=\tau_{k-1}+1}^{\tau_k}(i-1)X_i}{\tau_k-\tau_{k-1}} - \left[\frac{\sum_{i=\tau_{k-1}+1}^{\tau_k}X_i}{\tau_k-\tau_{k-1}}\right]\left[\frac{\sum_{i=\tau_{k-1}+1}^{\tau_k}(i-1)}{\tau_k-\tau_{k-1}}\right] > 0,$$

$$\frac{\sum_{i=\tau_k+1}^{n}(i-1)X_i}{n-\tau_k} - \left[\frac{\sum_{i=\tau_k+1}^{n}X_i}{n-\tau_k}\right]\left[\frac{\sum_{i=\tau_k+1}^{n}(i-1)}{n-\tau_k}\right] > 0,$$

i.e. that the least squares regression lines X_i on i, $i=1,\cdots,\tau_1,\tau_1+1,\cdots,\tau_2,\cdots,\tau_k+1,\cdots,n$, have positive slope. Since the calculation of the MLEs requires a numerical procedure which will be performed by a computer program, we suggest that (2.4) should always be tested before the change-points $\tau_1, \tau_2, \cdots, \tau_k$ are computed. In some cases, however, the MLE of N is not a satisfactory point estimator ; it has a serious bias. The difficulty has been discussed in Littlewood and Verrall(1981) and Joe and Reid(1985).

3. Numerical example

The data set compiled by Musa(1979) provides a convenient means of assessing performance of models of software reliability. The data set of 136 failure intervals is listed in Table 1.

Consider three models in this paper: JM model, one change-point model discussed by Zhao(1993) and multiple change-points model with only k=2 change-points. Of course, in case that attempts to capture the very complex reliability behaviour of software, we may consider multiple change-points model having more than 3 change-points.

For each of three models, the following results are obtained. The estimates for parameters (N, λ) in JM model are

$$\hat{N} = 141.90, \quad \hat{\lambda} = .3496882E\text{-}04.$$

In Kubat and Koch(1980), it was shown that the failure data fit the JM model fairly well.

The estimates for (τ, N, λ_1, λ_2) in one change-point model are

$$\hat{\tau}=16, \quad \hat{N}=144.89, \quad \hat{\lambda}_1= .1108633E\text{-}03, \quad \hat{\lambda}_2= .2992507E\text{-}04.$$

In Zhao(1993), it was shown that one change-point model was in favour of the JM model.

Also, the estimates for parameters ($\tau_1, \tau_2, N, \lambda_1, \lambda_2, \lambda_3$) in our two change-points model are

$$\hat{\tau}_1=16,\quad \hat{\tau}_2=78,\quad \hat{N}=164.920,\quad \hat{\lambda}_1= .966922089E-04,\quad \hat{\lambda}_2= .303803918E-04,$$
$$\hat{\lambda}_3= .155144826E-04.$$

We are interested in how well, on the basis of the given set of failure data in Table 1, three models considered here will predict future reliability behaviour. We have employed 3 distinct methods of assessing the predictive ability of the various models.

The first quality-of-prediction measure is the u-plot: it is well known that $U=F_X(X)$ has a uniform distribution on the interval [0,1]. Using this fact, if $\hat{F}_{i+1}(\cdot)$ is true distribution of X_{i+1} then $u_{i+1}=\hat{F}_{i+1}(x_{i+1})$ will be an observation from a U[0,1] distribution. Thus the empirical distribution formed from the u's should be closed to that of U[0,1]. If we observe n_0+n failures, starting to make predictions after the n_0th failure, the plot of

$$\{(u_{n_0+i},\ i/(n+1)),\quad i=1,2,\dots,n\}$$

is the u-plot. The maximum deviation of the u-plot from the identity function is a measure of quality-of-prediction. Figure 4.1 is the u-plots of three models starting to make predictions after 80 failures. Figure 4.1 shows that the predictive ability of our change-points model is better than others.

Table 1. Execution times between successive failures in seconds

3	30	113	81	115	9	2	91	112	15	138
50	77	24	108	88	670	120	26	114	325	55
242	68	422	180	10	1146	600	15	36	4	0
8	227	65	176	58	457	300	97	263	452	255
197	193	6	79	816	1351	148	21	233	134	357
193	236	31	369	748	0	232	330	365	1222	543
10	16	529	379	44	129	810	290	300	529	281
160	828	1011	445	296	1755	1064	1783	860	983	707
33	868	724	2323	2930	1461	843	12	261	1800	865
1435	30	143	108	0	3110	1247	943	700	875	245
729	1897	447	386	446	122	990	948	1082	22	75
482	5509	100	10	1071	371	790	6150	3321	1045	648
5485	1160	1864	4116							

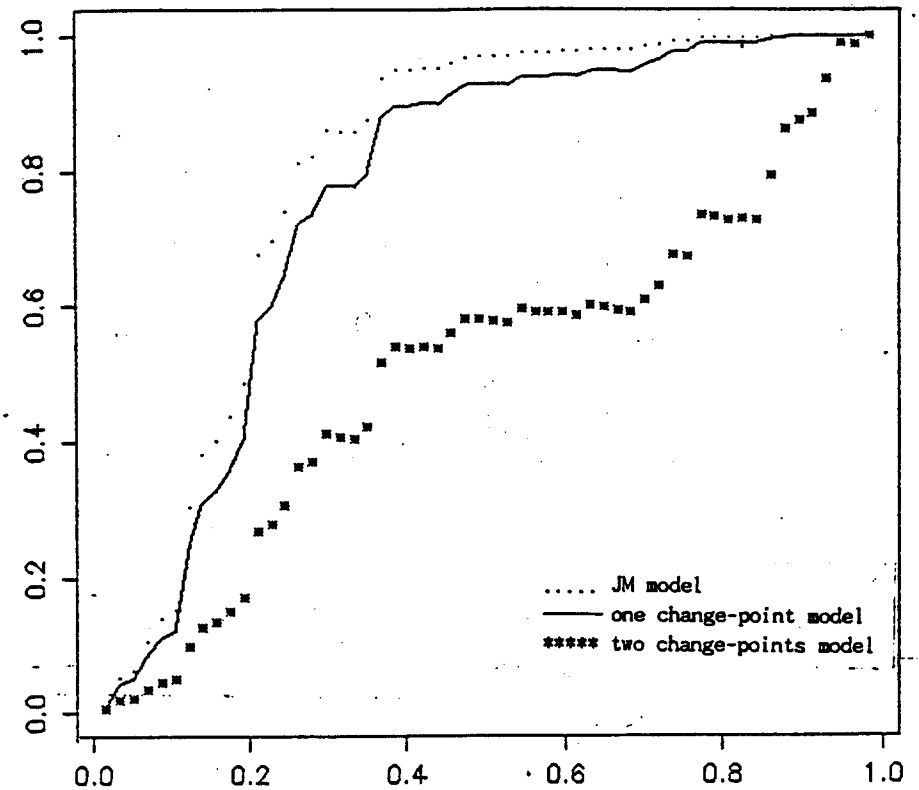

Figure 4.1 u-plots between two change-points model, one change-point model and the JM model

The second measure of quality-of-prediction is the y-plot: if the predictive distributions are good the u's should look like a random sequence of independent U[0,1] variables, and $-\log(1-u)$'s like exponential random variates. In this case, let

$$y_i = \sum_{j=1}^{i} \log(1-u_{n_0+j}) \ / \ \sum_{j=1}^{n} \log(1-u_{n_0+j}), \quad i=1,2,\cdots,n$$

and plot the pairs $\{(y_i,\ i/(n+1)),\ \ i=1,2,\cdots,n\}$. If the model has a good predictive ability, this plot should be close to the identity function. Figure 4.2 is the y-plots of three models starting 80 failures. Our two change-points model seems to be doing slightly better than one change point-model. The predictive ability of one change-point model and the JM model are almost equivalent.

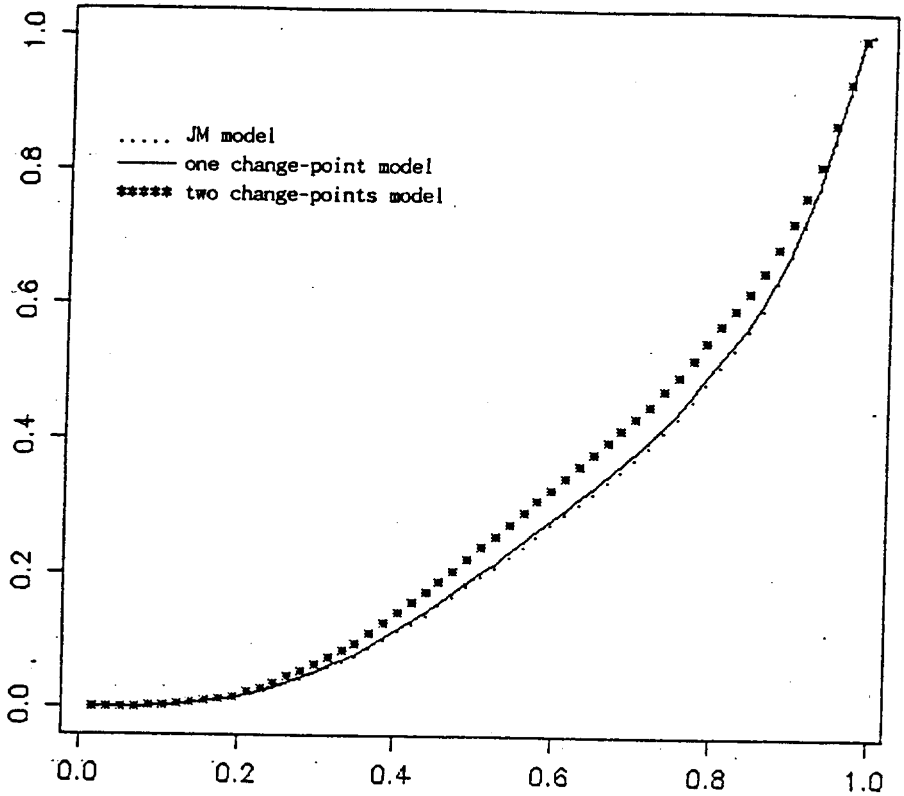

Figure 4.2 y-plots between two change-points model, one change-point model and the JM model

The third measure of quality-of-prediction is the prequential likelihood: based on Dawid(1984)'s generalization of likelihood to a sequential situation, we have

$$PL = \prod_{i=1}^{n} \hat{f}_{n_0+i}(x_{n_0+i}).$$

It can be shown that model A is favour of model B if the prequential likelihood ratio $PL_A \,/\, PL_B$ is consistently increasing as the predicting process. Figure 4.3 is the plot of the log prequential likelihood ratio of two change-points model and one change-point model, two change-points model and the JM model, one change-point model and the JM model, respectively. From Figure 4.3, we can see that our model has the largest prequential likelihood. For a detailed discussion of these three measures of quality-of-prediction, see Abdalla-Ghaly et al.(1986) and Keiller et al.(1983).

From the above discussion, the overall performance of our multiple change-points model is favourable.

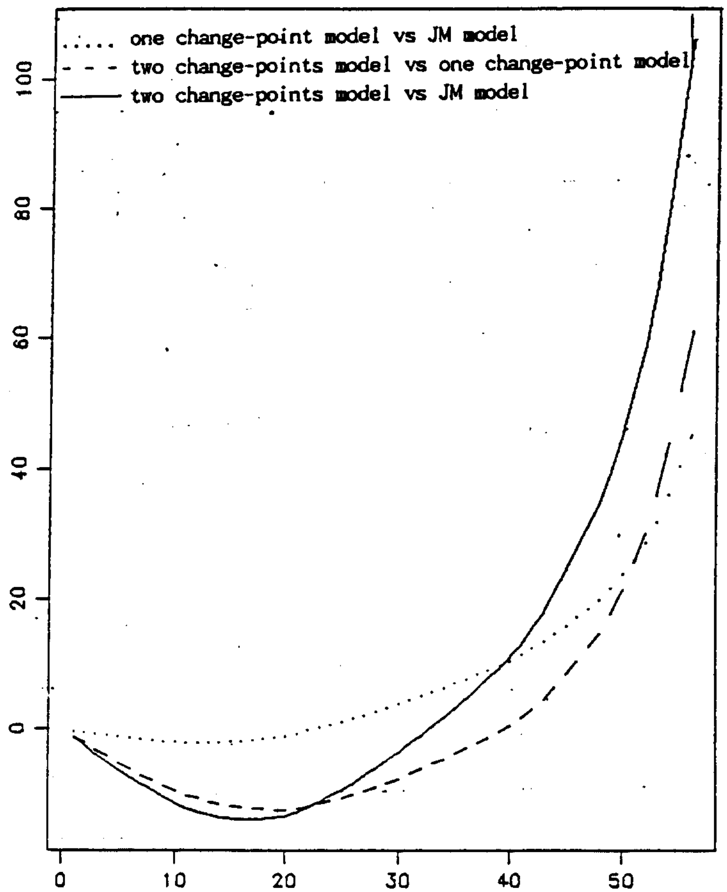

Figure 4.3 log prequential likelihood ratio of two change-points model and one change-point model, two change-points model and the JM model, one change-point model and the JM model.

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소프트웨어 신뢰도 모형에서 다중 변화점 문제

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요약

본 논문은 소프트웨어 신뢰도 모형에서 다중 변화점을 고려함으로서 미래의 관찰치에 대한 예측 성능을 높일 수 있는 새로운 모형에서 프로그램 에러수의 최우추정량이 유한일 조건을 제시하고, 변화점 추정 방법에 대해 논의한다. 또한, 제안된 모형의 타당성을 조사하기 위해 실제 예제를 통하여 모형 성능을 평가한다.

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