응용통계연구 제 7권 2호
1994년 9월 pp. 131-143

A Study on Bayes and Empirical Bayes Estimates of Poisson Means under Asymmetric Loss Functions[1]

Younshik Chung[2] and Chansoo Kim[3]

Abstract

Under the asymmetric losses (entropy loss and Stein loss), we find the classes of Bayes and empirical Bayes estimates for estimating the Poisson means when the distribution of means are believed a priori. Following the idea of Efron and Morris (1973), we have a computer simulation to compute a relative savings loss of proposed estimates as compared to the classical estimates.

1. Introduction

In the simultaneous estimation of mean $\theta = (\theta_1, \cdots, \theta_p)$ from p independent Poisson distributions the normalized squared error(NSEL) loss given in (1.1) is considered,

$$L(\delta,\theta) = \sum_{i=1}^{p} \theta_i^{-m_i}(\theta_i - \delta_i)^2 \qquad (1.1)$$

where m_i's are known nonnegative integers and $\delta = (\delta_1, \cdots, \delta_p)$. Clevenson -Zidek(1975) found estimators improving upon the minimum variance unbiased estimator X when $m_1 = \cdots = m_p = 1$ for $p \geq 2$, and Peng (1975) showed the inadmissibility of X for $p \geq 3$ and the admissibility for $p = 2$ in the case $m_1 = \cdots = m_p = 0$. Also, Tsui and Press (1982) found the improved estimators upon the maximum likelihood estimator X where $m_i > 0$ for some critical value of p. Indeed, Hwang (1982) considered a bigger class and obtained improved estimators for discrete exponential families under the NSEL (1.1). Also, Dey and Chung (1992) found that Clevenson-Zidek's estimator (1975) is robust under the loss (1.1) even when the Poisson distributions are dependent. Ghosh (1983) obtained the Bayes and empirical Bayes estimators under NSEL (1.1) and studied the ''relative savings loss'' of such estimates. But in all the above cases, the losses are symmetric. However, often in some practical problems, the use of asymmetric loss is

1) This research is supported by Korea Science and Engineering Foundation KOSEF # 923-0100-005-1.
2) Department of Statistics, Pusan National University, Pusan 609-735 KOREA.
Internet: yschung@hyowon.pusan.ac.kr
3) Department of Statistics, Pusan National University, Pusan 609-735, KOREA.

appropriate. In view of that we consider a loss based on the entropy distance (or Kullback-Leibler information number) between two distributions of p independent Poisson random variables, as indicated in Ghosh and Yang (1988). Ghosh and Yang(1988) obtained the simultaneous estimation of Poisson means under the entropy loss (1.2) which is defined as follows;

$$
\begin{aligned}
L_E(\theta,\delta) &= E_\theta\left[\ln \frac{\prod_{i=1}^{p} \frac{e^{-\theta_i}\theta_i^{x_i}}{x_i!}}{\prod_{i=1}^{p} \frac{e^{-\delta_i}\delta_i^{x_i}}{x_i!}}\right] \\
&= \sum_{i=1}^{p}\left(\delta_i - \theta_i - \theta_i \ln \frac{\delta_i}{\theta_i}\right) \qquad (1.2) \\
&= \sum_{i=1}^{p}\theta_i\left(\frac{\delta_i}{\theta_i} - \ln \frac{\delta_i}{\theta_i} - 1\right).
\end{aligned}
$$

where $\ln$ denotes the natural logarithm.

Since $\ln 0$ is not defined in loss (1.2), we consider the estimator (a correction version) $\delta^b(X) = X + b$, where $b = (b_1, \cdots, b_p)$, $b_i > 0$, $i = 1, \cdots, p$. Such a correction to the unbiased estimator or the maximum likelihood estimator is quite common for the binomial and the Poisson distributions when one observes zero counts. Using asymptotic considerations, Anscombe (1956) derived such a correction. From Ghosh and Yang (1988), it follows that $\delta^b(X)$ is a generalized Bayes rule.

Chung and Dey (1993) extended the Ghosh and Yang's estimators (1988) to a bigger class of improved estimators for Poisson and negative binomial distributions under the loss (1.2). Also, Chung (1992) found the improved estimator like trimmed version under the entropy loss (1.2).

Also Dey and Chung (1991) obtained the simultaneous estimate of the truncated Poisson mean under Stein's loss (1.3) which is given as follows;

$$
L_S(\theta,\delta) = \sum_{i=1}^{p}(\delta_i / \theta_i - \ln(\delta_i / \theta_i) - 1). \qquad (1.3)
$$

The loss (1.3) is first introduced by James and Stein (1961) for the estimation of the multinormal variance-covariance matrix. Later, Dey and Srinivasan (1985) investigated this loss (1.3) for the estimation of the multivariate normal variance-covariance matrix and it's inverse. Condsider p-independent Poisson random variables X_i's with means θ_i's. Since the loss (1.1) is symmetric, whether these estimates is overestimation or understimation, these estimations are penalized equally. The asymmetric losses given in (1.2) and (1.3) often arise in practice when the overestimation and underestimation are penalized differently. In practice, we may have different action according to the estimations since the loss is asymmetric. Also, in the dam construction, underestimation of the peak water level is usually more serious than overestimation. The application of the asymmetric loss for the Poisson mean estimation problem is useful in the software reliability assessment. In the software reliability example, usually the number of errors in a computer program will follow a Poisson distribution, where underestimation of the mean number of errors will involve a large amount of penality to the client.

In section 2, we find the class of the estimates which are Bayes (1st stage and 2nd stage prior) and empirical Bayes under the asymmetric loss (1.2) and (1.3). In section 3, following the idea of Efron and Morris (1973) we have the computer simulation to compute the ''relative savings loss'' of the proposed estimator and Ghosh and Yang's (1988) as compared to the classical estimator.

2. Bayes and empirical Bayes estimators

Consider p independent Poisson random variables $X_1, \cdots, X_p$ with respective parameters $\theta_1, \cdots, \theta_p$ where $\theta_i \in (0,1)$ for each $i=1, \cdots, p$. Consider the prior distribution of θ_i, $i=1, \cdots, p$ as follows.

Conditional on $u \in (0,1)$, $\theta_1, \cdots, \theta_p$ are independent, θ_i having probability density function

$$g(\theta_i|u) = \frac{1}{\Gamma(k_i)\left(\frac{1-u}{u}\right)^{k_i}} \theta_i^{k_i-1} \exp\left(-\theta_i \frac{u}{1-u}\right), \quad \theta_i > 0 , \tag{2.1}$$

where u has marginal probability density function

$$h(u) = u^{\alpha-1}(1-u)^{\beta-1} / B(\alpha,\beta) \ ; \quad \alpha,\ \beta > 0, \tag{2.2}$$

where B is the standard beta function. The main Theorem of this section generates a class of Bayes estimators of $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_p)$. We shall refer to the probability density function given in (2.1) as $\Gamma(k_i, (1-u)/u)$.

2.1. Entropy loss

Theorem 2.1 Under the prior given in (2.1) and (2.2), and the loss L_E given in (1.2), the Bayes estimate of θ is given by $\delta^E(X) = (\delta_1^E(X), \cdots, \delta_p^E(X))$ with

$$\delta_i^E(X) = \left(1 - \frac{k+\alpha}{\sum_{i=1}^{p} X_j + \beta + \alpha + 1 + k}\right)(X_i + k_i), \tag{2.3}$$

where $k = \sum_{i=1}^{p} k_i$.

Proof. First, the joint probability density function of X, θ, u is

$$f(x,\theta,u) = \prod_{i=1}^{p} \frac{e^{-\theta_i}\theta_i^{x_i}}{x_i\,!} \frac{1}{\Gamma(k_i)} \left(\frac{u}{1-u}\right)^{k_i} \theta_i^{k_i-1} e^{-\left(\frac{u}{1-u}\right)\theta_i} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1}(1-u)^{\beta-1}$$

$$= \prod_{i=1}^{p}\left(\frac{e^{\frac{-\theta_i}{(1-u)}} \theta_i^{x_i+k_i-1}}{x_i\,!\,\Gamma(k_i)} \right) u^{k+\alpha-1}(1-u)^{\beta-k-1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

Thus the Bayes estimate of θ is given by $\delta^*(X) = (\delta_1^*(X), \ldots, \delta_p^*(X))$ where $\delta_i^*(X)$ minimizes

$$\int_0^1 \int_0^\infty \cdots \int_0^\infty \left(\delta_i - \theta_i \ln \frac{\delta_i}{\theta_i} - \theta_i\right) f(x,\theta,u)\, d\theta_1 \cdots d\theta_p\, du \tag{2.4}$$

with respect to δ_i for each x. First compute

$$\int_0^\infty \left(\delta_i - \theta_i \ln \frac{\delta_i}{\theta_i} - \theta_i\right) e^{\frac{-\theta_i}{1-u}} \theta_i^{x_i+k_i-1} d\theta_i$$

$$= \delta_i \Gamma(x_i+k_i)(1-u)^{x_i+k_i} - \int_0^\infty \theta_i \ln \frac{\delta_i}{\theta_i} e^{-\frac{\theta_i}{1-u}} \theta_i^{x_i+k_i-1} d\theta_i$$

$$- \Gamma(x_i+k_i+1)(1-u)^{x_i+k_i+1}.$$

So, the integral given in (2.4) can be simplified as

$$\int_0^1 \prod_{j\neq i} \frac{\Gamma(x_j+k_j)(1-u)^{x_j+k_j}}{x_j\,!\Gamma(k_j)} u^{k+\alpha-1}(1-u)^{\beta-k-1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\times \int_0^\infty \left(\delta_i - \theta_i \ln \frac{\delta_i}{\theta_i} - \theta_i\right) \frac{1}{x_i\,!\Gamma(k_i)} e^{-\frac{\theta_i}{1-u}} \theta_i^{x_i+k_i-1} d\theta_i \quad du$$

$$= \prod_{j=1}^{p} \frac{\Gamma(x_j+k_j)}{x_j!\Gamma(k_j)} \int_0^1 (1-u)^{\sum_{j\neq i} x_j + \sum_{j\neq i} k_j} (1-u)^{\beta-k-1} u^{\alpha+k-1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\times \left(\delta_i(1-u)^{x_i+k_i} - \frac{1}{\Gamma(x_i+k_i)} \int_0^\infty \ln \frac{\delta_i}{\theta_i} e^{-\frac{\theta_i}{1-u}} \theta_i^{x_i+k_i} d\theta_i - (x_i+k_i)(1-u)^{x_i+k_i+1}\right) du \tag{2.5}$$

Differentiating (2.5) with respect to δ_i and equating to 0, then solve for δ_i. Thus,

$$\int_0^1 (1-u)^{\sum_{j\neq i} x_j + \sum_{j\neq i} k_j + \beta - k - 1} u^{k+\alpha-1} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\times \left[(1-u)^{x_i+k_i} - \frac{1}{\Gamma(x_i+k_i)} \frac{1}{\delta_i} d\theta_i \right] du = 0,$$

which implies

$$\frac{1}{\delta_i} \int_0^1 u^{k+\alpha-1} (1-u)^{\Sigma X_j + \beta} (x_i + k_i) \ du$$

$$= \int_0^1 u^{k+\alpha-1} (1-u)^{\Sigma X_j + \beta - 1} \ du.$$

Hence

$$\delta_i^E(X) = \left[1 - \frac{k+\alpha}{\sum x_j + \beta + \alpha + 1 + k}\right](x_i + k_i),$$

which complete the proof of the theorem.

Remark 1. Consider the Bayes rule (in one stage prior) under the entropy loss for $p=1$. So, the Bayes rule $\delta(X)$ minimizes

$$\int [\hat{\theta} - \theta \ln \frac{\hat{\theta}}{\theta} - \theta] f(\theta|x) \ d\theta. \tag{2.6}$$

Differentiating (2.6) with respect to $\hat{\theta}$ and equating to zero, we get

$$\int [1 - \frac{\theta}{\hat{\theta}}] f(\theta|x) \ d\theta = 0,$$

which implies

$$\hat{\theta} = \int \theta f(\theta|x) \ d\theta = E(\theta|x) .$$

Hence the first stage Bayes rule is $\hat{\theta} = E(\theta|x)$.

Remark 2. Let $X_1, \cdots, X_p$ be independent Poisson random variables with repective means $\theta_1, \cdots, \theta_p$. Consider the problem of estimating $\theta = (\theta_1, \cdots, \theta_p)$ under the entropy loss $L_E(\theta, \delta)$ given in (1.2). The prior distributions of the θ_i's are assumed to be independent $\Gamma(k_i, (1-u)/u)$, $u \in (0,1)$, where $k_i's$ are nonnegative integers. Then the posterior distribution of θ_i is $\Gamma(x_i + k_i , (1 + u/(1-u))^{-1})$ which is $\Gamma(x_i + k_i , 1-u)$. By Remark 1, the Bayes estimate of θ is given by $\delta(X) = (\delta_1(X), \cdots, \delta_p(X))$ where $\delta_i(X) = E(\theta_i|X_i)$.

Hence

$$\delta_i(X) = (1-u)(x_i+k_i). \tag{2.7}$$

Note that if u is unknown, we estimate u from the data $(x_1, \cdots, x_p)$ under the assumed prior where $X_1, \cdots, X_p$ are marginally independent probability mass function of X_i being given by

$$P(X_i=x_i) = \int_0^\infty \frac{\exp(-\frac{\theta_i}{1-u})\theta_i^{x_i+k_i-1}}{x_i\,!\Gamma(k_i)}\left(\frac{u}{1-u}\right)^{k_i} d\theta_i$$

$$= \binom{x_i+k_i-1}{x_i} u^{k_i}(1-u)^{x_i}, x_i = 0, 1, 2, \cdots$$

Thus the X_i's are marginally distributed as negative binomial and the minimal sufficient statistic for u is $T = \sum_{i=1}^{p} x_i$, where marginally T has the negative binomial probability mass function given by

$$P(T = t) = \binom{t+k-1}{t} u^k(1-u)^t, \qquad t = 0, 1, \cdots$$

where $k = \sum_{i=1}^{p} k_i$.

In view of (2.7), and the minimal sufficiency of T for u, an empirical Bayes estimate of θ is of the form

$$(\ (1-\widehat{u}(t))(x_1+k_1)\ ,\ \cdots\ ,\ (1-\widehat{u}(t))(x_p+k_p)\ ) \tag{2.8}$$

and the estimator given in (2.3) is precisely of this form.

Remark 3. Based on the marginal distribution of T, the minimum variance unbiased estimate of $1-u$ is given by $t/(t+k-1) = 1-(k-1)/(t+k-1)$, while the maximum likelihood estimate of $1-u$ is given by $t/(k+t) = 1-k/(k+t)$. So we have two types of empirical Bayes rules as follows.

$$\delta^{EB_1}(X) = (\ (1-\frac{k-1}{\sum X_j+k-1})(x_1+k_1), \cdots, (1-\frac{k-1}{\sum X_j+k-1})(x_p+k_p))$$

and

$$\delta^{EB_2}(X) = (\ (1-\frac{k}{\sum X_j+k})(x_1+k_1), \cdots, (1-\frac{k}{\sum X_j+k})(x_p+k_p))$$

Remark 4. Ghosh and Yang(1988) say as follows ;
Suppose $\delta^{GY}(X)=\delta^{b}(X)+\phi(X)$ where $\phi(X)=(\phi_1(X), \cdots, \phi_p(X))$ with

$$\phi_i(X)=\frac{-C(X)(X_i+b_i)}{\sum_{j=1}^{p}(X_j+d)} \tag{2.9}$$

where $d>0$ and $C(X)$ is nondecreasing in each coordinate and

i) $\sum b_j = B>1$

ii) $0<C(X)<\dfrac{2(B-1)pd}{pd+1+2(B-1)}$

Then $\delta^{GY}(X)$ dominates $\delta^{b}(X)$ under loss (1.2) in terms of risks. Also, by Remark 2, Ghosh and Yang's estimator $\delta^{GY}(X)$ given by (2.9) is considered as the one of empirical Bayes estimators in (2.8).

2.2. Stein's loss

Theorem 2.2. Under the prior given in (2.1) and (2.2) and the loss L_S given in (1.3), the Bayes estimate of θ is given by $\delta^{S}(X)=(\delta_1^{S}(X), \ldots, \delta_p^{S}(X))$ with

$$\delta_i^{S}(X)=\left(1-\frac{k+\alpha}{\sum x_i+k+\alpha+\beta-1}\right)(x_i+k_i-1). \tag{2.10}$$

Proof: The proof is omitted because it is similar to the proof of the theorem 2.1.

Remark 5. Consider the Bayes rule (in one stage prior) under the Stein loss for $p=1$. So, the Bayes rule $\delta(X)$ minimizes

$$\int\left[\frac{\hat{\theta}}{\theta}-\ln\frac{\hat{\theta}}{\theta}-1\right]f(\theta|x)\, d\theta. \tag{2.11}$$

Differentiating (2.11) with respect to $\hat{\theta}$ and equating to zero, we get the Bayes rule

$$\delta^{SB}(B)=\left[E\left(\frac{1}{\theta}|x\right)\right]^{-1}=(1-u)(x_i+k_i-1).$$

Remark 6. By the Remark 2, $T=\sum_{i=1}^{p}X_i$ is the minimal sufficient statistic for μ and so an empirical Bayes estimate of θ is of the form

$$((1-\widehat{u(t)})(x_1+k_1-1), \cdots, (1-\widehat{u(t)})(x_p+k_p-1)).$$

So we have two types of empirical Bayes rules as follows, by Remark 3,

$$\delta^{SEB_1}(X) = [(1-\frac{k-1}{\sum X_j+k-1})(X_1+k_1-1), \cdots, (1-\frac{k-1}{\sum X_j+k-1})(X_p+k_p-1)]$$

and

$$\delta^{SEB_2}(X)=[(1-\frac{k}{\sum X_j+k})(X_1+k_1-1),\cdots,(1-\frac{k}{\sum X_j+k})(X_p+k_p-1)].$$

3. Study for relative savings loss.

We consider a Bayesian view point analogous to Efron and Morris (1973). Consider once again the independent $\Gamma(k_i,(1-u)/u)$ priors for θ_i's with $k_i \geq 1$ for each $i=1, \cdots ,p$. Now, following Efron and Morris(1973), for any estimator $\delta^*(X)$ of θ, the ''relative savings loss'' is defined by

$$RSL(u,\delta^*) = \frac{\{ r(u,\delta^*)-r(u,\delta) \}}{\{ r(u,\delta_0)-r(u,\delta) \}} \tag{3.1}$$

where $\delta_0(X)$ is the classical estimator and $\delta(X)$ is (one stage) Bayes estimator and $r(u,\delta)$ is the Bayes risk of $\delta(X)$.

To obtain the relative saving loss of any estimate $\delta^*(X)$ over the classical estimate $\delta_0(X)$, it was necessary to use computer simulation studies based on 1000 observations of X per each value of u because these Bayes risks are not expressed in any analytic form. The simulation procedure is described below. At the first stage choose different values of α and β. At the second stage, one observation u from $B(\alpha,\beta)$ is generated. At the third stage, the number p of independent Poisson random variables is chosen. Next, p parameters θ_i are generated randomly from $\Gamma(k_i,(1-u)/u)$ for some k_i. Finally, one observation from each of the p Poisson distributions with parameter θ_i is generated. Then the losses under the relevant loss functions for the estimators are calculated. The whole process is repeated 1000 times and the average Bayes risks are calculated.

Consider the entropy loss and the classical estimator $\delta^1(X)=X+1$. In this case any concerned estimator gives no analytic expression for their Bayes risks. Under the entropy loss, Ghosh and Yang have the improved estimator $\delta^{GY}(X)$ over the classical estimator

$\delta^{b}(X)$. The Ghosh and Yang's estimator is given componentwise by

$$\delta_i^{GY}(X)=[1-\frac{C(X)}{\sum(X_j+d)}](X_i+b_i)$$

with some suitable conditions on $C(X)$ and d. This is one of the empirical Bayes estimators under entropy loss.

In the following tables, we have the relative saving loss for $\delta^{E}(X)$ and $\delta^{GY}(X)$ over $\delta^{b}(X)$ for various values of α, β, b and p.

Table 2.1

		α = 1.0	β = 1.0	
	b = 0.5	b = 1.0	b = 1.5	b = 2.0
p=3 δ^{E}	.2322	.1736	.1205	.0873
δ^{GY}	.6784	.3758	.2982	.2757
p=5 δ^{E}	.2016	.1300	.0887	.0605
δ^{GY}	.4392	.3052	.2651	.2599
p=10 δ^{E}	.1290	.0685	.0406	.0346
δ^{GY}	.3839	.2462	.2251	.2410

In table 2.1, consider $u \sim \mathrm{B}(1,1)$ which is uniform distribution.

Table 2.2

		α = 2.5	β = 1.0	
	b = 0.5	b = 1.0	b = 1.5	b = 2.0
p=3 δ^{E}	.1686	.1399	.0967	.0787
δ^{GY}	.7191	.3970	.3021	.2710
p=5 δ^{E}	.1366	.0947	.0765	.0599
δ^{GY}	.5401	.3236	.2586	.2452
p=10 δ^{E}	.0978	.0610	.0400	.0291
δ^{GY}	.4297	.2702	.2342	.2249

Table 2.3

		$\alpha = 1.0$ $\beta = 2.5$ b = 0.5	b = 1.0	b = 1.5	b = 2.0
p=3	δ^E	.2384	.1538	.1118	.0993
	δ^{GY}	.6013	.3106	.3036	.3371
p=5	δ^E	.1921	.1167	.0866	.0633
	δ^{GY}	.3904	.2625	.2874	.3174
p=10	δ^E	.1507	.0723	.0503	.0318
	δ^{GY}	.2751	.2045	.2430	.2897

Table 2.4

		$\alpha = 2.3$ $\beta = 2.7$ b = 0.5	b = 1.0	b = 1.5	b = 2.0
p=3	δ^E	.1349	.1190	.0811	.0766
	δ^{GY}	.6188	.3362	.2988	.3200
p=5	δ^E	.1370	.0984	.0650	.0606
	δ^{GY}	.4240	.2652	.2646	.2961
p=10	δ^E	.0930	.0647	.0399	.0328
	δ^{GY}	.2941	.2168	.2304	.2771

In table 2.2 - 2.4, we consider the symmetric (Table 2.3), left skewed (Table 2.2) and right skewed (table 2.4) distributions of u according to the different values of α and β.

Finally, using the similar method we have the relative saving loss of $\delta^s(X)$ given by (2.10) over $\delta^b(X) = X + b$ under Stein loss. (See Table 3.1 - 3.4).

Table 3.1

		$\alpha = 1.0$ $\beta = 1.2$ b=1.5	b=2.0	b=2.5	b=3.0
p=2	δ^S	.0188	.0395	.0401	.0511
p=5	δ^S	.0160	.0155	.0181	.0203
p=10	δ^S	.0109	.0099	.0128	.0111

Table 3.2

		α = 2.5		β = 1.2	
		b=1.5	b=2.0	b=2.5	b=3.0
p=2	δ^S	.0157	.0224	.0190	.0240
p=5	δ^S	.0097	.0146	.0128	.0121
p=10	δ^S	.0051	.0076	.0080	.0064

Table 3.3

		α = 1.0		β = 2.5	
		b=1.5	b=2.0	b=2.5	b=3.0
p=2	δ^S	.0150	.0454	.0528	.0456
p=5	δ^S	.0152	.0119	.0100	.0210
p=10	δ^S	.0117	.0113	.0098	.0115

Table 3.4

		α = 2.3		β = 2.7	
		b=1.5	b=2.0	b=2.5	b=3.0
p=2	δ^S	.0137	.0321	.0320	.0431
p=5	δ^S	.0192	.0111	.0169	.0179
p=10	δ^S	.0205	.0120	.0115	.0099

Table 3.1-3.4 shows that the proposed estimate (2nd stage Bayes) $\delta^E(X)$ is significant over $\delta^b(X)$ under the entropy loss L_E and the Ghosh and Yang's estimate $\delta^{GY}(X)$ is more significant than $\delta^E(X)$ in the sense that the RSL is larger. Also in table 3.1-3.4 the 2nd stage Bayes estimate $\delta^S(X)$ is significant but $\delta^S(X)$ is less improved than $\delta^b(X)$ because RSL is very small.

4. References

[1] Anscombe, F. J.(1956). On estimating binomial response relations, *Biometrika*, Vol. 43, 461-464.

[2] Chung, Y.(1992). Trimmed estimates in Simultaneous estimations of Poisson Means under Entropy loss, *Journal of Science*, 35-41, Pusan National University.

[3] Chung, Y. and Dey, K.(1994). Simultaneous estimation of parameters from power series distributions under asymmetric loss, *Journal of the Korean Statistical Society*, Vol. 23, 151-166.

[4] Clevenson, M.L. and Zidek, J.K.(1975). Simultaneous estimation of independent Poisson laws, *Journal of American Statistics Association* , Vol. 70, 698-705.

[5] Dey, D. K. and Chung, Y. (1991). Multiparameter estimation of the truncated power series distributions under the Stein's loss, *Communications in Statistics*, Vol. 20, 309-326.

[6] Dey, D. K. and Chung, Y. (1992). Compound Poisson distribution : Properties and Estimations, *Communications in Statistics*, Vol. 21, 3097-3122.

[7] Dey, D. K. and Srinivasan, C. (1985). Estimation of a covariance matrix under Stein's loss, *Annals of Statistics,* Vol. 13, 1518-1591.

[8] Efron, B. and Morris, C. (1973). Stein's estimation rule and its competitor - an empirical Bayes approach, *Journal of American Statistics Association*, Vol. 68, 117-130.

[9] Ghosh, M. (1983). Estimation of multiple Poisson means : Bayes and empirical Bayes, *Statistics and Decision,* Vol. 1, 183-195.

[10] Ghosh, M. and Yang, M.C.(1988). Simultaneous estimation of Poisson means under Entropy loss, *Annals of Statistics*, Vol. 16, 278-291.

[11] Hwang, J.T.(1982). Improving upon standard estimators in discrete exponential families with application to Poisson and negative binomial laws, *Annals of Statistics*, Vol. 10, 857-867.

[12] James, W. and Stein, C. (1961). Estimation with quadratic loss, *Proceeding Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Vol. 1, 316-379, University of California Press.

[13] Peng, J.C.M.(1975). Simultaneous estimation of the parameters of independent Poisson distribution, *Technical Report* 78, Department of Statistics, Stanford University.

[14] Tsui, K.W. and Press, S.J.(1982). Simultaneous estimation of several Poisson parameters under K-normalized squared error loss, *Annals of Statistics*, Vol. 10, 93-100.

비대칭 손실함수 아래서 포아송평균의 베이즈와 경험적베이즈 추정의 연구[4]

정윤식[5], 김찬수[6]

요약

엔트로피와 스타인손실들을 포함한 비대칭 손실함수아래서 포아송평균이 사전분포함수를 갖는 경우에서 베이즈적, 경험적 베이즈추정치들의 집합을 구하였다. 또한 Efron과 Morris의 접근방법에 의하여 앞에서 제안된 추정치와 기존의 추정치들의 상대절약 손실량을 모의실험을 통하여 비교하였다.

4) 본 연구는 1992년도 한국과학재단 연구비 지원에 의한 결과임. 과제번호 : 923-0100-005-1.
5) (609-735) 부산시 금정구 장전동 산30 부산대학교 자연과학대학 통계학과.
6) (609-735) 부산시 금정구 장전동 산30 부산대학교 자연과학대학 통계학과.