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k-Sample Rank Tests for Umbrella Location-Scale Alternatives

Hee Moon Park[1)]

Abstract

Some rank score tests are proposed for testing the equality of all sampling distribution functions against umbrella location-scale alternatives in k-sample problem. Only the case of known peak ℓ is considered. Under the null hypothesis and a contiguous sequence of umbrella location-scale alternatives, the asymptotic properties of the proposed test statistics are investigated. Also, the asymptotic local powers are compared with each others. The results show that the tests based on the Chen-Wolfe rank analogue statistic are more powerful than others for unequally spaced umbrella location-scale alternatives and robust.

1. Introduction

Let $X_{i1}, \cdots, X_{in}$ be k independent random samples from continuous distribution functions $F_i(x)$, $i=1,\cdots,k$, respectively. In this k-sample model, we want to propose test procedures for testing the null hypothesis

$$H_0 \ : \ F_1(x) = \cdots = F_k(x), \qquad \text{for all} \quad x. \tag{1.1}$$

Mack and Wolfe (1981) considered the umbrella alternatives, which is of the form

$$H_u \ : \ F_1(x) \geq \cdots \geq F_\ell(x) \leq \cdots \leq F_k(x), \qquad \text{for all} \quad x \tag{1.2}$$

with at least one strict inequality. ℓ is called the peak or point of umbrella. Mack and Wolfe proposed test statistic using the sum of an ordinary and a reversed Jonckheere (1954)-Terpstra (1952) statistics according to the direction of the peak of umbrella. Simpson and Margolin (1986) suggested general recursive procedures for the increasing dose-response relation when a downturn in response at high dose is possible. Hettmansperger and Norton (1987) proposed a linear rank test having the maximum local power and the greatest efficiency, when the pattern is specified. Shi (1988) suggested linear rank test which is referred to as the maximin efficient. Chen and Wolfe (1990) extended differently the Mack-Wolfe test to the unknown peak setting and also

1) Statistical Research Institute, College of Natural Science, Seoul National University, Seoul, 151-742, KOREA.

generalized the Chacko (1963) rank test. Park (1993) proposed two types of test. The first is a sum of an ordinary and a reversed linear rank tests according to the direction of the peak of umbrella, and the second is some weighted Mack-Wolfe test statistics. In these articles, they proposed rank tests against only differences in location not including scale parameters. But in usual k-sample problem, the experimenter often encounters with the cases that the variance increases as the mean increases. Hence the experimenter would do better to consider rank test which reacts well to the differences in both location and scale parameters. So, we let $F_i(x) = F((x-\theta_i)/\sigma_i)$, where θ_i and σ_i denote unknown location and scale parameter of ith population, $i=1,\cdots,k$, respectively, and consider the umbrella location-scale alternative, which is of the form

$$H_a : \theta_1 \leq \cdots \leq \theta_l \geq \cdots \geq \theta_k \text{ and } \sigma_1 \leq \cdots \leq \sigma_l \geq \cdots \geq \sigma_k, \text{ for some } l \qquad (1.3)$$

with at least one strict inequality. We want to find rank tests which have good performances on the asymptotic (local) power.

In Section 2, some rank tests based on (random vectors consisting of) the simple linear rank score statistics are proposed, where each of the linear rank score statistics is sensitive to the location alternatives or scale alternatives. Also, the asymptotic properties of the proposed test statistics are investigated under the null hypothesis. To investigate the local asymptotic properties under alternatives, a sufficient condition (which is referred in Shiraishi (1988)) for the contiguity of location-scale alternatives is introduced in Section 3. The local asymptotic powers of the proposed tests are also investigated. In Section 4, the proposed tests are compared by the asymptotic (or empirically estimated) local powers. The results show that the tests based on the Hettmansperger-Norton (1987) statistic are more powerful than others for equally spaced umbrella location-scale alternatives. However, the tests based on the Chen-Wolfe (1990) rank analogue statistic are more powerful for unequally spaced cases and highly stable for all the investigated cases.

2. Proposed tests and their asymptotic null properties

Let $X_{i1},\cdots,X_{in}$ be k independent random samples from continuous distribution functions $F_i(x) = F((x-\theta_i)/\sigma_i)$, where θ_i and σ_i denote unknown location and scale parameter of ith population, $i=1,\cdots,k$, respectively. We want to test the null hypothesis H_0 against the umbrella location-scale alternatives H_a. In order to propose some rank tests having good performances, we would do better to use rank tests sensitive to the differences in both location and scale parameters. Thus we define ranks and two score functions suitably for the umbrella location-scale alternatives. Let R_{ij} be the rank of X_{ij},

$i=1,\cdots,k, j=1,\cdots,n$, among the overall combined observations X_{ij}'s. Setting $N=kn$, let $a_N(\cdot)$ and $b_N(\cdot)$ be real valued functions defined on $\{1,\cdots,N\}$ satisfying

$$\lim_{N\to\infty}\int_0^1\{a_N(1+[uN])-\phi(u)\}^2du=0,\quad 0<u<1 \tag{2.1}$$

and

$$\lim_{N\to\infty}\int_0^1\{b_N(1+[uN])-\psi(u)\}^2du=0,\quad 0<u<1 \tag{2.2}$$

with $[x]$ denoting the largest integer not exceeding x, for some square integrable non-constant score generating functions ϕ and ψ. Furthermore, we assume, for $m=1,\cdots,N$,

$$a_N(N-m+1)=-a_N(m),\qquad b_N(N-m+1)=b_N(m) \tag{2.3}$$

and

$$a_N(1)\le a_N(2)\le\cdots\le a_N([N/2])\le 0,\qquad b_N(1)\ge b_N(2)\ge\cdots\ge b_N([N/2]). \tag{2.4}$$

From the definition of score $a_N(\cdot)$ in (2.3), one can show that $\sum_{m=1}^{N}a_N(m)=0$. Also, we assume that $\sum_{m=1}^{N}b_N(m)=0$ for simplicity of the score for scale alternatives. Then the equations (2.1) through (2.4) imply

$$\phi(1-u)=-\phi(u)\quad\text{and}\quad\psi(1-u)=\psi(u),\qquad\text{for}\quad u\in(0,1)$$

which give

$$\int_0^1\phi(u)du=0\quad\text{and}\quad\int_0^1\phi(u)\psi(u)du=0,\quad\text{for}\quad u\in(0,1).$$

These two scores $a_N(\cdot)$ and $b_N(\cdot)$ are often used in rank tests for the location and scale alternatives. Let

$$S=(S_1,\cdots,S_k)'\quad\text{and}\quad T=(T_1,\cdots,T_k)'$$

with

$$S_i=\frac{\sum_{j=1}^{n}a_N(R_{ij})}{\sqrt{n\sum_{m=1}^{N}a_N^2(m)/(N-1)}}\quad\text{and}\quad T_i=\frac{\sum_{j=1}^{n}b_N(R_{ij})}{\sqrt{n\sum_{m=1}^{N}b_N^2(m)/(N-1)}},$$

for $i=1,\cdots,k$. Then S_i (T_i) is the simple linear rank score statistic sensitive to the difference in location (scale) parameters, $\overline{S}=\sum_{i=1}^{k}S_i/k=0$ and $\overline{T}=\sum_{i=1}^{k}T_i/k=0$. Also, under H_0, $E_0(S)=E_0(T)=0$, $Var_0(S)=Var_0(T)=\Sigma_k$ and $Cov(S_i,T_j)=0$ for all i's and j's, where $E_0(\cdot)$, $Var_0(\cdot)$ and $Cov_0(\cdot,\cdot)$ denote the expectation, the variance-covariance matrix and the covariance, respectively, and $\Sigma_k = I_k - \mathbf{1}\cdot\mathbf{1}'/k$, where I_k is the unit matrix of order k, and $\mathbf{1}=(1,\cdots,1)'$.

The proposed test statistics based on (the random vectors consisting of) the simple linear rank score statistics S_i or T_i, $i=1,\cdots,k$, are as follows

$$LS=\sum_{i=1}^{k}(\hat{S}_i^2+\hat{T}_i^2), \tag{2.5}$$

$$HN^*=\frac{\sum_{i=1}^{\ell}(i-\bar{c})(S_i+T_i)+\sum_{i=\ell+1}^{k}(2\ell-i-\bar{c})(S_i+T_i)}{\sqrt{2}\sqrt{\sum_{i=1}^{\ell}(i-\bar{c})^2+\sum_{i=\ell+1}^{k}(2\ell-i-\bar{c})^2}}, \tag{2.6}$$

$$SH^*=\frac{2S_\ell-(S_1+S_k)+2T_\ell-(T_1+T_k)}{s_1}, \tag{2.7}$$

$$HN=\frac{\sum_{i=1}^{\ell}(i-\bar{c})S_i+\sum_{i=\ell+1}^{k}(2\ell-i-\bar{c})S_i}{\sqrt{\sum_{i=1}^{\ell}(i-\bar{c})^2+\sum_{i=\ell+1}^{k}(2\ell-i-\bar{c})^2}}, \tag{2.8}$$

$$SH=\frac{2S_\ell-(S_1+S_k)}{s_2}, \tag{2.9}$$

$$LN=\sum_{i=1}^{k}(S_i^2+T_i^2), \tag{2.10}$$

where $\hat{S}_1\le\cdots\le\hat{S}_\ell\ge\cdots\ge\hat{S}_k$ ($\hat{T}_1\le\cdots\le\hat{T}_\ell\ge\cdots\ge\hat{T}_k$) are the isotonic regression estimators of $S_1,\cdots,S_k$ ($T_1,\cdots,T_k$) with $\sum_{i=1}^{k}\hat{S}_i/k=0$ ($\sum_{i=1}^{k}\hat{T}_i/k=0$) and $\bar{c}=\{\sum_{i=1}^{\ell}i+\sum_{i=\ell+1}^{k}(2\ell-i)\}/k$. $s_1=2\sqrt{3}, s_2=\sqrt{6}$ for $\ell=1, k$ and $s_1=2$, $s_2=\sqrt{2}$ for $\ell\neq 1, k$. The algorithm to derive the isotonic regression estimators is discussed in Barlow et al. (1972). H_0 is rejected for large values of these test statistics. The characteristics of the proposed test statistics can be stated in the following ways. First, $\sum_{i=1}^{k}\hat{S}_i^2$ (denoted by $\bar{\chi}^2_{\ell,a(R)}$) is the rank statistic given by the rank analogue of the likelihood ratio test for the umbrella location alternatives not considering the umbrella scale alternatives (see Chen and Wolfe (1990)). Similarly, $\sum_{i=1}^{k}\hat{T}_i^2$ (denoted by $\bar{\chi}^2_{\ell,b(R)}$) is the rank statistic for the umbrella scale alternatives not considering the umbrella location alternatives. Thus, LS is the rank statistic given by the sum of two rank analogue statistics for the umbrella location-scale alternatives. Second, HN (HN^*) is the Hettmansperger-Norton (1987) type statistic for the umbrella location (location-scale) alternatives. Note that Hettmansperger and Norton proposed an optimal linear rank test $\sum_{i=1}^{k}(c_i-\bar{c})S_i / \{\sum_{i=1}^{k}(c_i-\bar{c})^2\}^{1/2}$ having the maximum local power and the greatest efficiency, when the patterns are specified. But generally the patterns are unspecified, so Hettmansperger and Norton recommended the weights $c_i=i$, for $i=1,\cdots,\ell$ and $c_i=2\ell-i$, for $i=\ell+1,\cdots,k$. Third SH (SH^*) is the Shirahata (1980) type statistic for the umbrella location (location-scale) alternatives, and SH is called the maximin efficient linear rank test when the peak ℓ is k (that is, for the ordered location alternatives). Finally LN is the sum of two Kruskal-Wallis type statistics having good performance for the general location-scale alternatives (no relations with the location and scale parameters).

Now, we consider the asymptotic distributions of the proposed test statistics under the null hypothesis H_0. From Bartholomew (1959), we have

$$\lim_{n\to\infty} \Pr{}_0\{\ \bar{\chi}^2_{\ell,a(R)} \geq t\} = \sum_{m=2}^{k} p(m,k)\cdot \Pr(\ \chi^2_{m-1}\geq t), \quad \text{for } t>0 \tag{2.11}$$

where $p(m,k)$ is the probability that the isotonic regression estimator $\hat{S}_i$, $i=1,\cdots,k$ leads to exactly m different values and χ^2_w denotes a random variable having the chi-square distribution with w degrees of freedom. Similarly, under H_0, from Shiraishi (1988),

$$\lim_{n\to\infty} \Pr{}_0(LS\geq t) = \Pr{}_0\{\ \bar{\chi}^2_{\ell,a(R)} + \bar{\chi}^2_{\ell,b(R)} \geq t\ \}$$

$$= \sum_{m=3}^{2k} \sum^{*} p(m_1,k)\cdot p(m_2,k)\cdot \Pr(\ \chi^2_{m-2}\geq t), \quad \text{for } t>0 \tag{2.12}$$

where $\sum^{*}$ denotes summation over all possible choices of (m_1,m_2) with $1\leq m_1$, $m_2\leq k$ and $m_1+m_2=m$. Also, HN^* (HN) and SH^* (SH) have asymptotically standard normal distribution and LN has asymptotically a chi-square distribution with $2k-2$ degrees of freedom.

3. Asymptotic properties under the contiguity

In order to investigate the asymptotic properties under local alternatives, consider the sequence of umbrella location-scale alternatives

$$H_{aN}\ :\ F_i(x) = F\left(\frac{x-\delta_i/\sqrt{n}}{e^{\omega_i/\sqrt{n}}}\right), \tag{3.1}$$

where

$$\delta_1\leq\cdots\leq\delta_\ell\geq\cdots\geq\delta_k \quad \text{and}$$

$$\omega_1 \le \cdots \le \omega_\ell \ge \cdots \ge \omega_k, \quad \text{for some} \quad \ell, \tag{3.2}$$

with at least one strict inequality. Let $p_n(x)$ and $q_n(x)$ are the joint density functions of observations X_{ij}'s under H_0 and H_{aN}, respectively, then

$$p_n(x) = \prod_{i=1}^{k} \prod_{j=1}^{n} f(x_{ij}),$$

$$q_n(x) = \prod_{i=1}^{k} \prod_{j=1}^{n} \frac{1}{e^{\omega_i/\sqrt{n}}} f\left(\frac{x_{ij} - \delta_i/\sqrt{n}}{e^{\omega_i/\sqrt{n}}}\right),$$

where $f(x) = F'(x)$ and assume that $f(x)$ is symmetric about 0. Setting $d_i(x,\theta) = f\{(x-\delta_i\theta)/e^{\omega_i\theta}\}/e^{\omega_i\theta}$, and we assume the following Assumption for sufficient condition for contiguity.

Assumption

$$\lim_{\theta \to 0} \int_{-\infty}^{\infty} \left\{ \frac{\sqrt{d_i(x,\theta)} - \sqrt{d_i(x,0)}}{\theta} - \frac{d_i'(x,0)}{2\sqrt{d_i(x,0)}} \right\}^2 dx = 0,$$

where $d_i'(x,0) = -\omega_i f(x) - (\delta_i + \omega_i x) f'(x)$.

With the Assumption, the family of densities $\{q_n(x)\}$ is contiguous to that of densities $\{p_n(x)\}$ as $N \to \infty$, from Shiraishi (1988). Moreover, the following theorem is obtained.

Theorem 1 Suppose that Assumption is satisfied. Then, under H_{aN} and as $n \to \infty$, $(S', T')'$ has asymptotically a multivariate normal distribution with mean $(\nu', \xi')'$ and variance-covariance matrix Γ, where

$$\nu = (\nu_1, \cdots, \nu_k)', \qquad \xi = (\xi_1, \cdots, \xi_k)',$$

$$\nu_i = -(\delta_i - \bar{\delta}) \int_{-\infty}^{\infty} f'(x)\, \phi(F(x))\, dx \,/ C(\phi), \tag{3.3}$$

$$\xi_i = -(\omega_i - \bar{\omega}) \int_{-\infty}^{\infty} x f'(x)\, \psi(F(x))\, dx \,/ C(\psi), \tag{3.4}$$

$$C(\phi) = \left\{ \int_0^1 \phi^2(u) du \right\}^{1/2}, \quad C(\psi) = \left\{ \int_0^1 \psi^2(u) du \right\}^{1/2},$$

$$\Gamma = I_2 \otimes \Sigma,$$

where $A \otimes B$ denotes the Kronecker products of A and B, $\bar{\delta} = \sum_{i=1}^{k} \delta_i / k$ and $\bar{\omega} = \sum_{i=1}^{k} \omega_i / k$.

With $\int_{-\infty}^{\infty} f'(x)\ \phi(F(x))\ dx<0$, $\int_{-\infty}^{\infty} x f'(x)\ \psi(F(x))\ dx<0$, (3.3) and (3.4), we rewrite the null hypothesis H_0 and the umbrella location-scale alternatives H_{aN} ((3.1) and (3.2)) to

$$H_0^* : \nu_i=\xi_i=0 \text{ for all } i=1,\cdots,k \tag{3.5}$$

$$H_a^* : \nu_1\leq \cdots \leq \nu_l \geq \cdots \geq \nu_k \text{ and } \xi_1\leq \cdots \leq \xi_l \geq \cdots \geq \xi_k \text{ for some } l \tag{3.6}$$

with at least one strict inequality and $\sum_{i=1}^{k}\nu_i=\sum_{i=1}^{k}\xi_i=0$.

To investigate the asymptotic properties of the proposed test statistics under the modified hypotheses and Assumption, we get the following results from Bartholomew (1959) and Shiraishi (1988). For $t>0$,

$$\lim_{n\to\infty} \Pr\{\ \bar{\chi}^2_{l,a(R)}\geq t\}=\sum_{m=2}^{k} p(m,k)\cdot \Pr\{\ \chi^2_{m-1}(\ \eta_1^2)\geq t\}, \tag{3.7}$$

$$\lim_{n\to\infty} \Pr(LS\geq t)=\Pr\{\ \bar{\chi}^2_{l,a(R)}+\ \bar{\chi}^2_{l,b(R)}\geq t\}$$

$$=\sum_{m=3}^{2k}\sum^{*} p(m_1,k)\cdot p(m_2,k)\cdot \Pr\{\ \chi^2_{m-2}(\ \eta_2^2)\geq t\}, \tag{3.8}$$

where $\chi^2_w(\ \eta_i^2)$, $i=1,2$, denote random variables having the noncentral chi-square distribution with w degrees of freedom and noncentrality parameters $\eta_1^2=\sum_{j=1}^{k}\nu_j^2$ and $\eta_2^2=\sum_{j=1}^{k}(\ \nu_j^2+\ \xi_j^2)$. The upper 100α percentile of the asymptotic distribution of LS are tabulated in Marcus et al. (1976). Also, for $t>0$, we get

$$\lim_{n\to\infty} \Pr(HN^*\geq t)=1-\Phi(t-\mu_{HN^*}), \tag{3.9}$$

$$\lim_{n\to\infty} \Pr(HN\geq t)=1-\Phi(t-\mu_{HN}), \tag{3.10}$$

$$\lim_{n\to\infty} \Pr(SH^*\geq t)=1-\Phi\{t-(2\nu_l-\nu_1-\nu_k+2\xi_l-\xi_1-\xi_k)\ /s_1\}, \tag{3.11}$$

$$\lim_{n\to\infty} \Pr(SH\geq t)=1-\Phi\{t-(2\nu_l-\nu_1-\nu_k)\ /s_2\}, \tag{3.12}$$

$$\lim_{n\to\infty} \Pr(LN\geq t)=\Pr\{\ \chi^2_{2k-2}(\ \eta_2^2)\geq t\}, \tag{3.13}$$

where

$$\mu_{HN^*}=\frac{\sum_{i=1}^{l}(i-\bar{c})(\nu_i+\xi_i)+\sum_{i=l+1}^{k}(2l-i-\bar{c})(\nu_i+\xi_i)}{\sqrt{2}\sqrt{\sum_{i=1}^{l}(i-\bar{c})^2+\sum_{i=l+1}^{k}(2l-i-\bar{c})^2}}$$

and

$$\mu_{HN}=\frac{\sum_{i=1}^{l}(i-\overline{c})\nu_i+\sum_{i=l+1}^{k}(2l-i-\overline{c})\nu_i}{\sqrt{\sum_{i=1}^{l}(i-\overline{c})^2+\sum_{i=l+1}^{k}(2l-i-\overline{c})^2}}$$

are the expectation of tests HN^* and HN under H_a^*.

4. Asymptotic power comparison

As the asymptotic distributions of the proposed test statistics have different types, we cannot compare all of the proposed tests by the Pitman asymptotic relative efficency. So we calculate the asymptotic powers of tests and compare with other tests. With the fixed common significance level α and the fixed asymptotic power β of the test based on LN, we choose some values of the pair (ν, ξ) satisfying $\eta_2{}^2=\sum_{j=1}^{k}(\nu_j{}^2+\xi_j{}^2)$ and

$$\Pr\{\chi^2_{2k-2}(\eta_2{}^2)\geq \chi^2_{2k-2,\alpha}\}=\beta,$$

where $\chi^2_{2k-2,\alpha}$ is the upper 100α percentile of the central chi-square distribution with $2k-2$ degrees of freedom. Applying the chosen value (ν, ξ) into the formula (3.7) to (3.12) with fixed α, we get the asymptotic powers of the others and compare them. Note that the asymptotic powers of the tests based on LS, HN^*, SH^* and LN are invariant to the changes of chosen values ν_i and ξ_i, $i=1,\cdots,k$. But the asymptotic powers of the tests based on $\overline{\chi}^2_{l,\alpha(R)}$, HN and SH are not, and does not depend on the difference of scale parameters ξ_i, $i=1,\cdots,k$. In other words, the tests based on $\overline{\chi}^2_{l,\alpha(R)}$, HN and SH are tests for the umbrella location alternatives without including scale alternatives. Thus letting $\gamma=\xi_i/\nu_i$ for $i=1,\cdots,k$, the asymptotic powers of the proposed test statistics based on LS, $\overline{\chi}^2_{l,\alpha(R)}$, HN^*, HN, SH^* and SH are given in Table 1 for $\alpha=0.05$; $\beta=0.5$; $\gamma=0.0$, 0.5, 1.0; k-sorts of specified umbrella location-scale alternatives; $k=3$, 4, 5. The asymptotic powers of test based on $\overline{\chi}^2_{l,\alpha(R)}$ and LS are empirically estimated by a small-sample Monte-Carlo simulation from 1,000 samples, which are generated by the IMSL subroutine RNMVN.

Table 1 shows that the asymptotic (or estimated) powers of the tests based on LS, HN^* and SH^* are larger than those of $\overline{\chi}^2_{t,a(R)}$, HN and SH, respectively, except $\gamma = 0.0$ which means the umbrella location alternatives having the same scale parameters. Also we know that the test based on HN^* is more powerful than others for equally spaced umbrella location-scale alternatives, otherwise the test based on LS is more recommended. Since the asymptotic powers based on the test LS are highly stable in the general alternatives, we can conclude that the test based on LS is robust.

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Table 1. Asymptotic Powers of LS, $\bar{\chi}^2_{\ell,\alpha(R)}$, HN^*, SH^*, HN and SH of Level $\alpha=0.05$ Relative to Asymptotic Power $\beta=0.5$ of LN

k	ℓ	ν	γ	LS	$\bar{\chi}^2_{\ell,\alpha(R)}$	HN^*	SH^*	HN	SH
			alternative: $\nu'=(-\nu,\ 2\nu,\ -\nu)$						
3	2	1.034	0.0	.718	.782	.558	.558	.812	.812
		0.766	0.5	.645	.615	.635	.635	.591	.591
		0.731	1.0	.727	.509	.813	.813	.558	.558
			alternative: $\nu'=(-\nu,\ \nu,\ 0.0)$						
3	2	1.790	0.0	.723	.793	.462	.462	.707	.707
		1.600	0.5	.746	.703	.677	.677	.623	.623
		1.267	1.0	.758	.553	.709	.709	.464	.464
			alternative: $\nu'=(-\nu,\ 0.0,\ \nu)$						
3	3	1.790	0.0	.696	.780	.558	.558	.812	.812
		1.600	0.5	.712	.663	.775	.775	.754	.754
		1.267	1.0	.752	.527	.813	.813	.558	.558
			alternative: $\nu'=(-\nu,\ 0.0,\ \nu,\ 0.0)$						
4	3	1.940	0.0	.776	.841	.616	.514	.864	.767
		1.730	0.5	.848	.805	.829	.726	.785	.683
		1.370	1.0	.838	.629	.863	.767	.614	.513
			alternative: $\nu'=(-3\nu,-\ \nu,\ 3\nu,\ \nu)$						
4	3	0.613	0.0	.765	.838	.577	.547	.830	.639
		0.548	0.5	.819	.776	.794	.600	.752	.557
		0.433	1.0	.810	.600	.830	.638	.576	.549
			alternative: $\nu'=(-3\nu,\ \nu,\ 3\nu,\ -\nu)$						
4	3	0.613	0.0	.769	.845	.577	.551	.829	.805
		0.548	0.5	.833	.782	.794	.767	.752	.755
		0.433	1.0	.858	.631	.830	.804	.576	.549
			alternative: $\nu'=(-3\nu,\ -\nu,\ \nu,\ \nu)$						
4	4	0.613	0.0	.745	.827	.615	.577	.863	.829
		0.548	0.5	.776	.753	.830	.794	.790	.752
		0.433	1.0	.815	.575	.863	.830	.614	.576

Table 1 (continued)

k	ℓ	ν	γ	LS	$\bar{\chi}^2_{\ell,\alpha(R)}$	HN^*	SH^*	HN	SH
		alternative:	$\nu' = (-4\nu,\ \nu,\ 6\nu,\ \nu,\ -4\nu)$						
5	3	0.350	0.0	.832	.886	.669	.647	.899	.887
		0.310	0.5	.862	.802	.868	.849	.829	.812
		0.250	1.0	.890	.656	.905	.893	.672	.653
		alternative:	$\nu' = (-2\nu,\ 0.0,\ 3\nu,\ \nu,\ -2\nu)$						
5	3	0.680	0.0	.847	.908	.643	.625	.885	.871
		0.610	0.5	.877	.825	.856	.840	.818	.801
		0.480	1.0	.881	.648	.884	.870	.642	.624
		alternative:	$\nu' = (-8\nu,\ -3\nu,\ 2\nu,\ 7\nu,\ 2\nu)$						
5	4	0.255	0.0	.819	.887	.659	.433	.896	.668
		0.225	0.5	.845	.795	.859	.619	.821	.576
		0.180	1.0	.869	.651	.896	.668	.658	.431
		alternative:	$\nu' = (-3\nu,\ 0.0,\ \nu,\ 3\nu,\ -\nu)$						
5	4	0.650	0.0	.819	.885	.568	.591	.821	.844
		0.580	0.5	.853	.806	.782	.805	.740	.765
		0.460	1.0	.891	.640	.821	.844	.568	.592
		alternative:	$\nu' = (-2\nu,\ -\nu,\ 0.0,\ \nu,\ 2\nu)$						
5	5	0.920	0.0	.763	.853	.659	.578	.897	.830
		0.820	0.5	.829	.794	.865	.792	.829	.750
		0.650	1.0	.851	.608	.896	.832	.659	.576

k-표본 우산형 위치-척도 대립가설에 대한 순위검정법의 연구

박희문[2)]

요약

본 논문에서는 k-표본 문제에서 우산형 위치-척도 대립가설에 대한 순위검정법들을 연구하였다. 위치모수와 척도모수의 변동에 민감한 순위점수에 기초한 검정통계량들을 제안하였다. 우산형 대립가설의 정점이 알려진 경우를 다루었으며 귀무가설과 대립가설하에서의 점근성질도 아울러 조사되었다. 모수들간의 간격이 같지않는 우산형 위치-척도모형에서 Chen-Wolfe의 동위회귀 추정량을 이용한 순위통계량에 의존한 검정법이 효율적이었으며 또한 아주 안정적이었다.

2) (151-732) 서울특별시 관악구 신림동 산 56-1 서울대학교 자연과학대학 통계연구소.