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연속 n중 k의 고장 연결 시스템에 있어서 최적 Redundancy 설계

Optimal Redundancy of the Consecutive k out of n Failure Lines Included or Excluded Sink-Source Parts.

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Abstract

A consecutive k out of n failure lines with sink-source parts is a system of components in sequence such that the system fails whether some k consecutive components are all fail.

Some object, be it a flow, is to be relayed from a source to a sink through a sequence of intermediate stations(components). Now care should be taken as to if the source and the sink are also considered components of the systems, i. e. , whether they serve the same function as the intermediate components(stations).

Such systems are different from ordinary consecutive k- out of n failure lines by adding the on source and sink pole[6]. The main properties of the reliability by the optimal redundancy of consecutive k out of n failure lines are presented under this modification.

1. Introduction

Consider an oil(water) pipeline system[6] with n pump stations. If one pump station is down, the flow of oil(water) would not be interrupted because the neighboring stations could the load. However, when two consecutive pump stations are down, then the oil(water) flow is stopped and the system fails.

This examples are what we call the consecutive components are a cut set[1], [6]. More generally, a system with n components in sequence is called consecutive k out of n failure system if the system fails whenever k consecutive components fail[1], [2], [3], [6]. Some object of this examples is it a flow on signal, a message, is to be relayed from a source to a sink through a sequence of intermediate stations. Now we must take care as to whether the source and the sink are also considered components of the syatems. As an example of the oil pipeline system and telecommunication system[1], [8], source, sink, and the intermediate stations are all the same kind of connected stations in one case and pumping stations in the other.

Thus both source and sink are considered components of the systems. However, in the special case of the oil pipeline system, the system includes the source as a component but excludes the sink if it is true the sink is just a storage unit. Such a system is named a closed line system if only the source is included, and opened line system if both source and sink are part of the system.

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In this paper we research the optimal redundancy of reliability about a closed system and opened system in the consecutive k out of n failure lines.

2. Notation

U_i	:	component(station) $_i$ (U_1 is source pole, and U_n is sink pole)
X_i	:	state of component(system) $_i$ (X_i is state of source pole, X_n is state of sink pole)
r_i	:	reliability of component(station)
Lc	:	closed system in the line
Lo	:	opened system in the line
Ø	:	a permutation of the set {1, ... ,n}
n	:	number of components(stations) in the line
n_i	:	size of cutset $_i$
RLc, RLo	:	reliability of Lc, Lo.
X	:	Vector of component(station) state.
L	:	a consecutive k out n failure line
RL	:	Reliability of L.

3. Reliability Computation of the Line System

Computing the reliability of the line with opened and closed system, all possible combinations of sequences of failed components with size less then k must be specified. [6]Shows a comparatively simple recursive formulation to compute the system reliability of line system. We further suppose that if or not the line system as a whole is operating is completely determined by the state vector X. The derivation of the recursive formulation is subject to the experimementing the first sequence of consecutive $X_i=0$ in the vector. If the number of consecutive $X_i=0$ in the first sequence is greater than or equal k, then the system is failed. It the number of consecutive $X_i=0$ in the first sequence is less than k, then the system reliability is a consecutive k out of n failure line system.

Nows we can get the recursive formula[6] as following[by using the Hwan-Oh table[2]].

$$RLc\ (U_1, \ldots, U_n) = r_1 * RL\ (U_2, \ldots, U_n) = \sum_{y=1}^{n-k+1} \sum_{z=y+1}^{y+k-1} RL\ (r, k, n-z)\ r^{y+1}\ (1-r)^{z-y} + r^{n-k+2} \qquad \text{.......(1)}$$

for the colsed line system case

$$RLo\ (U_1, \ldots, U_n) = r_1 r_n * RL\ (U_2, \ldots, U_{n-1}) = \sum_{y=1}^{n-k+1} \sum_{z=y+1}^{y+k-1} RL\ (r, k, n-z)\ r^{y+2}\ (1-r)^{z-y} + r^{n-k+3} \qquad \text{.......(2)}$$

for the opened line system case

$$RL\ (U_1, \ldots, U_n) = \sum_{y=1}^{n-k+1} \sum_{z=y+1}^{y+k-1} RL\ (r, k, n-z)\ r^{y}\ (1-r)^{z-y} + r^{n-k+1} \qquad \text{.......(3)}$$

for the general k out of n failure system with the excluded source-sink pole

For the equation (1), (2), (3), if and if only

RL (r, k, n-z) = 1, if $k > n-v \geq 0$
RL (r, k, n-z) = 0, if $n-z < 0$

and also

$$\sum_y \sum_z Pr[\Phi(x) = 1 \mid Y = y, Z = z] * Pr[Y = y, Z = z]$$

$$= \sum_{y=1}^{n-k+1} \sum_{z=y+1}^{n+k-1} Pr[\Phi(x) = 1 \mid Y = y, Z = z] * Pr[Y = y, Z = z] + r^{n-k+2}$$

$$= \sum_{y=1}^{n-k+1} \sum_{z=y+1}^{n+k-1} Pr[\Phi(x) = 1 \mid Y = y, Z = z] * (1-r)^{z-y} * r^{y+1} + r^{n-k+2} \quad \text{.......(4)}$$

for the closed line system case, and the opened line system case can be

described are to

$$\sum_{y=1}^{n-k+1} \sum_{z=y+1}^{n+k-1} Pr[\Phi(x) = 1 \mid Y = y, Z = z] * (1-r)^{z-r} * r^{y+2} + r^{n-k+3} \quad \text{.......(5)}$$

4. Optimal Closed Line System

For the equation (1), (2), (3), (4), (5) in Section 3 consecutive k out of n failure structure with the closed and opened line system were considered to i. i. d. components with reliability r. Now let us assume that the reliability of components are mutually different like as $r_1 \leq r_2 \leq \ldots\ldots r_n$. A permutation is optimal if it maximizes the reliability among all permutations. An optimal permutation is invariant if it depends only on the ordering of $\{r_i\}$ but not their values.

Any invariant optimal permutation should have U_n in the first position for $k \geq 2$, for $k = 1$, any permutation is clearly invariant optimal.

This is able to be proven like that : suppose to the contrary that W is an invariant optimal sequence with U_i in the first position and U_n in position j. Let W' be obtained from W by interchanging U_i and U_n. Then the interchange doesn't affect anything if either both or none fail. In the case one works and one is failed then the system having the failed component at the first position fails. Let W_{ij} be obtained from W be deleting the position 1 and setting the component at position j to fail. Then ––

$$RLc(W') - RLc(W) = RLc(W_{1j}) * r_n - RLc(W_{1j}) * r_i > 0, \text{ for } r_n > r_i \quad \text{.......(6)}$$

so W is invariant optimal. $U_{\phi(1)}, U_{\phi(2)}, \ldots, U_{\phi(n)}$ is an invariant optimal permutation for the closed line system if and only if $U_{\phi(1)}, = U_n$ and $U_{\phi(2)}, \ldots, U_{\phi(n)}$ is an invariant optimal permutation for a consecutive k out of (n-1) failure structure.

Theorem 1

An optimal the closed line system is $(1-r_i) = 1$ except for $i = jk+1$, $j = 0, 1, \ldots, L-1$ (L is the smallest integer not less than n/k, then $(1-r_i) = Q^{1/L}$.

(the problem is to determine the set $(1-r) \equiv ((1-r), \ldots, (1-r_n))$, under the constraint that $\prod_{i=1}^{n}(1-r_i) = Q$, a constant, such that an optimal permutation of (1-r) yields the largest reliability for a consecutive k out of n failure line.

Proof

Theorem 1 is trivially true for $n \leq k$.

Hence, assume $n \geq k+1$ for the rest of the proof. Let RLc^* denote an optimal closed line system which has $(1-r_1)$ in the first position. Then the remaining sequence must be an optimal consecutive k out of n failure sequence which can be assumed to be $(1-r)(k, n-1, Q/(1-r_1))$. Let f ≥ 1 denote the integral part of (n-1)/k. Then

$$RLc = r_1 [\ 1-\ (Q/(1-r_1))^{1/f}\ ]^h\ . \qquad \text{.................................}(7)$$

RLc is maximized where $(1-r_1) = Q^{1/L}$ is easily proved.

5. Optimal Opened Line System

We can also permute all components to obtain a new system with possibly different reliability

For k=1, any permutation is invariant optimal permutation. But for k=2, any invariant optimal permutation must have U_n and U_{n-1} in the first and last positione for the opened line system case. This opened line case is similarly with closed line case.

It is invariant optimal for a opened line if and only if $\{ U_{\emptyset(1)}, U_{\emptyset(n)} \} = \{ U_n, U_{n-1} \}$ and $U_{\emptyset(2)}, \ldots, U_{\emptyset(n-1)}$ is an invariant optimal permutation for a consecutive k out of (n-2) failure line,

Theorem 2

An optimal opened consecutive k out of n failure live is $(1-r_i) = 1$ except for i=n and i=jk+1, j=0,1, ... , b-1, where b is the smallest integer not less than (n-1)/k, then $(1-r_i) = Q^{1/(b+1)}$.

Proof

For $n \geq k+1$, since it is alway true for $n \leq k$ in the theorem 2. Let RLo denote an optimal opened consecutive k out of n failure line structure which has $(1-r_i)$ in the first position. Then the remaining sequence must be an optimal consecutive k out of n failure sequence which can be assumed to be $(1-r)(k, n-2, Q/(1-r_1)(1-r_n)$. Let $h \geq 1$ denote the integral part of $(n-1)/k$. Then

$$RLo = r_1 r_n [1 - (Q/(1-r_1)(1-r_n))^{1/h}]^h \qquad \text{..............(8)}$$

So, we can see that RLo is maximized where

$(1-r_i) = Q^{1/(b+1)}$ and $(1-r_n) = Q^{1/(b+1)}$.

(where $Q = \prod_{i=1}^{n}(1-r_i)$)

6. Optimal Redundancy in the Closed and Opened Line System.

Now, consider a sequence of s slots with unit distance separating any two adjacent slots, and a set of α components each of which can transmit is to assign the n components to the s slots to maximize the probability of a successful transmission from 1 to slot s.

Consider an assignment A of a close line system. Let A' be an assignment obtained from A by reassigning all components in positions jk+2, jk+3, ... , b-1 (where b is the smallest integer not less than $(n-1)/k$. These b positions, and position n in the opened line case are named, critical. Assignment A' works if and only if each of its critical positions works.

Failing of any critical position under assignment A' implies the failing of β consecutive positions (or position n for the opened line case) under assignment A, hence assignment A also fails. Therefore A' is at least as good as A and one needs to consider only assignment of the pattern A', which is named basic pattern.

A general consecutive k out of n failure line that has an assignment of the basic pattern can be viewed as a "series-parallel" system by deleting all noneritical positions.

Supposing to $r_i=r$, then an optimal assignment is to assign the n components to the critical positions in numbers as everly possible. However, considering a series-parallel, system with cutsets (Minimal Cutsets or Maximal pathsets) $C_1, C_2, \ldots, C_m$ sizes $n_1, n_2, \ldots, n_m$, respectively, such that $n_m \geq n_{m-1} \geq \ldots \geq n_2 \geq n_1$. Define $Q = \prod_{Ui \in Cj}(1-r_i)$. A result of shows that the reliability of such a system is maximized at an assignment at which $(Q_1, Q_2, \ldots, Q_m)$ is minimized in the sense of majorization. Such an assignment not exist.

7. Conclusion and Summary

What we called closed and opened consecutive k out of n failure line structures are really different from the many systems treated as consecutive k out of n failure systems in the general literature. Even though the reliability formula of line system for the both models are similar, the real reliabilities computed have a different values respectively. In the telecommunication system example and oil(water) pipeline system example[2], [3], [5], [6] source, sink, and the intermediate stations are all the same kind of relayed stations and pumping stations. In the mobile communication system example[1] it is assumed that the source and intermediate stations are all photo-transmitting spacecraft but not the sink (which could be just an antenna). Thus the system includes the source as a component but excludes the sink.

In these examples where the source and sink are also a components of a system, then the general consecutive k out of n failure line system model does not describe the system correctedly since the line system works only if the source(sink) works, regardless of the value of k. Such a system is called a closed and opened line system. Therefore one should choose the right model for line system reliability analysis. Some optimal assignment problems for the new model were considered.

The function of the closed and opened line system is more evenly spread over the whole system rather than emphasizing a successful consecutives from one particular component to another particular component.

Under these line systems, redundacy concept could be extended to the closed and opened line system case through the section 4, 5, and 6. Thus a redundant closed and opened line system had α_i components in position i, where position i is considered working if any of the α_i components work.

The main properties of the optimal redundancy of were preserved and suggested under this modification through this paper.

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