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Downdrag에 의한 군말뚝의 상호작용계수

Vertical Interaction Factors of Pile Groups due to Downdrag

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Abstract

The group effect which causes different downdrag distribution in individual piles within the group was investigated by using a numerical analysis and an analytical study. The interaction factors due to group spacing and total number of piles in a group were estimated by using a three dimensional non-linear finite element approach. Based on the results obtained, it is shown that the interaction factors of pile groups varies remarkably according to the group spacing, a major influencing parameter for the group effect. Also the downdrag prediction by the proposed method was compared with the other analytical methods through an example of calculations.

요 지

본 연구에서는 군말뚝에서 말뚝의 상대적인 위치에 따라 다른 downdrag 하중분포를 발생시키는 그룹효과를 이론적인 해석과 수치해석을 통하여 연구 검토하였다. 3차원 비선형 유한요소 해석으로 말뚝 간격과 갯수에 따른 군말뚝에서의 상호작용계수를 산정하였다. 연구결과 그룹안의 개개 말뚝들은 단독말뚝에 비해서 말뚝 간격에 따라 현저한 상호작용계수의 변화를 보였다. 또한 본 논문에서 제안된 방법에 의한 그룹안에서 말뚝들의 위치에 따른 downdrag 하중산정을 계산예를 통하여 그외 여러 해석적인 방법들과 비교 검토하였다.

1. Introduction

The behavior of pile groups subjected to downdrag is primarily influenced by the three-dimensional interaction between the soil and the piles. Most of the previous research has been conducted by using simplified assumptions for the geometry and material properties. To obtain detailed information on the behavior, however, it is necessary to simulate the three-dimensional geometry and nonlinear behavior of the soil.

For piles embedded in soft clay, substantial yielding may occur at the pil-soil interface. For such a condition, a non-linear analysis is more appropriate and realistic than a linear one. In the past, the non-linear behavior of soils has been examined mainly with the plasticity theory.

The overall objective of the present study is to investigate the interaction factors of pile groups by using a detailed numerical approach and compare it with the other analytical methods through

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an example of calculations.

2. Description and Validation of the Analytical Model

2.1 Model Description

The three-dimensional model includes standard finite element techniques. This approach consists of solving a boundary value problem by determining the displacement function that makes the corresponding functional stationary with respect to nodal unknowns. This leads to the following equilibrium equations.

$$[K]\cdot\{u\}=\{r\} \tag{1}$$

where $[K]$=global stiffness matrix, $\{u\}$=nodal displacements, $\{r\}$=global load vector.

The finite element mesh for a typical case is shown in the Fig. 1. The mesh consists of three-dimensional 8 noded solid hexahedral elements and is assumed to be resting on a rigid layer, and the vertical boundaries at the left- and right-hand sides are assumed to be on rollers to allow downward movement of soil layers due to external surcharge loading. Gravity(self weight) of the surrounding soil is used and the embankment is simulated by applying a surcharge loading to the ground surface.

The pile element is assumed to remain elastic at all times, while the soil is idealized as a linear elastic material or an Extended Drucker-Prager elastoplastic material. This model was selected from among the soil models in the library of ABAQUS.[1] All elements are 8 noded hexahedral. For certain group configurations, the use of symmetry reduced the size of the mesh. The actual size of the mesh is related to the pile length; the lower rigid boundary has been placed at a depth equal to 1.6 pile length and the side boundary is extended laterally to $r_m=2.5\ L\ (1-\nu)$.[12] The mesh was refined in the region surrounding the piles and near the pile head and tip until satisfactory results were reached.

2.2 Model Validation

The validity of the three-dimensional model was first tested by comparing some of the elastic results with the results of the previous elastic studies.

2.2.1 Single Piles

The downdrag force on an endbearing single pile (Fig. 2) was compared with the normalized results provided by Poulos and Mattes.[11] Their analysis was based on Mindlin's equations. The agreement between the two analysis is good. In Fig. 2, Pa is the axial force in the pile, S_o is the final surface settlement, L and d are the pile length and diameter, z is the depth along the pile, and E_s is the soil elastic modulus.

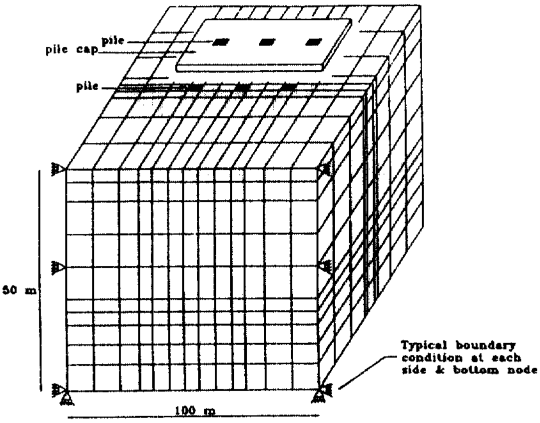


Fig. 1. Typical Finite Element Mesh.

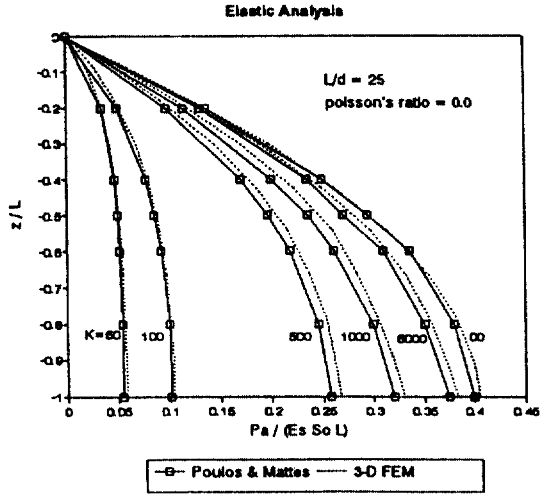
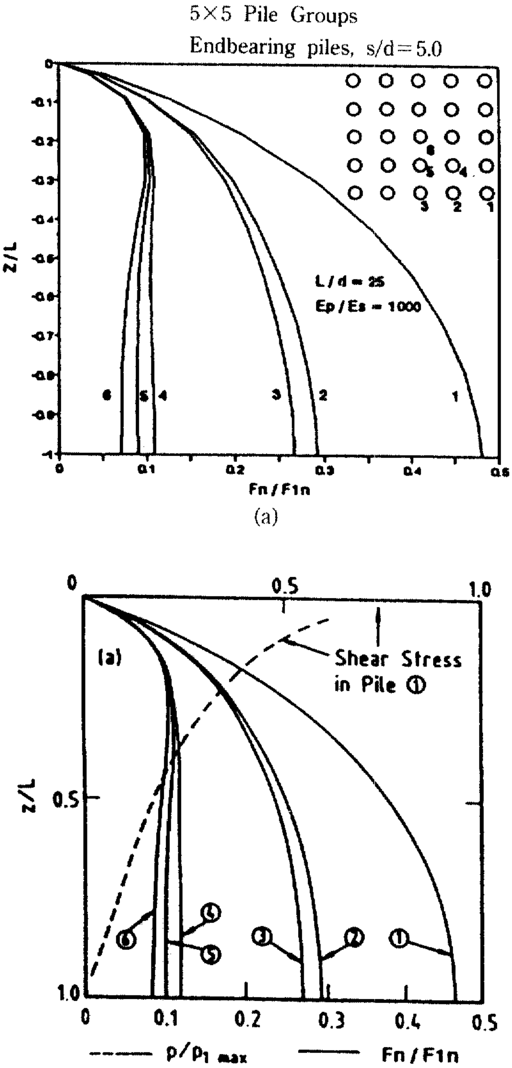


Fig. 2. Comparison of Endbearing Single Pile: Ultimate Axial Force.

2.2.2 Pile Groups

Fig. 3 is a comparison of the solution obtained in this study with the solution presented by Kuwabara and Poulos(9) for endbearing pile groups. They extended the analysis of a single pile to the analysis of pile groups using Mindlin's equation. Fig. 3 shows the distribution of the axial force in a pile group with a perfectly flexible pile cap: 25 piles are arranged in a square configuration with a spacing-to- diameter ratio equal to 5.0. The downdrag force, F_n, is normalized by the maximum downdrag force in a single isolated pile, F_{1n}. Resonably good agreement is obtained between the 3-dimensional solution (Fig. 3-a) and the solution presented by Kuwabara and Poulos (3-b).



(a)

(b)

Fig. 3. Comparison of Endbearing 25 Pile Groups with Flexible Pile Cap: (a) 3-d FEM, (b) Kuwabara and Poulos (1989).

3. Numerical Analysis

The behavior of a pile in a group is influenced by the presence of and loadings on neighboring piles when piles are closely spaced. This is referred to as group effect. A major parameter influencing the group effect is the spacing between piles. Experiments by Koerher and Mukhopadhyay(8) and Ito and Matsui(6) clearly show this influence through the use of small scale experiments: at center to center spacings larger than 5 diameters there is no evidence of group effect while below 2.5 diameters there is a definite group effect.

Based on the above literature review, a series of numerical analyses on pile groups were performed for different endbearing conditions, soil model and two different spacings (2.5, 5.0) between piles (Table 1). The cases of a single pile and of pile groups are analyzed. Table 2 shows the material properties and geometries used in this study.

The elastoplastic analyses were run to take into account the local yielding at the pile-soil interface and used an iterative and incremental analysis. For the solution of nonlinear problems, equation (1) is not satisfied at any stage of the computation

$$\psi=[K]\cdot\{u\}-\{r\}\neq 0 \tag{2}$$

where ψ is the residual force vector. For an elastoplastic state, the material stiffness is continually changing and instantaneously the incremental stress-strain relations is given by

$$\{d\sigma\}=[C^{ep}]\cdot\{d\varepsilon\} \tag{3}$$

where $[C^{ep}]$ is the elastoplastic constitutive matrix.

3.1 Analysis without Yielding

Fig. 4 and 5 present a comparison of the intera-

Table 1. Numerical Analysis for Pile Groups

Group	Endbearing Condition	Soil Model	Spacing (s/d)
1×1 (single)	Endbearing & Friction	Elastic	2.5 & 5
	〃	Elastoplastic	2.5 & 5
1×2 (G)	〃	Elastic	2.5 & 5
	Endbearing	Elastoplastic	2.5 & 5
1×4 (G)	〃	Elastic	2.5 & 5
1×6 (G)	〃	Elastic	2.5 & 5
1×8 (G)	〃	Elastic	2.5 & 5
1×10 (G)	Endbearing & Friction	Elastic	2.5 & 5
2×2 (G)	〃	Elastic	2.5 & 5
	〃	Elastoplastic	2.5 & 5
3×3 (G)	〃	Elastic	2.5 & 5
	〃	Elastoplastic	2.5 & 5
3×10 (G)	〃	Elastic	2.5 & 5
4×4 (G)	〃	Elastic	2.5 & 5
5×5 (G)	〃	Elastic	2.5 & 5
	〃	Elastoplastic	2.5 & 5
5×10 (G)	〃	Elastic	2.5 & 5
6×6 (G)	〃	Elastic	2.5 & 5
	〃	Elastoplastic	2.5 & 5
8×8 (G)	〃	Elastic	2.5 & 5
10×10 (G)	〃	Elastic	2.5 & 5

Notes: 3×10 (G)=3 rows and 10 columns of pile groups, s=center-to-center spacing between piles, d=diameter of pile, G=pile group

Table 2. Material Properties for Pile Groups

Pile	area	0.6 m×0.6 m
	length	30 m embedding depth
	E_p	20×10^6 kN/m² (concrete)
	ν	0.3
Soil	Elastic Properties	
	E_s	20×10^2 kN/m²
	E_b	20×10^4 kN/m²
	ν	0.4
	γ'	9.0 kN/m³
Soil	Plastic Properties	
	ϕ'	25 degree
	β	36.5 degree
	ψ	0 degree
	C′	3.0 kN/m²
Surcharge	q_c	250 kN/m²

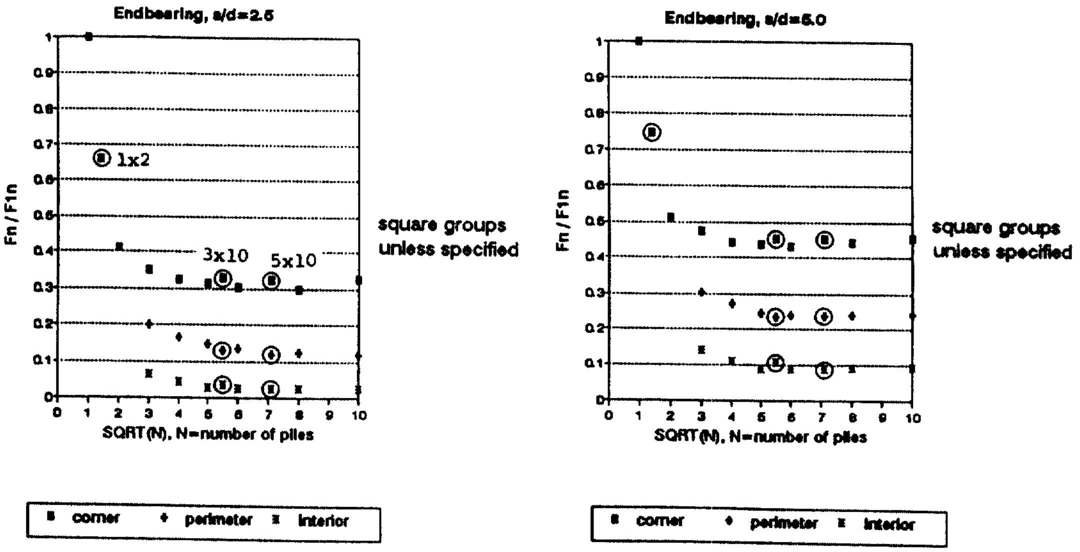
Notes: E_p: Young's modulus of piles; E_s: Young's modulus of soil; E_b: Young's modulus of bearing layer; γ': effective unit wt. of soil; ϕ': internal friction angle of soil; C′: cohesion of soil

ction for the 2.5 d and 5.0 d spacing between piles based on the elastic analysis of pile groups. The results are presented on table 3 in the form of downdrag interaction factors for endbearing and friction piles and for the two ratios. The downdrag interaction factor is defined as the ratio of the downdrag force on a pile in the group to the downdrag force on an isolated pile. It can be seen that the downdrag interaction factors of the piles within the group are much less than that of a single isolated pile.

3.2 Analysis with Yielding

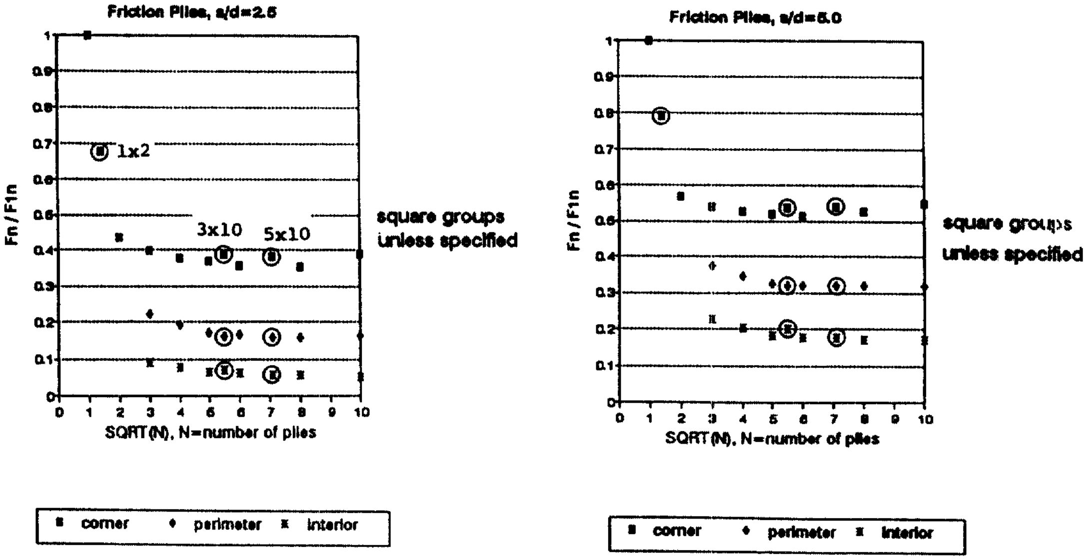
In the previoue section (3.1), all analyses were based on the linear elastic solution. The analysis of downdrag using a linear elastic soil model is of limited value when trying to represent the actual pile-soil-pile behavior. In general, the soil around most of the piles and the soil near the bottom of the endbearing piles are strained well into the plastic range, so that the direct application of an elastic solution is not valid.

To understand the true behavior, yielding of the soil at the pile-soil interface was considered by taking into account the effective strength parameters of a clay: the effective cohesion, C′ and



(a) (b)

Fig. 4. Effect of Group Spacing in Endbearing Piles: (a) s/d=2.5, (b) s/d=5.0.

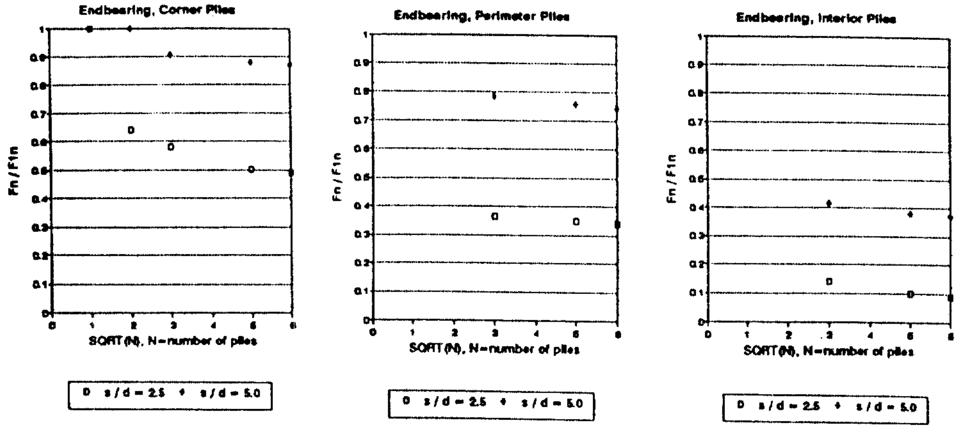


(a) (b)

Fig. 5. Effect of Group Spacing in Friction Piles: (a) s/d=2.5, (b) s/d=5.0.

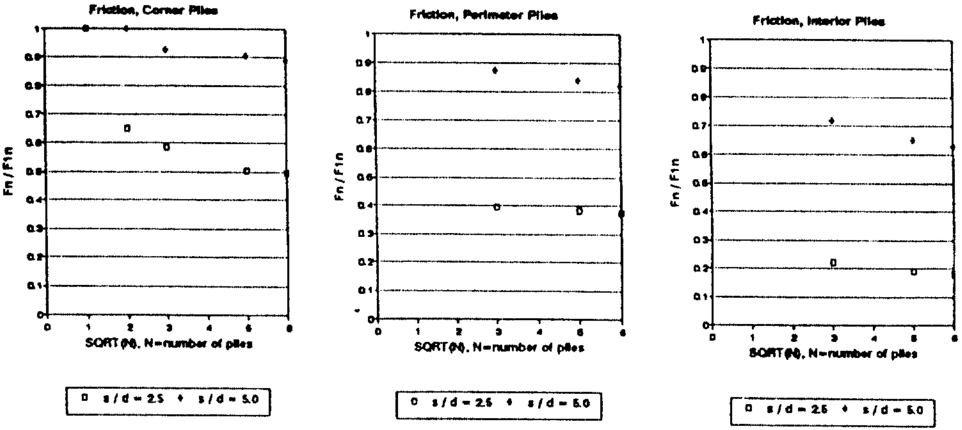
Table 3. Downdrag Interaction Factors (Without Yielding)

Groups	Interaction Factor					
	Endbearing Piles					
	s/d=2.5			s/d=5.0		
	Corner Pile	Perimeter Pile	Interior Pile	Corner Pile	Perimeter Pile	Interior Pile
Single	1.0	–	–	1.0	–	–
2×1	0.66	–	–	0.75	–	–
2×2	0.41	–	–	0.51	–	–
3×3	0.35	0.20	0.06	0.48	0.30	0.14
4×4	0.33	0.17	0.04	0.44	0.27	0.11
5×5	0.32	0.15	0.03	0.44	0.25	0.09
3×10	0.33	0.13	0.04	0.45	0.24	0.11
6×6	0.30	0.13	0.03	0.43	0.24	0.09
5×10	0.32	0.12	0.03	0.45	0.24	0.09
8×8	0.30	0.13	0.03	0.44	0.24	0.09
10×10	0.33	0.12	0.03	0.46	0.24	0.09
	Friction Piles					
	s/d=2.5			s/d=5.0		
	Corner Pile	Perimeter Pile	Interior Pile	Corner Pile	Perimeter Pile	Interior Pile
Single	1.0	–	–	1.0	–	–
2×1	0.68	–	–	0.79	–	–
2×2	0.44	–	–	0.57	–	–
3×3	0.40	0.22	0.09	0.54	0.37	0.23
4×4	0.38	0.19	0.08	0.53	0.34	0.21
5×5	0.37	0.17	0.06	0.52	0.32	0.18
3×10	0.39	0.16	0.07	0.54	0.32	0.20
6×6	0.36	0.17	0.06	0.51	0.32	0.18
5×10	0.38	0.16	0.06	0.54	0.32	0.18
8×8	0.35	0.16	0.06	0.53	0.32	0.17
10×10	0.39	0.16	0.05	0.55	0.32	0.17



(a) (b) (c)

Fig. 6. Effect of Group Spacing in Endbearingk Piles: (a) Corner Piles, (b) Perimeter Piles, (c) Interior Piles.



(a) (b) (c)

Fig. 7. Effect of Group Spacing in Friction Piles: (a) Corner Piles, (b) Perimeter Piles, (c) Interior Piles.

the effective friction angle, φ'.

Fig. 6 and 7 present a comparison of the interaction between piles with different spacings based on the non-linear analysis of pile groups. The results are presented on Table 4 in the form of downdrag interaction factors for endbearing and friction piles and for the two ratios.

The interaction factors on each pile decrease significantly as the pile spacing decreases.

4. Group Effect by Interaction Factors

The interaction factors obtained by the three-dimensional parametric analysis can play a significant role in determining the group effect between piles.

The maximum downdrag force on each pile within the group can be estimated by the product of the interaction factor and the downdrag force on an isolated pile determined from the analytical solution.(7)

5. Example of Calculations

An example of calculations(5) for a group of piles (Fig. 8) is given. This is a group of 12 piles (3×4);

Table 4. Downdrag Interaction Factors (With Yielding)

Groups	Interaction Factor					
	Endbearing Piles					
	s/d=2.5			s/d=5.0		
	Corner Pile	Perimeter Pile	Interior Pile	Corner Pile	Perimeter Pile	Interior Pile
Single	1.0	–	–	1.0	–	–
2×2	0.64	–	–	1.0	–	–
3×3	0.58	0.36	0.14	0.91	0.78	0.42
5×5	0.50	0.35	0.10	0.88	0.76	0.38
6×6	0.49	0.34	0.09	0.87	0.75	0.37
	Friction Piles					
	s/d=2.5			s/d=5.0		
	Corner Pile	Perimeter Pile	Interior Pile	Corner Pile	Perimeter Pile	Interior Pile
Single	1.0	–	–	1.0	–	–
2×2	0.65	–	–	1.0	–	–
3×3	0.58	0.40	0.22	0.93	0.88	0.72
5×5	0.50	0.38	0.19	0.90	0.84	0.65
6×6	0.50	0.38	0.18	0.89	0.82	0.63

each pile has a diameter of 0.5 m and a center to center spacing of 1.772 m. The water table is at the surface and the piles are driven through 20 m of very soft clay with an effective unit weight of $\gamma'=8\ \mathrm{kN/m^3}$ into a rigid bearing layer. The coefficient β is 0.3. A 10 m high embankment is placed on the ground surface and imposes a pressure q_o equal to 200 $\mathrm{kN/m^2}$ over a very wide area. Casings are used in the fill so that no downdrag exists between the fill and the pile.

Proposed Method

The parameter b is the average radius of influence for one pile:

$$\pi b^2 = 4 d^2 \tag{4}$$

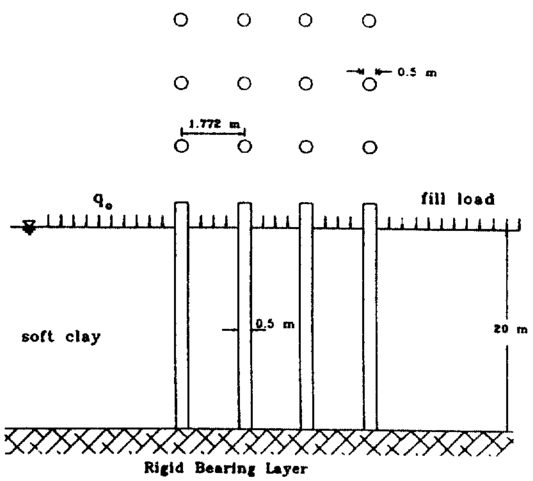


Fig. 8. An Example of Calculations for 12 Pile Group.

where $d=1/2\times1.772$; therefore $b=1$ m.

The proposed method consists of calculating first the downdrag on a single pile: the vertical effective stress at the single pile shaft proposed by the author[(6)] is

$$\sigma'(z,\ R)=\frac{\gamma'}{f(\psi,\ b)}+\exp^{-f(\psi,b)}\left[q_o-\frac{\gamma'}{f(\psi,\ b)}\right] \quad (5)$$

and the analytical solution σ′ (z, R) from equation (5) for a single pile will be

depth (m)	γ' (kN/m^3)	σ_o' (kN/m^2)	$\sigma'(z)$ (kN/m^2)	$\sigma'(z,\ R)$ (kN/m^2)
0	8.0	0	200	200
2.0	8.0	16	216	215.09
4.0	8.0	32	232	230.12
6.0	8.0	48	248	245.06
8.0	8.0	64	264	259.95
10.0	8.0	80	280	274.77
12.0	8.0	96	296	289.53
14.0	8.0	112	312	304.12
16.0	8.0	128	328	318.85
18.0	8.0	144	344	333.42
20.0	8.0	160	360	347.92

The maximum downdrag force of the single pile is:

$$\begin{aligned}F_n&=2\pi R\beta\int_0^H\sigma'(z,\ R)\ dz\\&=\frac{2\pi R\beta}{\pounds\,(\Psi,\infty)}\left[\sigma'(z)-\sigma'(H,\ R)\right]\\&=\frac{2\pi\ 0.25\ 0.3}{0.0022}\left[360-347.92\right]\\&=2568\ \text{kN}\end{aligned} \quad (6)$$

The downdrag on the group ($s/d=3.54$) is then calculated by choosing interaction factors halfway between the interaction factors for $s/d=2.5$ and $s/d=5.0$:

$$\begin{aligned}F_{n(corner)}&=0.8\times F_{n(single)}\\&=0.8\times2568=2054\ \text{kN}\end{aligned} \quad (7)$$

$$\begin{aligned}F_{n(side)}&=0.6\times F_{n(single)}\\&=0.6\times2568=1541\ \text{kN}\end{aligned} \quad (8)$$

$$\begin{aligned}F_{n(interior)}&=0.25\times F_{n(single)}\\&=0.25\times2568=642\ \text{kN}\end{aligned} \quad (9)$$

This leads to a total downdrag on the group of:

$$F_{n(group)}=4\times2504+6\times1541+2\times642=18746\ \text{kN} \quad (10)$$

Other Analytical Methods

The first method consists of calculating the downdrag on a single pile:

$$F_n=2\pi R\beta\int_0^H\sigma'(z)\ dz \quad (11)$$

where $\sigma'(z)=q_o+\gamma'z$ (12)

$$\text{Then, } F_n=2\pi R\beta(q_oH+\frac{1}{2}\gamma'H^2)$$

$$F_n=2\pi\times0.25\times0.30(200\times20+\frac{1}{2}\times8\times20^2)$$

$$F_n=2640\ \text{kN} \quad (13)$$

The downdrag on the group is then calculated as:

$$F_{n(group)}=12\times2640=31680\ \text{kN} \quad (14)$$

Terzaghi and Peck's method[(13)] makes use of the group perimeter (Fig. 8):

$$P=2\times5.82+2\times4.04=19.72\ \text{m} \quad (15)$$

The average friction over the 20 meter depth of clay is:

$$S=\frac{1}{H}\int_0^H\beta\sigma'(z)\ dz$$

$$S=\beta(q_0+\frac{1}{2}\gamma'H)$$

$$\begin{aligned}S&=0.3(200+0.5\times8\times20)\\S&=84\ \text{kN/m}^2\end{aligned} \quad (16)$$

The downdrag for the group is therefore:

$$\begin{aligned}F_{n(group)}&=S\ \ H\ \ P\\F_{n(group)}&=84\times20\times19.72\\F_{n(group)}&=33130\ \text{kN}\end{aligned} \quad (17)$$

Alternatively, if this method is applied to a short term condition with the undrained shear strength of the soft clay averaging 30 kN/m^2, then

$F_{n(group)} = S_u \ H \ P$

$F_{n(group)} = 30 \times 20 \times 19.72$

$$F_{n(group)} = 11832 \text{ kN} \tag{18}$$

In this case the long term downdrag is much more severe than the short term downdrag. The above results also tend to show that in the short term the shear plane for the downdrag maybe around the perimeter (11832 kN<31680 kN) while in the long term the shear planes may switch to individual piles (31680 kN<33130 kN).

Broms[(2)] method requires the consideration of three different cases. Case 1 is the case of individual pile action in the group and leads to $F_{n(group)} = 31680$ kN. Case 2 is the case of the long term approach in Terzaghi and Peck which gave $F_{n(group)} = 33130$ kN. In case 3 the following calculations apply:

Interior Piles

$F_{n(i)} = q_o \pi (b^2 - R^2)$

$F_{n(i)} = 200 \times 3.14(1^2 - 0.25^2)$

$$F_{n(i)} = 590 \text{ kN/pile} \tag{19}$$

for the 6 perimeter piles with a coefficient a=5

$$F_{n(p)} = q_o(\frac{H}{a} + d) \ 2d$$

$$F_{n(p)} = 200(\frac{20}{5} + 0.886) \times 2 \times 0.886$$

$$F_{n(p)} = 1730 \text{ KN/pile} \tag{20}$$

for the 4 corner piles

$$F_{n(c)} = q_o(\frac{H}{a} + d)^2$$

$$F_{n(c)} = 200(\frac{20}{5} + 0.886)^2$$

$$F_{n(c)} = 4775 \text{ kN/pile} \tag{21}$$

Table 5. Summary of Results for Example of Downdrag Calculations for a Pile Group

Downdrag Method	Downdrag Force on the Group $F_{n(group)}$ kN	Corner Pile $F_{n(c)}$ kN	Perimeter Pile $F_{n(p)}$ kN	Interior Pile $F_{n(i)}$ kN	Single Pile $F_{n(single)}$ kN
β method					2640
$F_{ng} = n \ F_{ns}$	31680				
Terzaghi & Peck (1948)	33130 (β σ' v approach) 11832 (S_u approach)				
Broms (1966)	Case 1: 31680 Case 2: 33130 Case 3: 22120	(4775) 2640	1730	590	
Broms (1976)	27580	2640	2640	590	
Zeevaert (1957)	26620	2640	2480	590	
Combarieu (1985)	10448	1265	758	420	
Proposed (1993)	18746	2054	1541	642	2568

This value is too high since it is larger than the downdrag on an isolated pile: 2640 kN. The downdrag on the group is:

$$F_{n(group)} = 2\times590 + 6\times1730 + 4\times2640$$
$$F_{n(group)} = 22120\ \text{kN} \qquad (22)$$

Broms method[3] leads to a somewhat higher answer. Indeed all parameter and corner piles are treated as isolated piles.

$$F_{n(group)} = 2\times590 + 10\times2640$$
$$F_{n(group)} = 27580\ \text{kN} \qquad (23)$$

Zeevaert's method leads to the following results[4]:

$$F_{n(i)} = 590\ \text{kN} \qquad (24)$$

$$F_{n(c)} = 2640\ \text{kN} \qquad (25)$$

$$F_{n(p)} = 2480\ \text{kN} \qquad (26)$$

The downdrag on the group is

$$F_{n(group)} = 26620\ \text{kN} \qquad (27)$$

Combarieu's method[5] starts with obtaining the parameter λ from β. Since $0.15<\beta<0.385$, $\lambda=0.385-\beta=0.085$. $Fn(\infty)$ which is the value of F_n for an isolated pile is 2450 kN while $F_{n(b)}$, the value of F_n for an interior pile, is 420 kN with a neutral point at a depth of 9.50m. Combarieu as a result of his theorerical work proposed some practical rules for pile groups:

$$\text{corner piles}\ F_{n(c)} = \frac{7}{12}F_{n(b)} + \frac{5}{12}F_n(\infty) = 1265\ \text{kN} \qquad (28)$$

$$\text{Perimeter piles}\ F_{n(p)} = \frac{5}{6}F_{n(b)} + \frac{1}{6}F_n(\infty) = 758\ \text{kN} \qquad (29)$$

$$\text{Interior piles}\ F_{n(i)} = F_{n(b)} = 420\ \text{kN} \qquad (30)$$

The downdrag on the group is:

$$F_{n(group)} = 4\times1265 + 6\times758 + 2\times420$$
$$F_{n(group)} = 10448\ \text{kN} \qquad (31)$$

Table 5 summarizes the results.

5. Conclusion

The downdrag on pile groups was investigated based on a numerical analysis and on analytical study. The case of a single pile and subsequently the response of groups were analyzed by developing interaction factors obtained from a three dimensional non-linear finite element study. On the basis of the findings of this study, the following main conclusions are drawn:

1. The pile-soil-pile interaction is much less favorable for a non-linear analysis than for a linear analysis. Thus, calculating downdrag in pile groups by using the results of linear analyses can substantially overestimate the degree of interaction in realistic situations.
2. The major parameters highly influencing the interaction factors are the group spacing, the total number of piles, and the relative position of the piles within the group.
3. The downdrag interaction factors of the piles within the group is much less than that of a single isolated pile.

REFERENCES

1. Abaqus User's and Theory Manuals, Version 4.9, *Hibbitt, Karlsson and Sorensen, Inc.*, Rhode Island, 1991.
2. Broms, B.B., "Design of Pile Groups with respect to Negative Skin Friction", *Proc. 7th Int. Conf. Soil Mech. and Found. Eng.*, Mexico, 1969, Special Session 8.
3. Broms, B.B., "Pile Foundation-Pile Groups", *Proc. 6th European Conf. on Soil Mech. and Found. Eng.*, Vienna, Austria, Vol. 2.1, 1976. pp. 103-132.
4. Combarieu, O., "Effet d'accrochage et methode d'evaluation du frottement negatif", Bulletin de liaison des P.&C. No. 71, 1974.
5. Combarieu, O., "Frottement negatif sur les pieux", Rapport de recherche LPC No 136, Laboratoire Centrale des Ponts et Chaussees, 1985, pp. 151.
6. Ito, T. and Matsui, T., "Negative Skin Friction Acting on Piles", *Proc. 5th Budapest Conf. Soil Mech. and Found. Eng.*, Akademiai Kiado, Budapest, 1976.
7. 정상섬, "단독말뚝과 군말뚝의 Downdrag", **대한토목학회 논문집**, 제13권 제4호, 1993, pp. 259-268.

8. Koerner, M.K. and Mukhopadhyay, C., "Behavior of Negative Skin Friction on Model Piles in Medium-Plastic Silt", *Highway Res. Rec.*, 405, 1972, pp. 34-50.
9. Kuwabara, F. and Poulos, H.G., "Downdrag Forces in Group of Piles", *J. Geotech. Eng.*, ASCE, Vol. 115. No. 6, 1989. pp. 806-818.
10. Mindlin, R.D. "Force at a Point in the Interior of a Semi-Infinite Solids", Physics, Vol. 7, 1936. pp. 195-202.
11. Poulos, H.G. and Mattes, N.S., "The Analysis of Downdrag in End-bearing Piles", *Proc. 7th Int. Conf. Soil Mech. and Found. Eng.*, Vol. 2, Mexico, 1969, pp. 203-208.
12. Randolph, M.F. and Wroth, C.P., "An Analysis of the Vertical Deformation of Pile Groups", *Geotechnique*, 29(4), 1979, pp. 423-439.
13. Terzaghi, K. and Peck, R.B., *Soil Mechanics in Engineering Practice*, Joha Wiley and Sons, New York. 1948.
14. Zeevaert, L., "Foundation Design and Behavior of Tower Latino Americana in Mexico City", *Geotechnique*, Vol. 7, No. 1, 1957, 115.

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