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수지형 하천에서의 부정류 흐름의 해석 알고리즘

Solution Algorithm of Unsteady Flow in a Dendritic Channel System

최 계 운*
Choi, Gye Woon

Abstract

This paper presents a simultaneous solution algorithm for one-dimensional unsteady flow routing through a dendritic channel system. This simulations solution algorithm is based on the double-sweep method and utilizes separate recursion equations for continuity, momentum and energy equations for each of the individual components of a dendritic channel system. Through separate recursion equations for each of the components, the new algorithm converts a dendritic channel network problem into a single-channel problem. The new algorithm is utilized in conjunction with a linearized unsteady flow model using full dynamic flow equations. The required computer storage for the coefficient matrix of the whole system is reduced significantly from the 2N×2N matrix to a 2N×4 matrix, where N is the number of cross sections used in the computation of flow variables in a dendritic channel system. The algorithm presented in this paper provides an efficient and accurate modeling of unsteady flow events through a dendritic channel system.

요 지

본 논문에서 수지형 하천내 1차원 부정류 흐름해석을 위한 알고리즘을 개발하였다. 이 해석 알고리즘은 수지형 하천내 부정류의 흐름을 각각의 지류내 단면간에는 연속방정식과 모우멘트 방정식을 적용하고 합류점과 분류점에서는 연속방정식과 에너지방정식을 적용하여 유한차분화한후, 이에 적합한 순환방정식을 도출하여 적용하였으며, 이때 순환계수들은 합류하는 하천단면, 분류하는 하천단면, 합류점, 분류점에 따라 각각 다르게 결정토록 하였다. 이와같은 순환계수 및 순환방정식을 이용하여 수지형 하천내 흐름해석을 단일 하천내 흐름해석과 동일하게 전진법에서는 순환계수를 계산하고, 후진법에서는 순환방정식의 해를 구하는 것이 가능하도록 하였다. 이에 따라 흐름해석을 위한 컴퓨터 저장용량도 2N×2N 행렬로부터 2N×4 행렬로 줄이도록 하였고 계산시간도 상당히 절약하였으며 이때 N은 수지형 하천내 흐름특성인 유량 및 수위를 결정해야하는 절점을 나타낸다. 이와같이 제안된 알고리즘을 이용하여 수지형 하천내 부정류 흐름을 개인용 컴퓨터등을 이용하여 효율적이며 정확하게 해석할 수 있다.

* 한국수자원공사 수자원연구소, 책임연구원, 공학박사

1. Introduction

Dendritic channel systems are frequently encountered in natural river basins and in man-made urban drainage systems. The dendritic channel system is composed of channel segments arranged in a branching configuration, with individual channel segments connected at junctions to form loops and treelike dendritic structures (Barkau *et al.*, 1989; Yen and Osman, 1976). Owing to backwater effects, the flow phenomena in a dendritic channel system are much more complicated than they are in a single channel (Akan and Yen, 1981; Yen and Osman, 1976). Many investigators in the past have simulated the dendritic channel system by considering these systems to be a combination of several independent channels or to be a main channel having tributaries with distributed lateral inflows (Barkau, 1985; Chen, 1973). These simplified solution algorithms have been used in an attempt to avoid solving complicated and/or extremely large coefficient matrices resulting from the full-system approach. Among the simplified methods, the overlapping Y-segment method suggested by Sevuk and Yen (1973) has been widely accepted (Yen, 1979; Yen and Osman, 1976). This method is based on the assumption that each channel segment is separate and independent from any other segments. Using this method, the downstream effect cannot propagate to the upper segments. In order to achieve the accuracy required in solving the flow problem of an entire dendritic system using this method, a large number of iterations are necessary. Barkau (1985) assumed the flow from the tributaries to be lateral inflow. This assumption is acceptable in the case of a small back-water and momentum contribution from tributary flows. However, Barkau's assumptions are restrictive in cases where the lateral momentum and flow contributions from tributaries are significant. In another approach, Tucci (1978) and Barkau *et al.* (1989) applied the skyline solution algorithm to simulate unsteady flows in a dendritic channel system (Barkau, 1985; Bathe and Wilson, 1976). The skyline solution algorithm, which was developed mainly to solve finite element equations, requires a large number of decision iterations during the reduction pass.

Although the simplified separate-segment iterative algorithms are popular and generate an acceptable overall solution, they fail to accurately describe the physical phenomena near channel junctions in a dendritic system. In order to overcome the simplifications used in these formulations, computational solution techniques are being developed for the simulation of entire channel networks as unit systems rather than as a combination of independent systems (Abbott, 1980; Tucci, 1978). In this paper, one such simultaneous solution algorithm is introduced. This direct solution algorithm, which is based on the double-sweep method, is applied for unsteady flow routing through a dendritic channel system (Richtmyer and Morton, 1967; Fread, 1971; Cunge *et al.*, 1980).

The matrix formed by the finite difference approximations to the simultaneous equations describing channel network flows loses its banded property when following the conventional double-sweep matrix solution. Solving the coefficient matrix of the simultaneous equations resulting from these approximations requires substantial computer memory for even moderately elaborate dendritic systems. For a dendritic channel system involving an N number of cross sections, the internal computer memory storage requirements using the conventional method for the solution of governing momentum and continuity equations with no provisions for memory storage reduction algorithms, is proportional to the size of the matrix ($2N \times 2N$). Sparse matrix solution methods which are in existence store only nonzero terms of the coefficient matrix to avoid inefficient memory storage utilization. In this paper, an efficient solution algorithm transforming the off-diagonal terms of the solution matrix to diagonal terms through recursion equations is introduced. As a result of this transformation, the storage requirements for the coefficient matrix for a network composed of N cross sections is reduced from ($2N \times 2N$) to ($2N \times 4$). Additionally, due to the banded nature of the proposed algorithm, the computational time required for the solution of this ($2N \times 4$) system is considerably faster than the nonbanded sparse matrix solution. In this paper the proposed algorithm is utilized in conjunction with a linear unsteady flow model for a dendritic channel system.

2. Governing Equations

The governing equations for simulating flows in a dendritic channel system are the conservation of mass equation and the conservation of momentum or energy equation. At the channel junctions, in addition to these equations, two equations for the conservation of mass and energy are needed. The governing flow equations for simulating unsteady one-dimensional flows in the individual channel segments are known as St. Venant's equation (Abbott, 1980; Chen, 1973; Cunge *et al.*, 1980) and are given as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial h}{\partial x} = gA(S_0 - S_f) \tag{2}$$

where A is cross-sectional area; Q is discharge; q is lateral inflow; h is water depth; S_0 is channel bed slope; and S_f is friction slope.

At channel junctions, assuming no change in storage volume (dS/dt=0, where S is storage), the continuity equation is given as (Sevuk and Yen, 1973; Yen, 1979)

$$\Sigma Q_i + \Sigma Q_0 = 0 \tag{3}$$

The conservation of energy equation for channel junction is

$$\frac{V_i^2}{2} + gh_i + gZ_i = \int \frac{dV}{dt} dx + \frac{V_0^2}{2} + gh_0 + gZ_0 + gh_{fi} \tag{4}$$

where V is velocity, A is elevation of the channel bed, and h_f is head losses due to friction and other local losses. The subscript i indicates the *i*th inflow channel at the junction and the subscript 0 represents the outflow channel at the junction. The first term in the right hand side of (4) represents energy losses due to acceleration of flow.

3. Basic Formulations

The governing dendritic channel equations (equations (1)-(4)) can only be solved numerically. The model presented in this pater employs the commonly used weighted four-point linear implicit finite difference method for approximating the governing equations in a dendritic channel system (Chen, 1973; Liggett and Cunge, 1975). In the implementation of the channel network solution algorithm, three different sets of recursion equations are utilized. These sets of equations pertain to three different flow regions encountered in a dendritic channel system. They are (1) interior channel cross sections; (2) converging channel junctions; and (3) diverging channel junctions.

3.1 Interior Channel Cross Sections

Flow in individual channel segments which make up the network is governed by (1) and (2). For interior channel cross sections along individual segments, the conservation of mass equation (equation (1)) and momentum equation (equation (2)) can be discretized using the weighted four-point implicit finite difference approximation. It is assumed that the friction slope used in the momentum equation can be obtained from Manning's equation. A Taylor series expansion of Manning's equation is used for discretizing the friction slope. The approximation equations for interior channel cross sections are

$$D_1Q_i^{n+1} + E_1h_i^{n+1} + F_1Q_{i+1}^{n+1} + G_1h_{i+1}^{n+1} = H_1 \tag{5}$$

$$D_2Q_i^{n+1} + E_2h_i^{n+1} + F_2Q_{i+1}^{n+1} + G_2h_{i+1}^{n+1} = H_2 \tag{6}$$

where D_1, E_1, F_1, G_1 and H_1 are known matrix coefficients in the conservation of mass equation and D_2, E_2, F_2, G_2 and H_2 are the known matrix coefficients in the conservation of momentum equation. The subscript i refers to the *i*th cross section and the superscript n refers to the *n*th time step.

3.2 Converging Channel Junctions

In the formulation used in this paper, each channel junction is allowed to have three upstream converging channel segments. The conservation of mass between the upstream channel sections and the downstream channel cross section is formulated as follows:

$$D_3Q_{u1}^{n+1}+E_3h_{u2}^{n+1}+F_3Q_{u3}^{n+1}+G_3h_d^{n+1}=H_3 \tag{7}$$

where D_3, E_3, F_3, G_3 and H_3 are known matrix coefficients in the conservation of mass equation at converging junction. Subscripts u and d refer to upstream and downstream stations from the junction.

The conservation of energy at channel junctions between upstream and downstream stations is formulated as follows (one equation for each branch):

$$D_4Q_u^{n+1}+E_4h_u^{n+1}+F_4Q_d^{n+1}+G_4h_d^{n+1}=H_4 \tag{8}$$

where D_4, E_4, F_4, and H_4 are known matrix coefficients in the conservation of energy equation at a converging junction.

3.3 Diverging Channel Junctions

In the formulation used in this study, each channel junction is allowed to have three downstream diverging channel segments. The conservation of mass between the upstream channel cross section and the downstream channel sections is formulated as follows:

$$D_5Q_u^{n+1}+E_5h_{d1}^{n+1}+F_5Q_{d2}^{n+1}+G_5h_{d3}^{n+1}=H_5 \tag{9}$$

where D_5, E_5, F_5, G_5 and H_5 are known matrix coefficients in the conservation of mass equation at diverging junction. The finite difference equation for the conservation of energy between the upstream cross section and downstream channel cross sections for diverging junctions is formulated in the same way as for converging junctions.

4. Network Solution Algorithm

Simultaneous equations resulting from the finite difference approximation of a dendritic channel system can be written in matrix notation as [A]{X}={B}, where [A] is coefficient matrix, {X} is vector of unknown variables (Q and h) and {B} is vector of intercept values. A typical nonbanded coefficient matrix [A] resulting from a dendritic channel system is given below:

$$\begin{bmatrix}
a_{1,1} & a_{1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & \cdots & \cdots & \cdots & \cdots & & & & \cdots & \cdots & \cdots \\
a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & \cdots & \cdots & \cdots & & & \cdots & & \cdots \\
\cdots & \cdots & a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdots & \cdots & \cdots & \cdots & a_{7,1} & a_{7,2} & & \cdot & \cdot & & \cdots & a_{7,3} & a_{7,4} & \cdots \\
\cdots & \cdots & \cdots & \cdots & & \cdot & a_{8,1} & a_{8,2} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdot & \cdots & a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdot & \cdot & \cdots & \cdots & \cdots & \cdots & \cdot \\
\cdots & \cdots & a_{12,1} & \cdots & a_{12,2} & \cdots & \cdots & \cdots & a_{12,3} & & \cdots & a_{12,4} & \cdot & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdots & \cdots & \cdots & \cdots & \cdot & \cdots & \cdots & \cdot & & \cdots & \cdots & \cdots & a_{2n,1} & a_{2n,2}
\end{bmatrix}$$

This matrix can be solved directly by the double sweep method by introducing recursion equations for each of the individual components of a dendritic channel system. This new noniterative approach reduces the computer memory storage requirments and also results in faster computational times. In the forward sweep of the algorithm, the new recurrent coefficient matrix is calculated. In the backward sweep, the unknown values of Q and h are obtained by back substitution into the recurrent equations. The recurrent matrix coefficients are given for the following four categories: converging channel sections, diverging channel sections, converging junctions, and diverging junctions. These coefficients are used to determine the unknown Q and h values (x_i, x_{i+1}, where $i=2\cdot$*(station number)÷1) from the following equations:*

$$x_i=\frac{Z_i-M_{i,4}x_{i+1}}{M_{i,3}}\;;\; i=1,\ 2,\ \cdots,\ 2n-1 \tag{10}$$

$$x_i=\frac{Z_i-a_{i,4}x_{i-2}-a_{i,3}x_{i+1}}{M_{i,2}}\;;\; i=2,\ 4,\ \cdots,\ 2n \tag{11}$$

The boundary conditions determine whether the odd-numbered or even-numbered x_i values are assigned to Q or to h. For example, if boundary conditions warrant the first x_i value to be assigned to Q_1 (as in the case of Q defined for the upstream boundary), then the second x_i value is assigned to h_1. All following odd-numbered x_i values would then be assigned to Q and even-numbered x_i values would be assigned to h.

For converging channel sections, the recurrent coefficients of the momentum equations are

$$M_{i,2} = -a_{i,1}\frac{M_{i-1,4}}{M_{i-1,3}} + a_{i,2} \quad (12)$$

$$Z_i = -a_{i,1}\frac{Z_{i-1}}{M_{i-1,3}} + b_i \quad (13)$$

in which the index i=2·(station number)−1.

For converging channel sections, the recurrent coefficients of the continuity equations are

$$M_{i,2} = -a_{i,1}\frac{M_{i-2,4}}{M_{i-2,3}} + a_{i,2} \quad (14)$$

$$M_{i,3} = -a_{i-1,3}\frac{M_{i,2}}{M_{i-1,2}} + a_{i,3} \quad (15)$$

$$M_{i,3} = -a_{i-1,4}\frac{M_{i,2}}{M_{i-1,2}} + a_{i,4} \quad (16)$$

$$Z_i = -M_{i,2}\frac{Z_{i-1}}{M_{i-1,2}} - a_{i,1}\frac{Z_{i-2}}{M_{i-2,3}} + b_i \quad (17)$$

in which the index i=2·(station number).

For the diverging channel sections, the recurrent coefficients of the momentum equations are

$$M_{i,2} = -a_{i,4}\frac{M_{i+2,4}}{M_{i+2,3}} + a_{i,3} \quad (18)$$

$$M_{i,3} = -a_{i+1,2}\frac{M_{i,2}}{M_{i+1,2}} + a_{i,2} \quad (19)$$

$$M_{i,4} = -a_{i+1,1}\frac{M_{i,2}}{M_{i+1,2}} + a_{i,1} \quad (16)$$

$$Z_i = -M_{i,2}\frac{Z_{i+1}}{M_{i+1,2}} - a_{i,4}\frac{Z_{i+2}}{M_{i+2,3}} + b_i \quad (21)$$

in which the index i=2·(station number)−1.

For the diverging channel sections, the recurrent coefficients of the continuity equations are

$$M_{i,2} = -a_{i,4}\frac{M_{i+1,4}}{M_{i+1,3}} + a_{i,3} \quad (22)$$

$$Z_i = -a_{i,4}\frac{Z_{i+1}}{M_{i+1,3}} + b_i \quad (23)$$

in which the index i=2·(station number).

The recurrent coefficients of the energy equations at channel junctions are

$$M_{i,2} = -a_{i,1}\frac{M_{i-1,4}}{M_{i-1,3}} + a_{i,2} \quad (24)$$

$$Z_i = -a_{i,1}\frac{Z_{i-1}}{M_{i-1,3}} + b_i \quad (25)$$

in which the index i=2·(station number).

For the converging channel junctions, the recurrent coefficients of the continuity equations are

$$B_{2u_k} = -M_{2u_k,2}\frac{M_{2u_k-1,3}}{M_{2u_k-1,4}} \quad (26)$$

$$D_{2u_k} = Z_{2u_k} - M_{2u_k,2}\frac{Z_{2u_k-1}}{M_{2u_k-1,4}} \quad (27)$$

$$M_{i,3} = a_{i,4} - \sum_{k=1}^{NC} a_{i,k}\frac{a_{2u_k,3}}{B_{2u_k}} \quad (28)$$

$$M_{i,4} = -\sum_{k=1}^{NC} a_{i,k}\frac{a_{2u_k\cdot 4}}{B_{2u_k}} \quad (29)$$

$$Z_i = b_i - \sum_{k=1}^{NC} a_{i,k}\frac{D_{2u_k}}{B_{2u_k}} \quad (30)$$

in whcih NC is number of upstream branches, u_k is upstream section number along branch number k, and the index i=2·(station number)+1.

For diverging channel junctions, the recurrent coefficients of the continuity equations are

$$M_{i,3} = \sum_{k=2}^{NC+1} -a_{i,k}\frac{a_{2d_k-1,2}}{M_{2d_k-1,3}} \quad (31)$$

$$M_{i,4} = a_{i,1} - \sum_{k=2}^{NC+1} a_{i,k}\frac{a_{2d_k-1,1}}{M_{2d_k-1,3}} \quad (32)$$

$$Z_i = b_i - \sum_{k=2}^{NC} a_{i,k}\frac{Z_{2d_k-1}}{M_{2d_k-1,3}} \quad (33)$$

in which d_k is downstream section number at branch number k and the index i=2·(station number).

The recurrent coefficients of the energy equations at channel junctions are

$$M_{i,2} = -a_{i,4}\frac{M_{2d_k,4}}{M_{2d_k,3}} + a_{i,3} \quad (34)$$

$$Z_i = -a_{i,4}\frac{Z_{2d_k}}{M_{2d_k,3}} + b_i \quad (35)$$

in which the index i=2·(station number)−1.

Following the forward sweep, in the back substitution sweep the conventional method is applied to compute unknown values.

For converging sections, for stations just upstream from the junction, the general recursion equation utilizes the junction Q and h values instead of the corresponding Q and h values on other branches. Similarly, for diverging sections at stations just downstream from the junction, the general recursion equation uses the junction Q and h values rather than the values of these variables corresponding to other branches.

5. Applications

In order to show applicability of the simultaneous dendritic channel solution algorithm presented in section 4, a hypothetical dendritic system composed of five tributary channel segments and three main channel segments is selected. A linear discharge variation with a maximum discharge of 65 m^3/s (2300 ft^3/s) was provided as input data. The tributary and main channel lengths were each 9.7 km (6.0 miles). The channel widths were varied from 30 m (100 feet) for tributaries 1 and 2, 61 m (200 feet) for the reach downstream from tributaries 1 and 2, 122 m (400 feet) for the reach downstream from tributaries 3 and 4, to 152 m (500 feet) for the downstream main channel. The slope in each channel segment was 0.002 m/m (foot/foot) and the Manning's roughness coefficient was 0.04. The initial discharge per unit width was constant throughout the network. The network was linked at three channel junctions. A time increment of 10 min was used in the simulation. The cross sections in the tributaries and the main channel segments were spaced 1.6 km(1 mile) apart from each other. At the junctions, however, the distance between upstream and downstream stations were set to 30 m (100 feet)(in order to satisfy the condition of no change in storage). The sequential models used for comparison in this example treat the channel network as a combination of single channel segments. Starting from the upstream segments, flows are computed in each branch and are input into the main channel segments. Two sequential formulations are utilized in this comparison: the kinematic wave formulation and the full dynamic wave formulation. For the initial conditions, the same water flow depths are assumed in the branchs and in the main channel segments. The input hydrographs which were used as the upstream boundary condition in the simulation are shown in Fig. 1 for each of the branches.

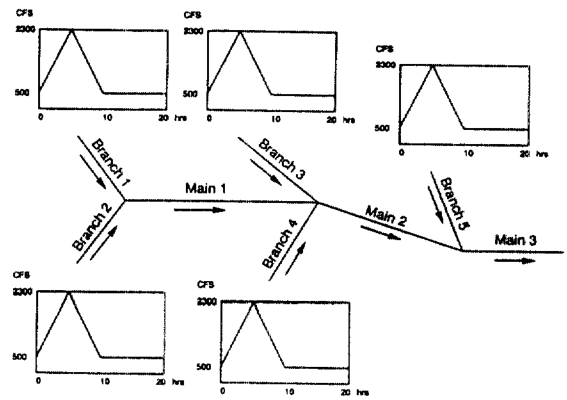

Fig 1. Input data used in a dendritic channel simulation

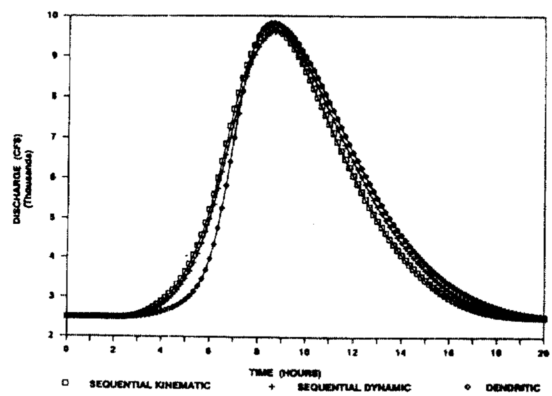

Fig 2. Comparison of sequential and dendritic simulations

The computed outflow hydrographs at the downstream end of the channel are shown in Fig. 2. As shown in Fig. 2, the outflow hydrograph produced by the dendritic simulation differs slightly from the sequential kinematic and sequential full dynamic simulations. Even though the peak discharge by full dynamic wave approximation is usually less than that by kinematic wave approximation(due to the faster attenuation of the dynamic wave), this application shows the peak discharge produced by the dendritic full dynamic wave simulation model to be greater than the peak discharge produced by the sequential dynamic simulation models and of the same order of magnitude as the sequential kinematic wave model. This is because the dendritic simulation model propagates the downstream backwater effect, whereas the sequential models do not. For the example dendritic simulation problem using 120 time steps and 48 cross sections, the computational time is just a few seconds using an 386 IBM PC compatible microcomputer.

6. Conclusions

In this paper, a simultaneous solution algorithm is presented for a dendritic channel system. This algorithm is used in a linear unsteady one-dimensional model. Using the algorithm, the coefficient matrix storage requirements can be reduced from 2N×2N to 2N×4. This significant reduction of the computer storage requirements makes possible the simulation of a large dendritic channel system using even personal computers. Additionaally, due to the banded nature of the proposed algorithm, the computational time required for the solution of this (2N×4) system is considerably faster than the nonbanded sparse matrix solution algorithms. With the reduction of computer storage requirements and faster computational times, a more efficient and accurate simulation of a dendritic channel system is accomplished. The proposed algorithm contains a limitation on the number of upstream converging or downstream diverging channel segments at each junction. For the large majority of the channel network problems this limitation poses no restrictions. However, for cases involving larger than three converging/diverging channel segments per junction formulations would require modification to include larger numbers of channel segments.

APPENDIX: FORMULATIONS CITED IN TEXT

$$D_1 = -4\theta\Psi \tag{A1}$$

$$E_1 = 2T^n_{i+1/2} \tag{A2}$$

$$F_1 = 4\theta\Psi \tag{A3}$$

$$G_1 = 2T^n_{i+1/2} \tag{A4}$$

$$H_1 = 2T^n_{i+1/2}h^n_i + 2T^n_{i+1/2}h^n_{i+1} + 4(1-\theta)\Psi Q^n_i - 4(1-\theta)\Psi Q^n_{i+1} + 4\theta\Delta t q^{n+1}_{i+1/2} - 4(1-\theta)\Delta t q^n_{i+1/2} \tag{A5}$$

$$D_2 = 1 + \frac{2g\theta A^n_{i+1/2}\Delta t S^n_{fi}}{Q^n_i} - 2\theta\Psi V^n_i \tag{A6}$$

$$E_2 = -2g\theta\Psi A^n_{i-1/2} - \frac{2g\Delta t A^n_{i+1/2} t S^n_{fi}}{K^n_i}\,\frac{K^n_i}{A^n_i}\left(\frac{5T^n_i}{3} - \frac{4T^n_i}{3}\right) \tag{A7}$$

$$F_2 = 1 + 2\theta\Psi V^n_{i+1} - \frac{2g\theta\Delta t A^n_{i+1} S^n_{fi+1}}{Q^n_{i+1}} \tag{A8}$$

$$G_2 = -2g\theta\Psi A^n_{i+1/2} - \frac{2g\theta\Delta t A^n_{i+1/2} S^n_{fi+1}}{K^n_{i+1}}\,\frac{K^n_{i+1}}{A^n_{i+1}}\left(\frac{5T^n_{i+1}}{3} - \frac{4R^n_{i+1}}{3}\right) \tag{A9}$$

$$\begin{aligned} H_2 = {} & Q_i + Q^n_{i+1} - 2(1-\theta)\Psi V^n_{i+1}Q^n_{i+1} \\ & + 2(1-\theta)\Psi V^n_i Q^n_i + gA^n_{i+1/2}\theta\Delta t S^n_{fi+1} \\ & - 2gA_{i+1/2}(1-\theta)\Psi h^n_{i+1} + 2gA^n_{i+1/2}\cdot(1-\theta)\Psi h^n_i \\ & + gA^n_{i+1/2}\Delta t(S^n_{oi} + S^n_{oi+1}) \\ & - \frac{2g\theta A^n_{i+1}\Delta t S^n_{fi+1} h^n_{i+1}}{K^n_{i+1}}\,\frac{K^n_{i+1}}{A^n_{i+1}} \\ & \cdot\left(\frac{5T^n_{i+1}}{3} - \frac{4R^n_{i+1}}{3}\right) + g\theta A^n_{i+1/2}\Delta t S^n_{fi} \\ & - \frac{2g\theta A^n_{i+1/2}\Delta t S^n_{fi} h^n_i}{K^n_i}\,\frac{K^n_i}{A^n_i}\left(\frac{5T^n_i}{3} - \frac{4R^n_i}{3}\right) \end{aligned} \tag{A10}$$

$$D_3 = E_3 = F_3 = \theta \tag{A11}$$

$$G_3 = -\theta \tag{A12}$$

$$H_3 = -(1-\theta)(Q^n_{u1} + Q^n_{u2} + {}^n_{u3} - Q^n_{d1}) \tag{A13}$$

$$D_4 = \frac{2\theta V^n_u}{A^n_u} - \frac{1}{\Psi A^n_u} - \frac{2g\theta\Delta x S^n_{fu}}{Q^n_u} \tag{A14}$$

$$E_4 = -\frac{2\theta(V^n_u)T^n_u}{A^n_u} + 2\theta g + \frac{10g\theta\Delta x T^n_u S^n_{fu}}{3A^n_u} \tag{A15}$$

$$F_4 = -\frac{1}{\theta A^n_D} - \frac{2\theta V^n_D}{A^n_D} - \frac{2g\theta\Delta x S^n_{fD}}{Q^n_D} \tag{A16}$$

$$G_4 - \frac{2\theta(V^n_D)^2 T^n_u}{A^n_D} - 2g\theta + \frac{10g\theta\Delta x T^n_D S^n_{fD}}{3A^n_D} \tag{A17}$$

$$\begin{aligned} H_4 = {} & \theta(V_u^n)^2 - \frac{2\theta(V_u^n)^2 T_u^n}{A_u^n} h_u^n - (1-\theta)(V_u^n)^2 \\ & - 2(1-\theta) g h_u^n - \frac{V_u^n}{\theta} - \frac{V_D^n}{\theta} - \theta(V_D)^2 \\ & + \frac{2\theta(V_D^n)^2 T_D^n}{A_D^n} h_D^n + (1-\theta)(V_D^n)^2 + 2g(1-\theta) h_D^n \\ & - g\theta\Delta x S_{fu}^n + \frac{10 g\theta\Delta x T_u^n S_{fu}^n}{3A_u^n} h_u^n - g\theta\Delta x S_{fD}^n \\ & + \frac{10 g\theta\Delta x T_D^n S_{fD}^n}{3A_D^n} h_D^n - (1-\theta) g\Delta x S_{fu}^n \\ & + (1-\theta) g\Delta x S_{fD}^n - 2gS_0\Delta \times \end{aligned} \qquad \text{(A18)}$$

$$D_5 = \theta \qquad \text{(A19)}$$

$$E_5 = F_5 = G_5 = -\theta \qquad \text{(A20)}$$

$$H_5 = -(1-\theta)(Q_{u1}^n - Q_{d1}^n - Q_{d2}^n - Q_{d3}^n) \qquad \text{(A21)}$$

Notation

- θ : weighting factor.
- T : top width of flow.
- Ψ : $\Delta t/\Delta x$.
- Δt : time increment.
- Δx : distance increment.
- q : lateral inflow.
- g : acceleration of gravity.
- Q : discharge.
- V : velocity.
- S_f : frictional slope.
- A : area.
- R : hydraulic radius.
- Q_{ui} : upstream discharge at channel junction.
- Q_{di} : downstream discharge at channel junction.
- K : conveyance.

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