大韓土木學會論文集
第14卷 第6號 · 1994年 11月
pp. 1365~1373

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난류전단 흐름에서의 기포응집에 관한 수치모의: 2. 모형의 적용

Numerical Simulation of the Coalescence of Air Bubbles in Turbulent Shear Flow: 2. Model Application

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Abstract

A Monte-Carlo simulation model, developed to predict size distribution of air bubbles in turbulent shear flow, is applied to a laboratory-scale problem. Sensitivity to various numerical and physical parameters of the model is analyzed. Practical applicability of the model is explored through comparisons of results with experimental measurements. Bubble size increases with air-water discharge ratio and friction factor. Bubble size decreases with increasing mean flow velocity, but the total bubble surface area in the aeration region remains fairly constant. The effect on bubble size distribution of the longitudinal length increment in the simulation model is negligible. A larger radial length increment yields more small and large bubbles and fewer in between. Bubble size distribution is significantly affected by its initial distribution and the location of air injection. Collision efficiency is introduced to explain the discrepancy between collisions with and without coalescence.

요 지

난류전단 흐름에서의 기포 크기분포를 예측하기 위하여 개발된 Monte-Carlo 모의모형을 실험실 크기의 문제에 적용하였다. 각종 모형 매개변수 및 물리적 변수들에 대한 민감도 분석을 수행하였으며, 실험 관측치와의 비교를 통하여 모형의 실제 적용성에 관하여 조사하였다. 공기와 물의 유량비 또는 마찰계수가 증가함에 따라 기포의 크기가 커지는 것으로 나타났다. 평균유속이 증가함에 따라 기포의 크기는 작아지지만, 폭기구간내 기포의 총표면적은 거의 일정함을 보였다. 모형의 종방향 거리증분에 따른 기포 크기분포의 변화는 거의 없었으며, 횡방향 거리증분을 크게 할수록 기포가 크거나 작은 쪽으로 치우쳐 중간정도의 크기를 갖는 기포의 수가 감소하였다. 기포의 크기분포는 그 초기분포 및 공기의 주입위치에 크게 영향을 받는 것으로 나타났다. 기포의 충돌과 응집을 구분하기 위하여 충돌효율을 도입하였다.

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1. Introduction

For aeration systems that utilize air bubbles such as classical diffused aeration[1] or hydroturbine aeration[2-5] systems, bubble size distribution is one of the most important factors that govern the oxygen transfer process. In part 1 of this work,[6] a Monte-Carlo simulation model to predict size distribution of air bubbles in turbulent shear flow is developed and described in detail. The simulation model consists of generating a population of air bubbles into the initial positions at each time step and tracking them by simulating motions and checking collisions. The radial displacement of air bubbles in the simulation model is produced by numerically solving an advective diffusion equation. Longitudinal displacements are generated from the logarithmic flow velovity distribution and the bubble rise velocity. Collision of air bubbles for each time step is detected by a geometric test using their relative positions at the beginning of the time step and relative displacements during the time step. At the end of the time step, the total number of bubbles, their positions, and sizes are updated. The numerical model can give detailed information on the bubble size distribution at any time and at any location in the aeration region.

In this paper the simulation model is applied to a laboratory-scale problem. A series of numerical simulations is made to investigate the effects of various physical variables and numerical parameters on bubble size distributions. A comparison is made of the numerical results with experimental measurements, and practical applicability of the model is discussed.

2. Model Application and Sensitivity Analysis

2.1 General

Numerical experiments were performed to simulate bubble size distributions for various flow conditions and model parameters. The size distribution was not computed until the bubbles generated at the first time step reach the outlet of the aeration system. Once they did, a statistical

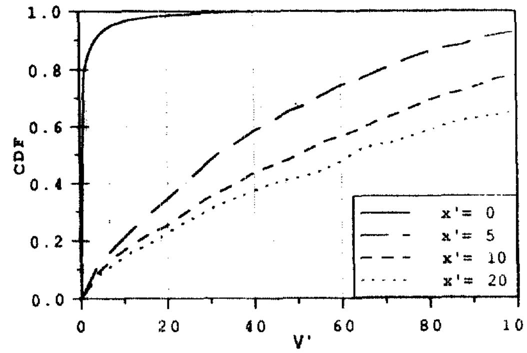

Fig. 1. Bubble Size Distribution at Various Downstream Locations

steady state was reached. To obtain reasonable bubble size statistics, the size distribution was averaged over about 40 time steps.

The following geometry, flow conditions and parameters were used for the simulation unless they are specified otherwise: pipe radius, $a=3.8$ cm; pipe length, $L=76.2$ cm; radial distance to the inner boundary of the annular air source, $a_1=0.9a$; radial distance to the outer boundary of the annular air source, $a_2=a$; mean flow velocity, $U=1.5$ m/s; air to total flow rate, $\gamma=0.02$; friction factor, $f=0.02$; unit bubble size, $R_0=1$ mm; longitudinal length increment, $\Delta x=0.25a$; radial length increment, $\Delta r=0.1a$; $\Delta t=0.05$ sec. The simulation results for these conditions are shown in Figs. 1 and 2. The basis for the selection of model parameters such as Δx, Δr and Δt and sensitivity to R_0 and f are described later. The flow and pipe characteristics are based on the experimental study of Jun and Jain.[7]

Fig. 1 presents the cumulative distribution function (CDF) for bubble volume at various downstream positions x', which is the longitudinal distance normalized by the pipe radius. The CDF is defined as the fraction of the volume of bubbles which are smaller than a certain size to the total volume of bubbles. V' is a bubble volume normalized with respect to the unit volume, V_0. All bubbles at $x'=0$ are smaller than $V'=30$. Note that the input CDF at $x'=0$ is generated by randomly distributing the unit bubbles in a subdomain of D(0) and by coalescing the bubbles which collided with each other. About 93, 78, and 65 percent of

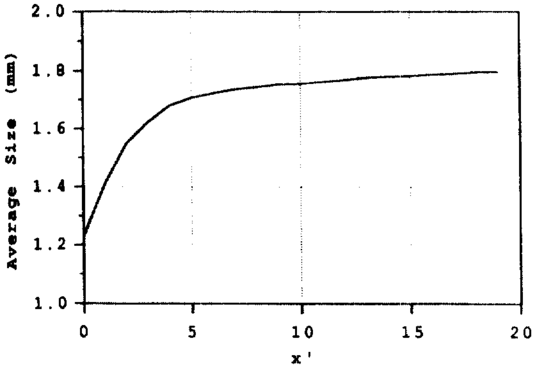

Fig. 2. Average Bubble Size at Various Downstream Locations

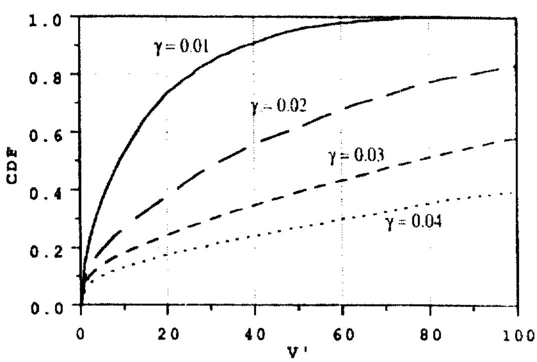

Fig. 3. Bubble Size Distribution for Various Air Flow Rates

air bubbles at $x'=5$, 10, and 20, respectively, are smaller than $V'=100$. The evolution of average bubble sizes with downstream distance for the same condition is shown in Fig. 2. It is observed that the average size of bubbles keeps increasing but the rate of increase becomes smaller as they travel further downstream. This implies that collisions, which frequently occur between bubbles near $x'=0$, become less and less frequent so that the bubbles approach an equilibrium size distribution, which is the ultimate steady-state distribution where no further significant collisions occur, at far downstream locations.

2.2 Results for Various Physical Variables

With other conditions unchanged, the air to total discharge ratio was varied from $\gamma=0.01$ to $\gamma=0.04$. Results are shown in Fig. 3. The CDF in the following figures of the present paper is for the whole system, i.e., for the region from $x'=0$ to

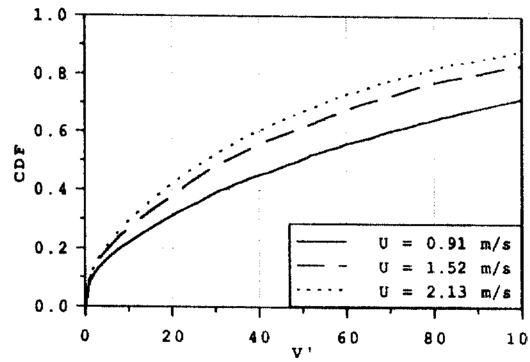

Fig. 4. Bubble Size Distribution for Various Flow Velocities

$x'=20$. The maximum bubble size for $\gamma=0.01$ is less than $V'=100$. In case of $\gamma=0.02$, 0.03, and 0.04, about 83, 58, and 40 percent of the air volume, respectively, belongs to bubbles which are smaller than $V'=100$. It means that bubble size becomes larger for increased air flow rates as would be expected for a larger number of bubbles coalescing. Also observed is that the size range becomes wider for larger air flow rates meaning that collisions between larger bubbles become more significant. These conclusions are in agreement with the experimental results presented later in Figs. 10 and 11. The total bubble surface area in the system was computed for each case and normalized by that for $\gamma=0.01$; the computed normalized values are 1.00, 1.49, 1.82 and 2.00 for $\gamma=0.01$ to $\gamma=0.04$. If the size distributions were identical for each case, the normalized total bubble surface area would have been 1, 2, 3 and 4. In other words, the rate of increase of total bubble surface area decreases as increases because of the increased rate of coalescence and ultimately larger bubble sizes for increased values.

Simulation results for various mean flow velocities (U) are shown in Fig. 4. It is seen that 72, 82, and 88 percent of air volume, respectively, for $U=0.91$, 1.52, and 2.13 m/s belongs to bubbles which are smaller than $V'=100$; the bubble size is larger for lower flow velocity. The total bubble surface area in the system also was computed. The values normalized by that for $U=0.91$ m/s are 1.000, 0.983 and 0.979 for each cases of $U=0.91$, 1.52 and 2.13 m/s, respectively. In spite of

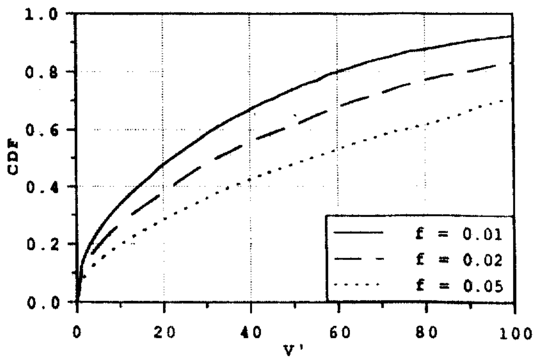

Fig. 5. Bubble Size Distribution for Various Friction Factors

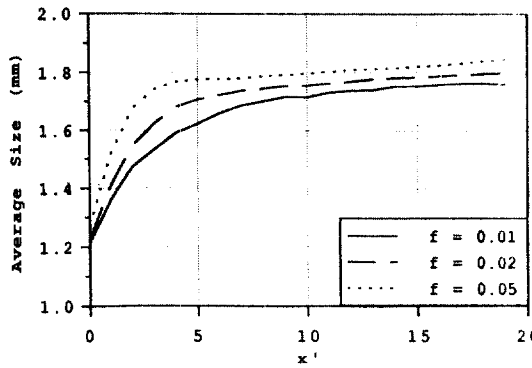

Fig. 6. Bubble Size Evolution for Various Friction Factors

the larger average bubble size at the lower flow velocity, it gives somewhat larger bubble surface area. The difference between the flow velocity and bubble rise velocity,[8] and thus between the flow velocity and the velocity of bubbles, is relatively large in the case of low flow velocity, giving more air volume in the system. In fact, this is why the bubble size is larger for the lower flow velocity since more air volume in the system results in more frequent collisions between air bubbles. These two factors, increased air volume and bubble size, compensate each other and as a consequence, the total surface area is not very sensitive to the flow velocity.

The advective diffusion equation in the nondimensional form[9] shows that friction factor is the only parameter which controls the radial turbulent diffusion of air bubbles. Simulation results for friction factors of 0.01, 0.02 and 0.05 are shown in Figs. 5 and 6. Bubble size becomes larger as friction factor increases. Bubbles are mixed more quickly over the cross section for a larger friction factor, which apparently implies less chance of collisions. However, for the same mean flow velocity the volume concentration near the location of air injection is higher for a larger friction factor due to the lower flow velocity near the wall.[9] A higher concentration implies more chance of collisions because bubbles are placed nearer to each other. Fig. 6 shows that the growth of bubble size for large friction factor is remarkable for $x'<5$. This says that collisions for small x' are more significant for a large friction factor. This is the reason bubble size becomes larger for larger friction factors.

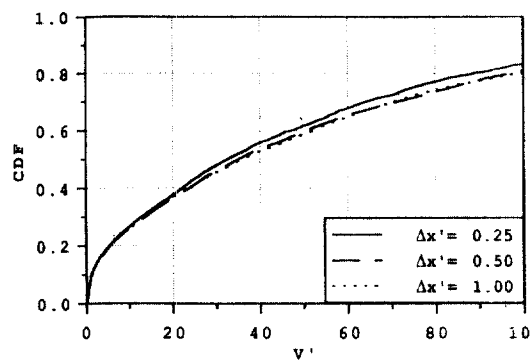

Fig. 7. Bubble Size Distribution for Different Δx

2.3 Sensitivity to Numerical Parameters

There are three model parameters; Δx, Δr, and Δt. Discretizing the flow domain with a smaller Δx enables the prescribed cross-sectional volume distribution to be more accurately incorporated. Results for different values of Δx are shown in Fig. 7. No particular tendency is found, suggesting that Δx has no more significance than the accurate incorporation of the prescribed cross-sectional volume distribution.

The parameters, Δr and Δt, have physical significance as representative length and time scales of turbulent eddies; Δr is taken as the smallest length scale important for the large scale mixing and Δt, the corresponding time scale. Considering that the radial turbulent velocity component is of the same order as the shear velocity(u_*),[10] Δt is

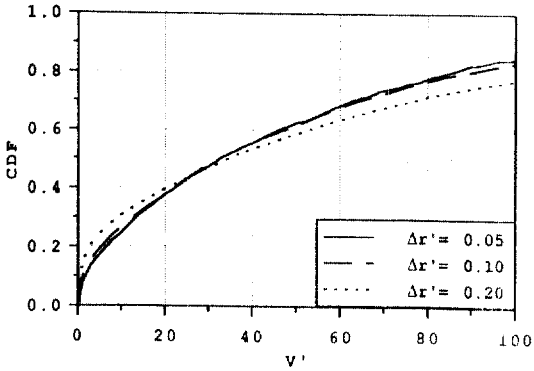

Fig. 8. Bubble Size Distribution for Different Δr

taken as

$$\Delta t = \frac{\Delta r}{u_*} \qquad (1)$$

To examine the sensitivity of the model to the above parameters, simulations were performed for $\Delta r' = 0.05$, 0.1 and 0.2, where $\Delta r'$ is Δr normalized by the pipe radius. Results are shown in Fig. 8. For small V' the value of CDF becomes larger as Δr increases. The opposite is observed for large V'. For example, for $\Delta r' = 0.05$, 0.1 and 0.2, respectively, about 25, 27, and 31 percent of air bubbles are smaller than $V' = 10$; and about 16, 17, and 22 percent of air bubbles are larger than $V' = 100$. This means that a larger $\Delta r'$ gives more small bubbles as well as more large ones and fewer in between. Bubbles moving with a larger scale tend to grow more since they have a better chance to meet and collide with more bubbles. This is the reason an increased Δr gives more large bubbles. Moreover, an increased Δr gives more small bubbles, some of which would have collided with a smaller scale motion if there had been no bubbles of larger scale motion. In other words, an increased Δr may let one bubble collide twice and another never in a given time step, whereas both would have collided once if Δr had not been increased. This explains why there are more small as well as large bubbles for increased Δr. The total bubble surface areas, for each case normalized by that for $\Delta r' = 0.1$, are 0.98, 1.00, and 1.06. The total bubble surface area increases with Δr, but is not very sensitive to it.

2.4 Sensitivity to Unit Bubble Size

The unit bubble size (V_0) is also a model parameter in some sense. It is introduced to the model to simulate the unknown initial bubble size distribution. If the process of bubble formation from the air pocket were well understood so that the size distribution of bubbles released from the air pocket could be properly prescribed, the introduction of V_0 would not be necessary. Instead, the initial bubble size distribution would be given as an input to the model. It is apparent that results from the present model may be sensitive to V_0, which is arbitrarily chosen.

To examine the model sensitivity to V_0, simulations were performed for various unit bubble sizes and air flow rates. Results are shown in Fig. 9, where V_0' is the unit bubble volume normalized to that for a 1 mm radius. It is seen that bubble size becomes larger as V_0' does. The total bubble surface area for each case normalized with respect to that for $V_0' = 1$ is given in Table 1. These results show that the model is more sensitive to the unit bubble size than to any other model parameter. Though the sensitivity becomes less as γ increases, the effect of V_0' on bubble size distribution is significant.

3. Practical Applicability of the Model

Discussed in the following sections are concerns which are associated with the application of the present model to practical problems. Certain limitations of the current model for the application to real world problems are presented, and the need for future studies is described.

3.1 Initial Conditions

Table 1. Total Bubble Surface Area for Various Unit Bubble Sizes

	$V_0' = 1$	$V_0' = 2$	$V_0' = 4$
$\gamma = 0.01$	1	0.89	0.78
$\gamma = 0.02$	1	0.91	0.84
$\gamma = 0.03$	1	0.92	0.85
$\gamma = 0.04$	1	0.96	0.88

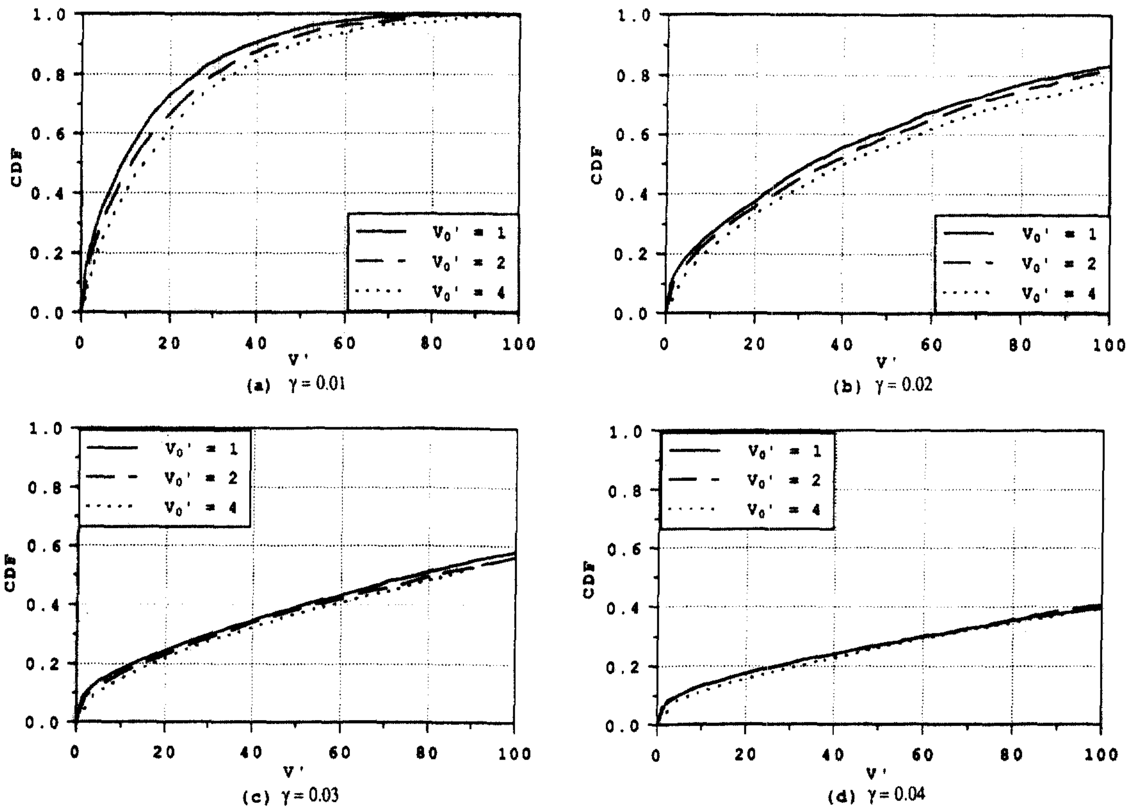

Fig. 9. Bubble Size Distribution for Various Unit Bubble Sizes

The uncertainty in initial conditions is one of the most significant issues to be resolved to ensure better performance of the model. Initial bubble size distribution and the cross-sectional area in which bubbles are introduced need to be prescribed. To do this, the formation of the air pocket, and bubble formation from the air pocket, must be well understood. The significance of the initial size distribution is already shown before. The significance of the size of air pockets which determines the initial position of air bubbles is illustrated in Fig. 10, which shows the comparison between the simulation model results and the experimental measurements at $x'=8$ for four runs (A-1, B-1, C-1 and D-1). The experimental details are described in Jun and Jain.[7] The main difference among the four runs is the flow velocity which is 2.04, 1.75, 1.47, and 1.26 m/s for Runs A-1, B-1, C-1, and D-1, respectively. The range of the measured bubble sizes increases with decreasing flow velocity from Run A-1 to Run D-1. Three different cases of initial bubble positions, a_1' (a_1 normalized by pipe radius) were tried. A large initial area of air injection (small a_1') gives smaller bubble sizes owing to the better initial mixing; i.e., the less chance of collisions. The value of a_1', which simulates the bubble size distributions close to the measured ones, is 0.7.

It may be possible to use a_1 or V_0 as model parameters in case that they cannot be properly specified as model inputs. However, scaling laws for these parameters are needed.

3.2 Collision Efficiency

It has been assumed in the model that two bubbles coalesce into one whenever they collide. However, the collision of bubbles does not always imply coalescence. Sufficient contact area and colliding momentum are required for coalescence. Surface tension is also a factor working against

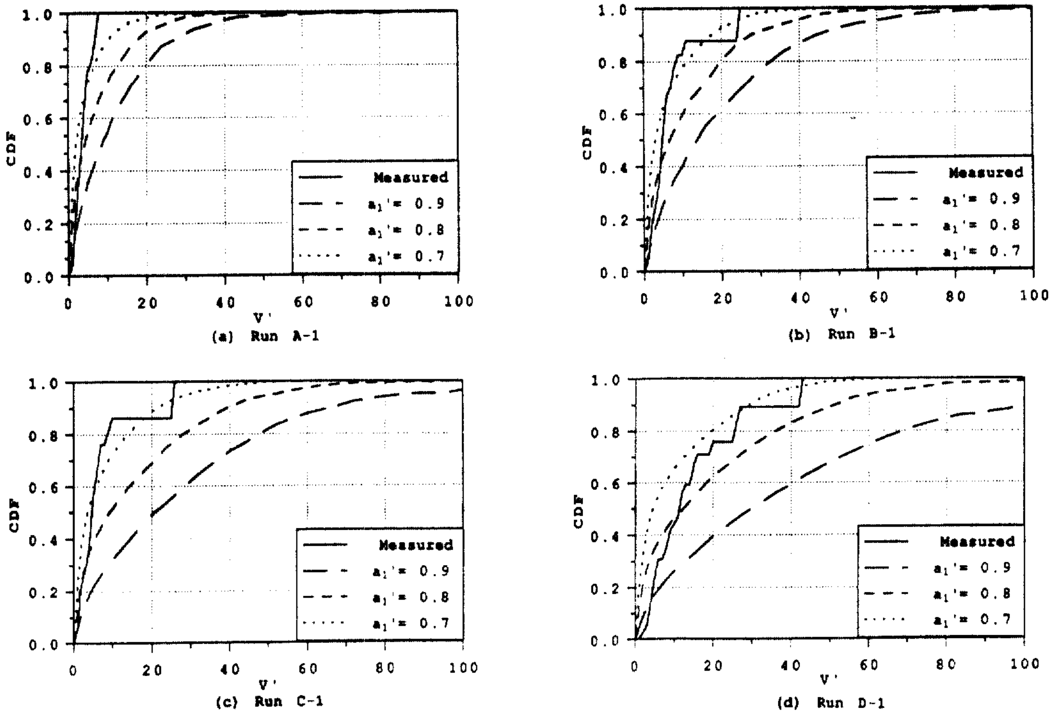

Fig. 10. Bubble Size Distribution for Various Size of Air Pockets

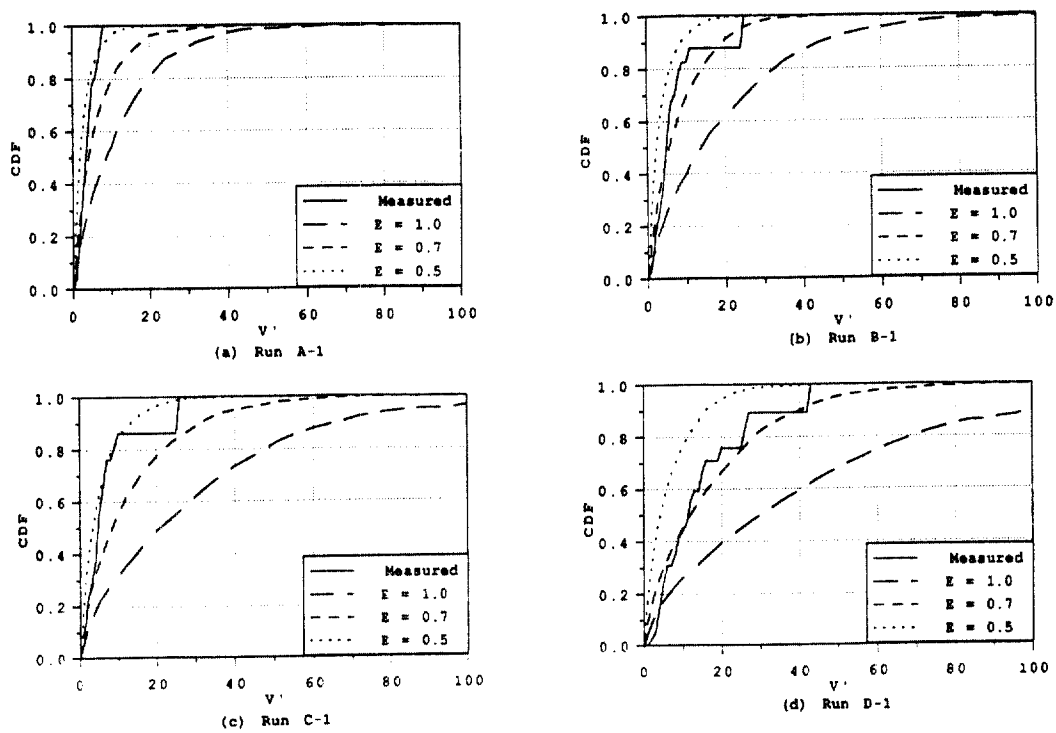

Fig. 11. Bubble Size Distribution for Different Collision Efficiency

coalescence. Smaller bubbles are less likely to coalesce on collision. The discrepancy between collisions with and without coalescence can be resolved by introducing a parameter (E) termed collision efficiency which has a value between 0 and 1. It is defined in such a way that two bubbles are considered to coalesce only if the distance between the centers of them is shorter than a certain fraction of the sum of their radii. $E=1$ implies that bubbles coalesce if they make contacts; and $E=0$ implies that bubbles do not coalesce unless their centers are coincident.

Model sensitivity to the collision efficiency is shown in Fig. 11. For the computation $a_1'=0.9$ was assumed. As is expected, bubble size becomes larger for higher collision efficiency. The value of E, for which the agreement between the simulated and measured bubble size distributions becomes better, increases with decreasing flow velocity. This is expected because more air volume with larger bubbles in the system at lower flow velocities results in more frequent collisions with coalescence between air bubbles. If the initial bubble size distribution and size of the air pocket can be given as model inputs so that the simulation model does not need V_0 and that a_1 is known, the collision efficiency would then be the most significant parameter for calibrating the simulation model.

4. Conclusions

The Monte-Carlo simulation model for the bubble size distribution is not sensitive to model parameters relevant to the displacements of bubbles. Increasing air to total discharge ratio (γ) gives larger bubble sizes due to more frequent collisions, which agrees with experimental results. At the same γ, a lower flow velocity gives more air volume in the system because the difference of bubble- to water-flow velocity is relatively larger. However, this allows more frequent collisions and thus larger bubble sizes. These two factors compensate each other; consequently, the total bubble surface area is not sensitive to the variation of the flow velocity. Bubble size increases with friction factor because the larger friction factor brings higher volume concentration of air bubbles near the diffuser, where collisions are most significant in the system.

The model is sensitive to the unit bubble size used to simulate the unknown initial bubble size distribution. The size of air pockets, from which bubbles are formed and released, is also significant to the bubble size distribution. The discrepancy between collisions with and without coalescence can be resolved by introducing the collision efficiency. If initial conditions are completely prescribed, the collision efficiency is the most significant parameter for the calibration of the model. There is a need for characterizing the behavior of the air volume injected through the diffuser in terms of relevant flow variables so that initial conditions (size of air pocket and initial size distribution of bubbles) can be prescribed as model inputs.

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(接受 : 1994. 6. 9)