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관수로 시스템의 최적설계

Optimal Design of Municipal Water Distribution System

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Abstract

The water distribution system problem consists of finding a minimum cost system design subject to hydraulic and operational constraints. Since the municipal water distribution system problem is nonconvex with multiple local minima, classical optimization methods find a local optimum. An outer flow search - inner optimization procedure is proposed for choosing a better local minimum for the water distribution systems. The pipe network is judiciously subjected to the outer search scheme which chooses alternative flow configurations to find an optimal flow division among pipes. Because the problem is nonconvex, a global search scheme called Stochastic Probing method is employed to permit a local optimum seeking method to migrate among various local minima. A local minimizer is employed for the design of least cost diameters for pipes in the network. The algorithm can also be employed for optimal design of parallel expansion of existing networks. In this paper one municipal water distribution system is considered. The optimal solutions thus found have significantly smaller costs than the ones reported previously by other researchers.

요 지

관수로시스템 문제는 수리학적 및 시스템운영 제약조건아래서 시스템의 전체비용을 최소비용으로 구하는 것이다. 관수로시스템 문제는 수많은 국지해(local minimum)을 갖는 비볼록(nonconvex) 이므로 종래의 최적화 기법은 임의의 국지해만을 구할 수 있다. 따라서 본 연구에서는 좀더 나은 국지해를 구하기 위해 외부탐사 및 내부최적화 단계 즉 2단계 분해기법을 제안하였다. 외부탐사 단계에서는 관로들의 최적유량을 찾기 위해 여러 국지해 사이를 이동하면서 좀더 나은 국지해를 찾는 방법인 추계학적탐사방법(stochastic probing method)을 이용 하였고 내부최적화 단계(local minimizer)에서는 외부탐사 단계에서 구한 국지해를 증진시킨다. 이 제안한 방법은 신설 관수로시스템 설계와 기존 관수로시스템의 확장에 적용할 수 있으며, 제안한 방법의 효율성을 검증하기 위해 어느 관수로시스템을 표본으로 채택하여 제안한 방법을 적용한 결과 먼저 발표된 연구자들의 결과보다 적은 비용으로 설계할 수 있었다.

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1. Introduction

The classical *steady state* approach to designing water distribution systems involves the following subproblems: (i) planning, (ii) design, and (iii) analysis. The *planning* aspect involves the determination of the requirements of the system based on population projections, including fire demand estimates, in particular. The *design* problem involves the selection of an optimal topological layout as well as the sizes of the distribution system components. The *analysis* step involves the evaluation of flows and pressures in the system, based on the given layout and sizes of the system components. The problem consists of solving a set of simultaneous nonlinear equations involving the energy balance relationships, along with a set of linear equations representing the continuity of flow. The real design aspects of layout selection and choosing suitable diameters for the pipes however, are quite cumbersome.

A comprehensive review of optimization of water distribution systems is given in Lansey and Mays (1989). Rowell and Barnes (1982) and Morgan and Goulter (1985) offered two different routes for selecting an optimal layout for a water distribution system. While the Rowell and Barnes scheme selects the minimum acceptable layout configuration for a network which is a tree and adds loop-forming links for the reliability of the system, Morgan and Goulter's heuristic scheme deletes links from a potentially saturated network. Loganathan *et al.* (1990) reported encouraging results by removing the constant head gradient assumption used by Rowell and Barnes and offered a computationally attractive TREESEARCH algorithm.

The algorithm TREESEARCH iteratively constructs a tree pipe network, which is a spanning tree. The importance of such a spanning tree in determining an optimal solution for a looped network has also been pointed out by Kessler and Shamir (1989) and Bhave and Sonak (1992). Bhave and Sonak (1992) and Sonak and Bhave (1993) emphasize the importance of branching configuration but have not offered a viable procedure for generating these tree configurations whereas the algorithm TREESEARCH fills that void in a heuristic manner. Once the optimal tree network is found, a set of loop-forming (redundant) links should be chosen to provide sufficient connectivity in the case of failure of a tree link. Note that a tree becomes disconnected even if a single link fails. The algorithm REDUNDANCY of Loganathan *et al.* (1990) determines an optimal set of loop-forming (redundant) links that cover the set of all possible tree link failures. The optimal redundant link set augmenting the optimal tree network, yields a looped network that should be optimized. Fujiwara and Khang (1990) and Kessler and Shamir (1991) proposed a two-phase decomposition method. These methods attempt to find the global optimum. It is not uncommon to find real problems which have cost or profit functions defined over a nonconvex feasible region involving local optima.

In this paper a municipal water distribution system is considered. The general mathematical optimization model for a pipe network is nonlinear and nonconvex, which may have several local optima. While classical optimization methods find only local optima, global optimization schemes adapt the local optimum seeking methods to migrate among local optima to find the best one. Of course, without presupposing the nature of local optima, a global optimum can not be guaranteed except to declare a relatively best optimum. A variety of general global optimization strategies have been suggested and excellent reviews are given in Torn and Zilinskas (1987) and Rinnooy Kan and Timmer (1989). A two-stage search scheme herein is employed on the municipal water distribution system to find better optima.

2. Model Formulation

The following mathematical programming formulation Problem (P1) is adopted for the general pipe network optimization:

Problem (P1): Minimize

$$\sum_{(i,j)} \left[C_{1(i,j)} x_{(i,j)} + C_{2(i,j)} x_{2(i,j)} \sum_{t=s(i,j)}^{S(i,j)} C_{3(i,j)} x_{3(i,j)t} \right]$$
$$+ \sum_{i \in s} C_{4i} a_i Hs_i + \sum_{l} \sum_{i \in s} C_{5i} H_{bi,l} + \sum_{l} \sum_{i \in s} \frac{C_6 \gamma \Delta T_{i,l}}{\eta_{i,l}} Q_{bi,l} H_{bi,l}$$

$$+\sum_{l}\sum_{i\in S} C_{7i}Hp_{i,l}+\sum_{l}\sum_{i\in S}\frac{C_8\gamma\Delta T_{i,l}}{\eta_{i,l}}Qp_{i,l}Hp_{i,l} \qquad (1)$$

Subjecto to

$$-\sum_{k:(i,k)\in L} Q_{(i,k),l}+\sum_{k:(i,k)\in L} Q_{(k,i),l}=q_{i,l}$$

$$\text{for all } i\in N \text{ and } l\in £ \qquad (2)$$

$$-\sum_{(i,j)\in r(k)} \pm[J_{1(i,j),l}x_{1(i,j)}+J_{2(i,j),l}x_{2(i,j)}$$

$$+\sum_{t=s(i,j)}^{S(i,j)} J_{3(i,j)t,l}x_{3(i,j)t}]+(\pm)[Hs_i+Hs_i^{ele}]$$

$$+(\pm)[H_{bi,l}+H_{bi}^{ele}]+(\pm)[Hp_{i,l}+Hp_i^{ele}]\geq H_{k,l}^{min}$$

$$\forall\ i\in s,\ k\in N,\ l\in £,\text{ and } i\in S \qquad (3)$$

$$\sum_{(i,j)\in p} \pm[J_{1(i,j),l}x_{1(i,j)}+J_{2(i,j),l}x_{2(i,j)}$$

$$+\sum_{t=s(i,j)}^{S(i,j)} J_{3(i,j)t,l}x_{3(i,j)t}]+(\pm)[Hs_i+Hs_i^{ele}]$$

$$+(\pm)[H_{bi,l}+H_{bi}^{ele}]+(\pm)[Hp_{i,l}+Hp_i^{ele}]=b_{p,l}$$

$$\text{for all } p\in \not p \text{ and } l\in £ \qquad (4)$$

$$x_{1(i,j)}+x_{2(i,j)}+\sum_{t=s(i,j)}^{S(i,j)} x_{3(i,j)t}-L_{(i,j)}=0$$

$$\text{for all } (i,j)\in L \qquad (5)$$

$$j_{min}\leq J_{(i,j)}\leq j_{max} \qquad \text{for all } (i,j)\in L \qquad (6)$$

$$Q_{(i,j),l}\geq Q_{(i,j),l}^{min} \qquad \text{for all } (i,j)\in L \qquad (7)$$

$$x_{1(i,j)}\geq 0,\ x_{2(i,j)}\geq 0,\ x_{3(i,j)t}\geq 0,\ Hs_i\geq 0,\ H_{bi,l}\geq 0,\ Hp_{i,l}\geq 0$$

where:

$C_{1(i,j)}$ = unit cost for existing pipe in a link (i, j)
$C_{2(i,j)}$ = unit cost for cleaning existing pipe in a link (i, j)
$C_{3(i,j)t}$ = unit cost for the t th new diameter segment in a link (i, j)
$x_{1(i,j)}$ and $x_{2(i,j)}$ = length of existing diameter segment in a link (i, j)
$x_{3(i,j)t}$ = length of t th new diameter segment in link (i, j)
C_{4i} = unit cost of tank at node i
C_{5i} = booster pump capital cost per pumping head at node i
C_{7i} = pump capital cost per pumping head at node i
C_6 and C_8 = unit energy cost (\$/kwh)
Hs_i = additional head for tank node i
a_i = bottom area of tank at node i
$H_{bi,l}$ = operating head of a booster pump at node i for the lth loading
L = set of loadings
$Hp_{i,l}$ = operating head of a pump at node i under the lth loading
Hs_i^{ele} = elevation of tank at node i
H_{bi}^{ele} = elevation of booster pump at node i
Hp_i^{ele} = elevation of pump at node i
$Q_{bi,l}$ = pumping rate of a booster pump at node i under the lth loading
$Qp_{i,l}$ = pumping rate of a source pump at node i under the lth loading
$\Delta T_{i,l}$ = pumping period for pump at node i under the lth loading
γ = specific weight of water
$\eta_{i,l}$ = pump efficiency for pump at node i under the lth loading
s = set of booster pump or tank nodes
S = set of source pump nodes
$q_{i,l}$ = consumptive use or demand at node i for the lth loading
$H_{k,l}^{min}$ = minimum pressure head required at node k under the lth loading
$Q_{(i,j),l}$ = link flow for the lth loading
r(k) = a path through the network connecting a source node (source pump or tank) and demand node k
$\not p$ = set of loops and paths connecting source head nodes
$b_{p,l}$ = zero in the loop and is head difference between the source heads (source pumps or/and tanks) for path p connecting them
L = number of links in the network
$L_{(i,j)}$ = length of a link (i, j)
J_{max} and J_{min} = upper and lower bounds of the hydraulic gradient J
N = number of nodes in the network except source nodes (source pumps and tanks).

$S_{(i,j)}$ and $s_{(i,j)}$ are the indices of maximum and minimum diameters of a link (i, j) for velocity restrictions:

$$S_{(i,j)}=\max\{t \mid \frac{4Q_{(i,j),l}}{\pi d_t^2}\leq V^{min},\ t=1,\cdots,T\},\ d_{max}=\sqrt{\frac{4Q_{(i,j),l}}{\pi V^{min}}}$$

$s_{(i,j)} = \min\{t \mid \frac{4Q_{(i,j),l}}{\pi d_t^2} \le V^{max},\ t=1, \cdots, T\}$, and $d_{min} = \sqrt{\frac{4Q_{(i,j),l}}{\pi V^{max}}}$. where, d_{min} and d_{max} are themaximum and minimum diameter for velocity restrictions. V^{min} and V^{max} are the minimum and maximum velocity, T is the number of distinct pipe diameters. The optimal diameters are chosen from available pipe diameters for a link (i, j) which lie on between the maximum diameter and the minimum diameter, say $d_1 \le d_{min}, \cdots, d_{max} \le d_T$.

The hydraulic gradient for the lth loading from the Hazen-Williams equation is used along with the SI system of unit: $J_{1,l(i,j)} = k1_{f(i,j)} Q_{(i,j),l}^{1.852} D_{1(i,j)}^{-4.87}$, $J_{2,l(i,j)} = k2_{f(i,j)} Q_{(i,j),l}^{1.852} D_{1(i,j)}^{-4.87}$ and $J_{3(i,j)t,l} = k3_{f(i,j)} Q_{(i,j),l}^{1.852} D_{3(i,j)t}^{-4.87}$ for each option of pipe in which $k1_f = 10.7/C_e^{1.852}$, $k2_f = 10.7/C_e^{1.852}$, $k3_f = 10.7/C_n^{1.852}$, C_e is the Hazen- Williams coefficient for existing pipe, C_c is the Hazen-Williams coefficient for cleaning existing pipe, C_n is the Hazen-Williams coefficient for new pipe, $Q_{(i,j),l}$ is the given flow in link (i, j) for the lth loading, $D_{1(i,j)}$ is the diameter for existing pipe in link (i, j) and $D_{3(i,j)t}$ is the new diameter of tth segment in link (i, j).

The objective function of Problem (P1) is the summation of the pipe network cost, the storage tank cost, the booster pump capital cost, the pumping cost for booster pump, the pump capital cost, the operating cost for source pump, respectively. Constraints (2) represents the flow continuity equations under the lth loading, constraints (3) represents the hydraulic head requirement at each node under the lth loading, constraints (4) represents the sum of head losses in a path under the lth loading, constraints (5) represents the length constraints, constraints (6) represents the hydraulic gradient bounding constraints, and constraints (7) represents the flow bounds under the lth loading. In constraints (3) the positive sign is taken only if the path direction coincides with the flow direction in link (i, j). In constraints (4) the positive sign is taken only if any path connecting two source pumping head nodes coincides with the flow direction in link (i, j). If there are S source nodes (source pumps and tanks) in a network, ${}_sC_2$ number of constraints (4) should be account for. The constraints will have to be duplicated for each demand pattern if more than one demand pattern are to be considered.

3. Two-Stage Decomposition Method

Since Problem (P1) previously stated is nonlinear, nonconvex programming problem which has several local minima, assuring global optimal solution is an extremely difficult task because it requires that there is no better point than a global minimum in every neighborhood of the global minimum. A two-stage method for the solution of Problem (P1) is suggested for the optimal design of new water distribution systems as well as parallel expansion of existing networks. The pipe network problem is decomposed into a two-stage problem comprising of an outer search strategy for selecting link flows and a local minimizer for optimal design of pipes in terms of selecting optimal segment lengths of known diameters. This decomposition does have merit provided efficient global search schemes for the outer search strategy can be found. The following two stage strategy Problem (P2) is suggested for the solution of Problem (P1).

$$\text{Problem (P2): } \min_{Q(i,j)} \left[\min_{x \in X} f(x) \right] \tag{8}$$

in which $Q_{(i,j)}$ are the perturbed flows of an underlying near optimal spanning tree of the looped layout satisfying constraint (2), X is the feasible region made up of constraints (3)~(7), and f(x) is the objective function of Problem (P1) for a fixed link flows. It is worth noting that the optimal layout tends to be a tree layout. Deb (1973) showed that a typical pipe cost objective was a concave function which attained its minimum on a boundary point resulting in a tree solution. Gessler (1982) and Templeman (1982) also argued that because optimization had the tendency to remove any redundancy in the system the optimal layout should be a tree. However, a tree gets disconnected even when one link fails.

It is observed that the inner Problem (P3) of Problem (P2) given by

$$\text{Problem (P3): } \min_{x \in X} f(x), \text{ for fixed flows} \tag{9}$$

is a linear program which can be solved efficiently by using commercially available codes. This phase represents the local minimizer.

3.1 Outer Search Strategy

To choose the flows in the outer problem of (P2) a Stochastic Probing method is adopted. The solution behavior over the feasible region is easily assessed. To make the search efficient, first a set of flows corresponding to a near optimal spanning tree of the network is found. The flows in the looped network are taken as the perturbed tree link flows by

$$Q_{(i,j)}(\text{loop}) = Q_{(i,j)}(\text{tree}) + \sum \pm \Delta Q_{p(i,j)} \qquad (10)$$

in which the sum is taken over loops 'p' which contain link (i, j) with positive loop change flow $\Delta Q(=\varepsilon)$ used for clockwise flow. The stochastic probing method is used to search the location of the global loop flow vector ε^*. The method begins with the construction of a probing probability distribution, with the density function $P \sim N(\varepsilon, \sigma)$, where ε is the location parameter (loop flow vector) and σ is a scaling parameter, respectively. The costs of $f(\varepsilon)$ are evaluated at a few loop flows ε sampled from the density function. The updating location of loop flow ε and scale σ are based on Gibbs-like distribution and the entropy of the current distribution, respectively (Laud *et al.*, 1992). The Stochastic Probing Method is given in detail in the following.

Step 0. (Initialization) Select an initial location of loop flow ε^0 and a scale factor σ^0 of a probing distribution.

Step 1. (Generation step) At stage n ($n \geq 0$), generate k independent, identically distributed loop flows $\varepsilon_{n1}, \ldots, \varepsilon_{nk}$ from $P_r \sim N(\varepsilon^0, \sigma^0)$. Let $\varepsilon_{no} = \varepsilon_n$ and $\varepsilon^n = (\varepsilon_{no}, \cdots, \varepsilon_{nk})$.

Step 2. (Update location) The updated location ε_{n+1} is chosen from a point that is the highest probability in the following Gibbs-like distribution;

$$P_r(\varepsilon_{n+1} = \varepsilon_{ni}) = \frac{1}{Z} e^{-B(f,i,\varepsilon^n)}, \quad i = 0,\ 1,\ \cdots,\ k$$

where, $Z = \sum_{i=0}^{k} e^{-B(f,i,\varepsilon^n)}$ and

$$B(f,\ i,\ \varepsilon^n) = \frac{f(\varepsilon_{ni}) - f(\varepsilon_{no})}{\min(f(\varepsilon_{ni}),\ f(\varepsilon_{no})) - f_{min}}$$

in which: f is the objective of the linear program (P1).

Step 3. (Updated scale) The rule for scale reduction is of the form

i) $\varepsilon_{n+1} = \varepsilon_n$ and $\sigma_{n+1} = \sigma_n$ if $f(\varepsilon_{n+1}) \geq f(\varepsilon_n)$.

ii) $\sigma_{n+1} = w_n \sigma_n$ if $f(\varepsilon_{n+1}) < f(\varepsilon_n)$, where

$$w_n = \frac{\text{Ent}(n)}{\log(k+1)}$$

The scale reduction factor w_n is based on the entropy of the current distribution:

$$\text{Ent}(n) = -\sum_{i=0}^{k} P_r(\varepsilon_{n+1} = \varepsilon_{ni}) \log P_r(\varepsilon_{n+1} = \varepsilon_{ni})$$

where $P_r(\cdot)$ is given in the current distribution.

Step 4. (Stopping rule)

i) If $\sigma_{n+1} < \sigma_n$, increase n by one and go to step 1.

ii) If $\sigma_{n+1} = \sigma_n$ and no improvement has been obtained in the last few interations, save ε_n^* and $f(\varepsilon_n^*)$ and stop; otherwise go to Step 1.

3.2 Local Minimizer

If a set of flows is specified on a network feasible to the continuity equation (2), then Problem (P1) becomes a linear programming problem and there exists the global optimum associated the link flows. The linear Program (P1) yields the optimal segment lengths $x_{1(i,j)}$, $x_{2(i,j)}$ and $x_{3(i,j)}$ for various diameters $d_{1(i,j)}$, and $d_{3(i,j)}$ and the optimal objective function value f^*. These values are dependent on the selected link flows satisfying continuity which are derived from the perturbing loop flows for l loadings: $\varepsilon_l = [\Delta Q_{1,l}, \ldots, \Delta Q_{p,l}]$. The following description to search for an appropriate ε_l locally is due to Fujiwara *et al.* (1987). Define $f^*(\varepsilon_l)$ be the optimal value and $x(\varepsilon_l)$ be the optimal solution for Problem (P1). We are interested in evaluating the rate of change of the optimal value function f^* with the changing ε_l values.

Fujiwara *et al.* (1987) suggest the quasi-Newton procedure with Broyden-Fletcher-Goldfarb-Schanno (BFGS) Hessian update as implemented in Dennis and Schnabel (1983) for using the above

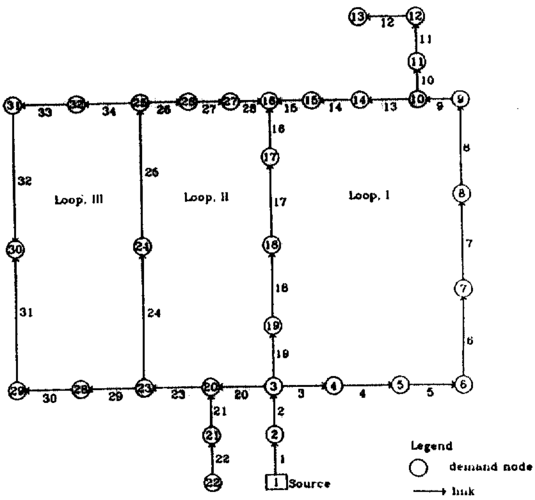

Fig. 1. Hanoi Water Distribution System (No Scale)

direction to update the flows. In the proposed methodology, first a global search strategy is used to select different configuration of link flows. Second, the linear programs (P1) is solved for each configuration of link flows. Only at this stage the BFGS routine is implemented for possible refinement about the best flow configuration from the global search.

4. Analysis of Example Network

The first example network is the planned water distribution network (Fujiwara and Khang, 1990; Sonak and Bhave, 1993) in Hanoi, Vietnam, which is solved using the proposed procedure. Figure 1 shows the network consisting of 1 source node, 31 demand nodes, 34 links, and three loops. As given in the previous studies, the following data are used: Hazen-Williams; $C=130$ for all links; conversion factor $K=162.5$ for flows in cubic meters per hour and diameters in inches; exponents for discharge and diameter are 1.85 and -4.87 respectively. Commercially available diameters are 12, 16, 20, 24, 30, and 40 inches and the cost per unit length of pipes is given by $1.1d^{1.5}$ in which d is the diameter in inches. The minimum required flow in a link is 5 cubic meters per hour. The link length, demand, minimum hydraulic heads at nodes are given in Table 1.

Table 1. Node and Link Data for Hanoi Water Distribution Network

Node data			Link data	
Node number	Demand m^3/hour	Minimum heand, m	Link number	Length, m
1	19,940	100	1	100
2	890	30	2	1,350
3	850	30	3	900
4	130	30	4	1,150
5	725	30	5	1,450
6	1,005	30	6	450
7	1,350	30	7	850
8	550	30	8	850
9	525	30	9	800
10	525	30	10	950
11	500	30	11	1,200
12	560	30	12	3,500
13	940	30	13	800
14	615	30	14	500
15	280	30	15	550
16	310	30	16	2,730
17	865	30	17	1,750
18	1,345	30	18	800
19	60	30	19	400
20	1,275	30	20	2,200
21	930	30	21	1,500
22	485	30	22	500
23	1,045	30	23	2,650
24	820	30	24	1,230
25	170	30	25	1,300
26	900	30	26	850
27	370	30	27	300
28	290	30	28	750
29	360	30	29	1,500
30	360	30	30	2,000
31	105	30	31	1,600
32	850	30	32	150
			33	860
			34	950

The decision variables of the Hanoi optimization model based on Problem (P1) are the unknown segment lengths of known six different candidate

Table 2. Optimal Tree Link Flows for Hanoi System

Link Number	Flow, m^3/hour	Link Number	Flow, m^3/hour
1	19940.0	18	2210.0
2	19050.0	19	2270.0
3	8015.0	20	7915.0
4	7885.0	21	1415.0
5	7160.0	22	485.0
6	6155.0	23	5225.0
7	4805.0	24	3530.0
8	4255.0	25	2710.0
9	3730.0	26	1270.0
10	2000.0	27	370.0
11	1500.0	28	0.0
12	940.0	29	650.0
13	1205.0	30	360.0
14	590.0	31	0.0
15	310.0	32	360.0
16	0.0	33	465.0
17	865.0	34	1270.0

diameters. The algorithm TREESEARCH (Loganathan *et al.*, 1990) is applied to obtain the optimal tree layout and the optimal tree link flows. The global tree network obtained by deleting links 16, 28, and 31 has a cost of $5,812,889. The optimal tree link flows given in Table 2 are then perturbed to obtain the flows for the looped network. The procedure Stochastic Probing is implemented to search the feasible region in the outer problem of (P1) beginning with the perturbed optimal tree link flows as the initial flows. The optimal loop flows $(\Delta Q_1, \Delta Q_2, \Delta Q_3) = (229.71, 15.80, 10.98)$ with a cost of $6,032,548 is then obtained. For the local minimizer of (P1), the loop flows is further refined by the gradient search (BFGS) to obtain the optimal loop flow $(\Delta Q_1, \Delta Q_2, \Delta Q_3) = (229.71, 15.80, 0.0)$ producing a cost of $6,031,807. For the gradient search, the IMSL subroutine UMING is used. However, the latter solution to the local minimizer of (P1) violates the minimum required flow, 5 m^3/hour. Fujiwara and Khang (1990) have suggested a two phase procedure. In the first phase cost minimization problem has a nonlinear convex objective in terms of head loss with a linear constraints region and yields optimal head losses. Then in the spirit of the LPG (Linear Programming Gradient) procedure, an improving direction is generated with the aid of the Lagrange multiplier of the Phase I constraints called NLPG (Nonlinear Programming Gradient) direction. Sonak and Bhave (1993) emphasize the importance of a branching configuration and find the best tree network using a heuristic manner. Once the optimal tree network is found, a set of loop-forming links satisfying just the minimum required flow is chosen to provided sufficient connectivity of the network. The present approach yields a cost of $6,032,548 which is an improvement over $6,319,000 of Fujiwara and Khang, and $6,045,500 of Sonak and Bhave. The optimal solution with minimum flow of 5 m^3/hour is given in Table 3.

5. Conclusions

Optimization of water distribution systems by conventional optimization techniques can not guarantee a global optimum because the problem is nonconvex with multiple local minima. A two stage decomposition which uses readily available linear programming routines within the framework of a global search for solving the nonconvex pipe networks is put forward. The two-stage decomposition method employs the Stochastic Probing method for the outer search conducted over flows and the inner linear program designs pipes for the selected flows. It is found that utilizing optimal tree link flows with perturbations to obtain looped network flows enhances the outer search efficiency. Since Problem (P1) has several local minima, each perturbation moves towards a local optimum which indicated a set of loop flows. The various locally optimal designs obtained as the outer search progresses, greatly aid in understanding the feasible region in terms of the objective function surface. As a practical matter, the methodology helps the designer in understanding how close the various designs are in terms of cost.

The results of the well established test problems for municipal water distribution system (Ha-

Table 3. Optimal Solution for Hanoi System (flow: m^3/hour)

Node		Link			
Node Number	Optimal Head, m	Link Number	Diameter, inches	Length, m	Looped Link Flow
1	100.00	1	40	100.00	19940.00
2	97.17	2	40	1350.00	19050.00
3	62.00	3	40	900.00	7785.29
4	57.52	4	40	1150.00	7655.29
5	51.97	5	40	1450.00	6930.29
6	46.15	6	40	450.00	5925.29
7	44.80	7	40	850.00	4575.29
8	43.22	8	40	850.00	4025.29
9	41.97	9	30	547.70	3500.29
10	39.16	9	40	252.30	3500.29
11	37.61	10	30	950.00	2000.00
12	34.20	11	24	1200.00	1500.00
13	30.00	12	24	3500.00	940.00
14	33.18	13	16	569.98	975.29
15	30.20	13	20	230.02	975.29
16	30.00	14	12	500.00	360.29
17	36.19	15	12	550.00	80.29
18	53.57	16	12	2730.00	213.91
19	59.10	17	16	1463.07	1078.91
20	50.67	17	20	286.93	1078.91
21	35.15	18	24	800.00	2423.91
22	30.00	19	24	400.00	2483.91
23	44.33	20	40	2200.00	7930.80
24	38.56	21	16	508.58	1415.00
25	34.82	21	20	991.42	1415.00
26	30.51	22	12	500.00	485.00
27	30.01	23	40	2650.00	5240.80
28	37.57	24	30	1230.00	3534.82
29	30.01	25	30	1300.00	2714.82
30	30.00	26	20	813.32	1285.80
31	30.21	26	24	36.68	1285.80
32	32.14	27	16	300.00	385.80
		28	12	750.00	15.80
		29	16	1500.00	660.98
		30	12	944.14	370.98
		30	16	1055.86	370.98
		31	12	1600.00	10.98
		32	16	150.00	349.02
		33	16	860.00	454.02
		34	20	246.49	1259.02
		34	24	703.51	1259.02

noi network) attest to the efficiency of the method in finding better local optima. The method accommodates various pipe fittings in terms of their minor losses, pumps, and elevated tanks. Multiple loadings are included by appropriate addition of the constraints corresponding to each loading pattern while retaining the pipe length variables to be the same in all loadings. The pipe length variables remain the same because the same pipes are utilized under all loadings. Moreover, the global method is capable of choosing an optimal layout with two connectivity for a reliable network. In conclusion, the proposed method is comprehensive and efficient in providing an optimal network design.

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