大韓土木學會論文集
第14卷 第5號 · 1994年 9月
pp. 1097～1104

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구조물의 확률론적 지진손상평가

Probabilistic Seismic Damage Assessment of Structures

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Abstract

The external loads applied to a real structure may cause a severe damage and may eventually lead to total failure. It is thus the requirement that the structure must be designed to fulfil its safe function under any anticipated loads and must have the desired level of safety. The purpose of the present study is to propose a method of damage accumulation under seismic loadings to utilize it in the safety assessment of a reinforced concrete structure. To this end, the nonlinear hysteretic behavior of reinforced concrete structures is first modeled and the equivalent linearization technique is employed to solve numerically the probabilistic characteristics of response under random seismic loadings.

요 지

구조물은 그 수명기간동안 여러 하중을 받게 되고 이로 인해 손상을 입을 수 있으며 때로는 극심한 외부하중에 대해 구조물이 제기능을 상실하거나 파괴에 도달하게 되므로 구조물 설계과정에서 이에 대한 안전성확보가 중요한 요건이 된다. 지진하중을 받는 철근 콘크리트 구조물은 비선형 이력거동을 하게 되며 심한손상을 받을 수 있다. 본 연구에서는 지진하중을 받는 구조물의 추계적 응답특성을 이용하여 손상에 대한 확률적 누적방법을 제안하고 지진하중에 대한 구조물의 손상수준을 손상지수로서 표현하여 안전성평가에 이용하고자 하였다.

1. Introduction

During its lifetime the structure is subjected to loads (or actions), i.e., forces or forced displacements. The loads may cause a change of condition or state of the structure, going from the undamged state to a state of deterioration, or wear, damage in varying degrees, failure, or collapse.(1)

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Reinforced concrete structures subjected to strong earthquakes may have the hysteretic behavior and undergo the severe damage state. Seismic damage arises from the fact that structure subjected to cyclic loading experiences permanent deformation and degradation of strength and stiffness. The responses of the structures are time dependent and excurse into inelastic range by degrading strength and stiffness. Thus, the seismic response analysis for concrete structures considering inelastic, hysteretic, degradation effects is required. The responses of structures to earthquakes have the probabilistic properties due to the randomness of motion. Considering the characteristics of the seismic response, it is rational to introduce the probabilistic concept in accumulating damage for the damage assessment of the concrete structures. In this study, the probabilistic accumulation method of seismic damage, using the peak value distribution of the maximum responses, is suggested and damage index is calculated from the probabilistic accumulation of damage.

2. Earthquake Load

In comparison with wind and wave loads, the loads from earthquakes are rare events. This leads to greater uncertainty in their description, and also makes it useful to consider earthquake loads conditioned on some seismic event, and then to attempt an independent assessment of the probability of the seismic event.

The equation of motion can be stochastically solved by use of Kanai-Tajimi spectrum.[6] Fig. 1 shows the typical one-sided K-T spectrum used in the present analysis.

Envelope function is introduced to account for the nonstationarity of earthquakes. In the present study, seismic response analysis is conducted using the power spectral density and envelope function.

3. Nonlinear structural analysis model

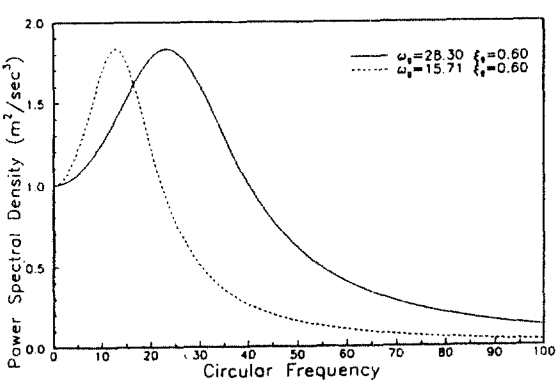

Fig. 1. One-sided Kanai-Tajimi spectrum

3.1 Analytic Modeling of Degrading System

Bouc's hysteresis as extended by Wen[6] can represent a wide variety of hardening or softening, smoothly varying or nearly bilinear hysteresis with a considerable range of cyclic energy dissipations.

Consider the MDOF shear beam structure in Fig. 1. Noting that the quantities u_i are the relative displacement of the i-th and (i+1)-th stories, the equations of motion may be written in the form as Eq. (1).

$$m_i\left(\sum_{j=1}^{i}\ddot{u}_j+\ddot{x}_B\right)+q_i-q_{i+1}=0, \qquad i=1,\ 2,\ \cdots,\ n \tag{1}$$

in which, m_i is the mass of the i-th floor, $\ddot{x}_B$ is the ground acceleration, and q_i is the i-th restoring force, including viscous damping (Fig. 3).

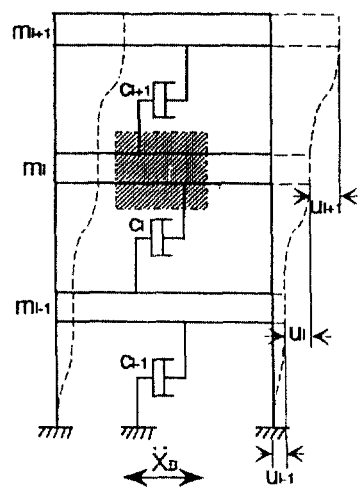

Fig. 2. MDOF shear beam structure

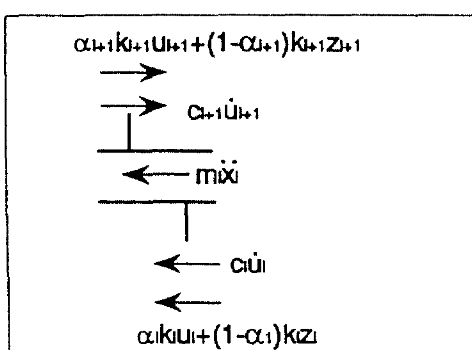

Fig. 3. Internal forces diagram

Restoring force, q_i is given by

$$q_i = c_i \dot{u}_i + \alpha_i k_i u_i + (1-\alpha_i) k_i z_i \quad (2)$$

in which, c_i = the viscous damping; k_i controls the initial tangent stiffness; α_i controls the ratio of postyield stiffness to preyield stiffness; and z_i = the i-th hysteresis.

The time-history dependent hysteretic restoring force is modeled by the first order, nonlinear different equation.

$$\dot{z}_i = \frac{A_i \dot{u}_i - \nu_i(\beta_i |\dot{u}_i| |z_i|^{n_i - 1} z_i + \gamma_i \dot{u}_i |z_i|^{n_i})}{\eta_i} \quad (3)$$

in which A_i, ν_i, β_i, γ_i, η_i, and n_i are parameters that control the hysteresis shape and degradation of system.

Response statistics for the MDOF shear beam structure can be obtained by linearization of z.

$$\dot{z}_i = C_{ei} \dot{u}_i + K_{ei} z_i \quad (4)$$

where, $C_{ei} = E[\partial \dot{z}_i / \partial \dot{u}_i]$ and $K_{ei} = E[\partial \dot{z}_i / \partial z_i]$

The typical hysteretic shape is shown in Fig. 4. From this, it is seen that z is a transformed displacement variable, such that the restoring force given by Eq. (3) exhibits smooth hysteretic behavior.

System degradation may be introduced into the model for z_i by allowing the parameters of the model to vary as a function of the response duration and severity.

3.2 Zero time lag covariance equation

Eq. (1) and Eq. (4) may be rewritten, symbolically, as

$$\{\dot{y}\} + [G]\{y\} = \{b\} \quad (5)$$

in which $\{y\}^T = [y_1, y_2^T, \cdots, y_n^T]$
and $y_i^T = [u_i, \dot{u}_i, z_i]$

If the excitation is passed through a Kanai filter, y contains an additional subvector

$$y_0^T = [u_g, \dot{x}_B] \quad (6)$$

Finally, zero time lag covariance response equation is,

$$[\dot{S}] + [G][S] + [S][G]^T = [B] \quad (7)$$

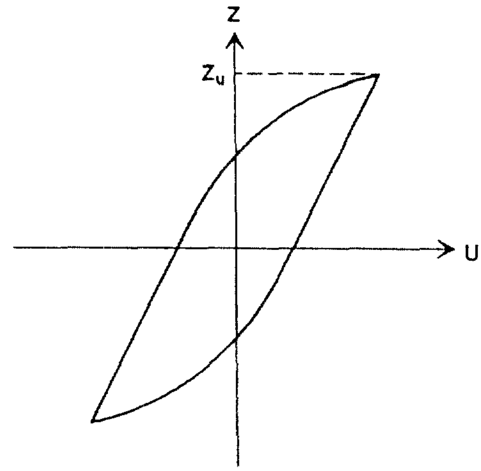

Fig. 4. Typical Hysteresis

where, $[B]$ = excitation matrix
$B_{ii} = 0$ except for $i = n$.
$B_{nn} = I(t) = 2\pi S_0$
for stationary random process
$= 2\pi S_0 \Psi^2(t)$
for nonstationary random process
$\Psi(t)$ = envelope function

Since $[G]$ in Eq. (7) depends on the response statistics $[S]$, Eq. (7) is a system of nonlinear ordinary differential equations which can be solved by numerical integration. In the integration, the $[G]$ matrix is updated at each time step to include the changes in the system that have taken place such decrease in stiffness and increase in damping.

4. Distribution of maximum responses

The maximum structural response is one of the most important quantities in the analysis and design of structures under earthquake excitations.

In the theory of random vibration, the maximum structural response is a statistical variable. Asymptotic approximation is established for the distribution of the maximum nonstationary response using extreme value theories.[7]

Let Y_m be the absolute global maximum of Y(t) in the time interval (T_1, T_2). Then, with the assumptions that the extrema $\eta_j (j = 1, 2, \cdots, n)$ are statistically independent and also that the total number, n, of the extrema in (T_1, T_2) is large, it can be shown that the distribution of Y_m, $F_{Y_m}(u)$, approaches asymptotically to Eq. (8). Its type is Gumbel.

$$F_{Y_m}(u)=F_{n_\eta}\ (u:T_1,\ T_2) = \exp[-\exp[-K^{\alpha-1}(\frac{u}{\sigma}-K)]] \qquad (8)$$

in which, $K=(\alpha \ln n)^{1/\alpha} \cong [\alpha \ln \int_{T_1}^{T_2} \nu(t)dt]^{1/\alpha}$
$\nu(t)$ = the expected zero crossing rate of $Y(t)$

The mean value and standard deviation of the absolute global maximum, Y_m, are obtained from Eq. (8).

$$\mu_{Y_m}=(K+0.5772K^{1-\alpha})\sigma \qquad (9)$$

$$\sigma_{Y_m}=\frac{1.28\sigma}{K^{\alpha-1}} \qquad (10)$$

The mean value, μ_η, and the coefficient of variation, V_η, of an extremum, η, in (T_1, T_2), can be obtained from solution of zero time lag covariance response equation, whereas Gumbel parameters and are determined using Eq. (11)~(12).

$$V_\eta=\frac{[\Gamma(\frac{2}{\alpha}+1)-\Gamma^2(\frac{2}{\alpha}+1)]^{1/2}}{\Gamma(\frac{1}{\alpha}+1)} = \frac{\sqrt{\frac{2}{n}\sum_{j=1}^{n}\sigma_j^2-\frac{\pi}{2n^2}\left(\sum_{j=1}^{n}\sigma_j\right)^2}}{\sqrt{\frac{\pi}{2}}\frac{1}{n}\sum_{j=1}^{n}\sigma_j} \qquad (11)$$

$$\mu_\eta=\sigma\alpha^{1/\alpha}\Gamma(\frac{1}{\alpha}+1)=\sqrt{\frac{\pi}{2}}\frac{1}{n}\sum_{j=1}^{n}\sigma_j \qquad (12)$$

in which $\Gamma(\cdot)$ = the gamma function.

5. Damage Assessment

5.1 Probabilistic Aspects of Damage

When the earthquake ground motion is modeled as a stochastic process, the maximum structural response is a statistical variable. An approximate distribution for the maximum response of peaks is proposed by the Gumbel type.[7]

Beyond the yield level for maximum response of structure, damage of member occurs and the damage state is influenced by the probabilistic nature of maximum responses.[4]

Cumulated damage during earthquake motion is dependent on the current path of the structural response whose nature is probabilistic due to random excitations. The incurred damage in any one cycle is different from the other because of the varying amplitude of responses and the damage levels vary with the response levels, that is, damage beyond the yield deformation increases with the response amplitude increasing.

5.2 Damage Accumulation Method[4]

Damage potential, D_p is the total capacity of the component to sustain damage and damage consumption, D_c is the portion of the allowable capacity that is lost or dissipated during the course of the applied load history, which is composed of strength damage, D_s that is the loss of damage potential due to strength deterioration and hysteretic dissipated energy and deformation damage, D_d that is the remainder of the loss of the damage potential. Strength damage and deformation damage is defined in Fig. 5.

Strength damage and deformation damage are evaluated as follows.

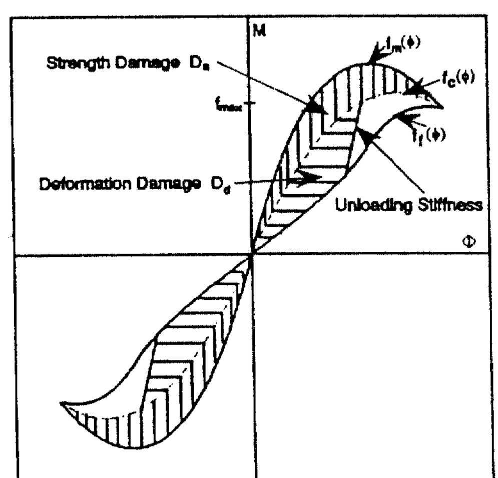

Fig. 5. Conceptual evaluation of structural damage

$$D_s = \int_{-\phi_u}^{+\phi_u} [f_m(\phi) - f_c(\phi)d\phi \quad (13)$$

$$D_d = \int_{-\phi_\alpha}^{+\phi_\alpha} [f_c(\phi) - f_f(\phi)d\phi \quad (14)$$

in which, $f_m(\phi)$ = monotonic envelope of the moment-curvature diagram

$f_c(\phi)$ = current envelope

$f_f(\phi)$ = failure envelope

And damage consumption and damage potential are

$$D_c = D_s + D_d \quad (15)$$

$$D_p = \int_{-\phi_u}^{+\phi_u} [f_m(\phi) - f_f(\phi)d\phi \quad (16)$$

From above, damage index is D_c/D_p.

It is noticeable from the several experimental results in low-cycle fatigue that after some initial inelastic deformation, the deformation damage remains essentially constant while the member continues to dissipate energy through hysteretic behavior and that the dominant damage state variable near failure is the dissipated energy.

Deformation damage controls the damage index for the case of a little of damage. However, near failure, the index is strongly influenced by strength damage.(2) Therefore, strength damage may be rational for the failure prediction of the member.

The several studies for damage evaluation in the past don't take into account of the probabilistic nature of the seismic responses and thus the probabilistic accumulation of damage is presented in this work.

Dissipated energy during the earthquake motions increases with the exciting duration and exciting level increasing, and absorption capacity of energy varies with displacement level or curvature level. As the dissipated hysteretic energy increases, the structure undergoes the strength degradation and stiffness degradation and the degradation beyond a certain level results in the failure.

Strength deterioration is the result of the cumulated damage during the excitation and thus is chosen as the preferred variable of the damage evaluation in this study.

The accumulation of damage is affected by the

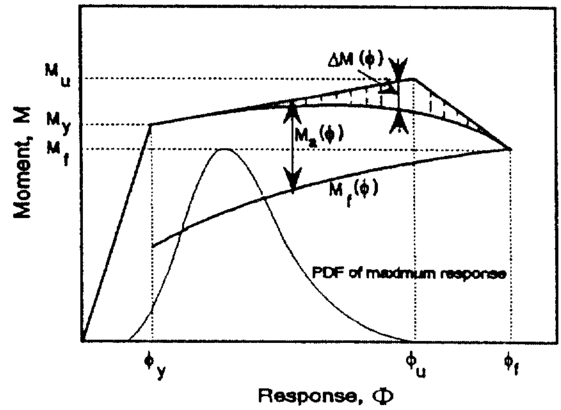

Fig. 6. Damage accumulation using PDF of maximum response

distribution of maximum responses whose basic variables are the yield level, predominant frequency of response, standard deviation of response (displacement and velocity), duration of excitation, intensity of excitation, critical damping ratio, etc. Cumulated damage, therefore, can be represented by the type of an increasing function of certain variables as above.

Strength degradation determined from the several experimental results is the increasing function of response level. In this study, strength deterioration relationship, proposed by Chung,(3) is adopted to accumulate the incremental damage in one load cycle. The accumulation of incurred damage and allowable damage is affected by the distribution type of maximum response (curvature) as shown in Fig. 6.

The total damage incurred during external loadings is ten summed up as the response levels happen to upcross the yield level. This is represented as follows.

The mean of damage for a given curvature, ϕ, in a single load cycle is $\mu_{\Delta M}$ and the mean of upcrossing number over the yield level is n. Therefore, the total damage is $\mu_{\Delta M} \cdot n$ as shown in Eq. (17).

$$\int_t \nu_{\phi_y} dt \cdot \int_\phi f_{\phi_y} \cdot \Delta M(\phi) d\phi = \mu_{\Delta M} \cdot n \quad (17)$$

in which, ν_{ϕ_y} = upcorssing rate of response over yielding

$f_{\phi y}$ = distribution of response over yielding

n = mean of upcorssing number of response over yielding

$= \overline{\nu_{\phi y}} \cdot t$

t = exposure time to load.

The mean of allowable damage is as follows;

$$\int_{\phi} f_{\phi y} \cdot M_a(\phi) d\phi = \mu_{M_a(\phi)} \tag{18}$$

in which, $M_a(\phi)$ = allowable damage at

response level $\phi[= M(\phi) - M_f(\phi)]$

Damage index is the ratio of cumulated damage to allowable damage.

5.3 Overall damage index

The damage of a structure is obviously a function of the damages of its constituent components. Any measure of the overall damage of a structure should reflect the potential damage concentration on the weakest part of a structure. It is well-known that the damage distribution is closely correlated with the distribution of the absorbed energy.[5] Therefore, the overall damage of a structure may be expressed as the sum of the damage indices of the components weighted by the energy absorbing contribution factor.

In the viewpoint of structural reliability, the parallel system reliability should be account for the overall damage index, but for the series system structural failure depends on the limit state in a critical section and damage in that section is dominant in the overall components' damage.

6. Numerical examples

Numerical studies of SDOF system were conducted to demonstrate the damage assessment presented in this study. In this phase, seismic response analysis, determination of distribution of response and seismic damage assessment were continuously performed.

For the SDOF stiff system which has the hysteretic properties such as $\alpha = 0.05$, $\beta = \gamma = A = n = 1.0$, $\delta_\eta = 0.5$ (for degrading system), analysis results

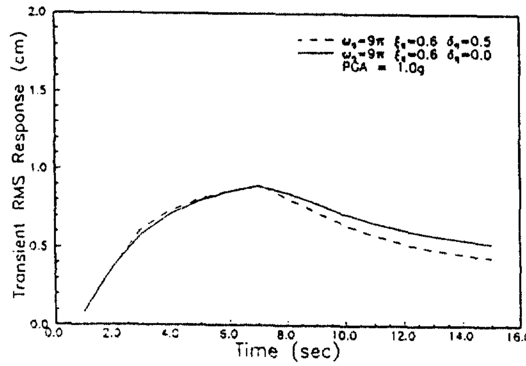

Fig. 7. Transient RMS response

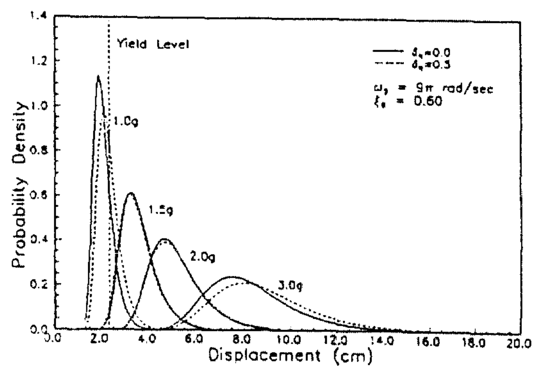

Fig. 8. PDF of maximum response

are shown in Fig. 7~Fig. 9. This system is not realistic but chosen to present the procedure of damage assessment.[4]

Fig. 7 shows the RMS response history for the nondegrading and degrading system and in Fig. 8 PDF of maximum response is presented.

PDF is used in accumulating damage of structural system or member under seimic loadings. And then damage index is calculated as shown in Fig. 9. From this, it can be known that the damage index increase reapidly as the level of PGA increase. Damage is, that is, sensitive to the maginitude of response beyond yield level. Therefore the rational prediction of strength deterioration for one-cycle loading helps the structural member's damage to be assessed realistically.

1-DOF system is chosen to assess seismic damage and compare with Park's study.[5] Material properties are shown in Table 1.

Yield displacement of system is 0.67 cm, ultimate displacement is 1.85 cm, and displacement

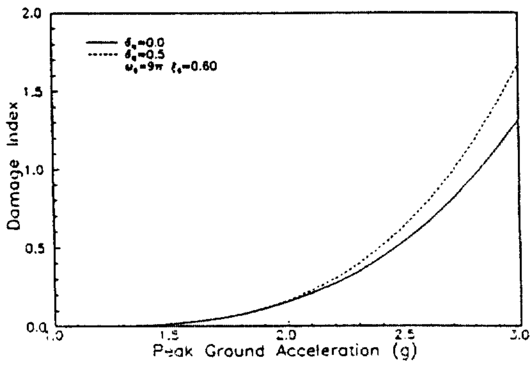

Fig. 9. Damage index of system

Table 1. Material properties of 1-DOF system (unit: kg/cm for K kg·sec^2/cm for M)

k	M	ζ	α	β	γ	n	A	ω_0
14905	34.0	0.05	0.05	5.5	5.5	1	1	3.3

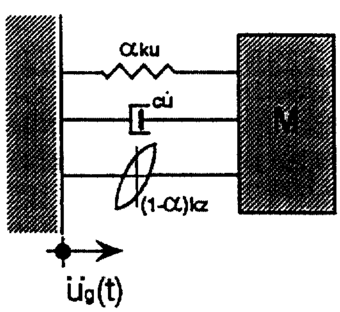

Fig. 10. 1-DOF system model

Table 2. Characteristics of Earthquake Loads

Spectrum	Effective Duration	ξ_g		ω_g	Envelope Type
		Mean	COV		
Kanai-Tajimi	3, 6 sec	0.6	0.4	5π	Combined

at failure is 2.00 cm.

System is modeled as Fig. 10.

Characteristics of earthquake loads are shown in Table 2.

Seismic response is obtained by solving covariance equation stochastically, in which system degradation effect is considered.

In Fig. 11 the transient RMS response is shwon for the PGA level and in Fig. 12 for the effective

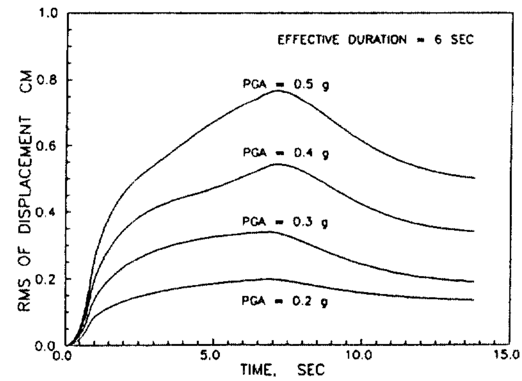

Fig. 11. Transient RMS response for the different PGA level

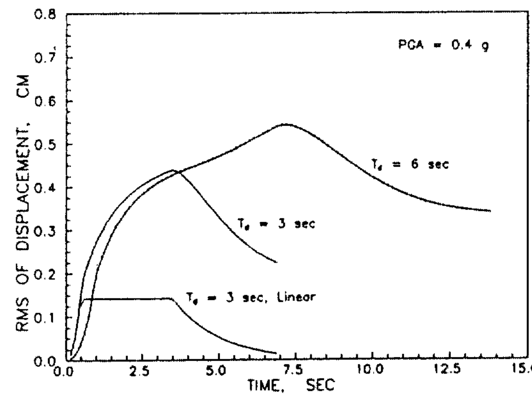

Fig. 12. Transient RMS response for the different effective duration

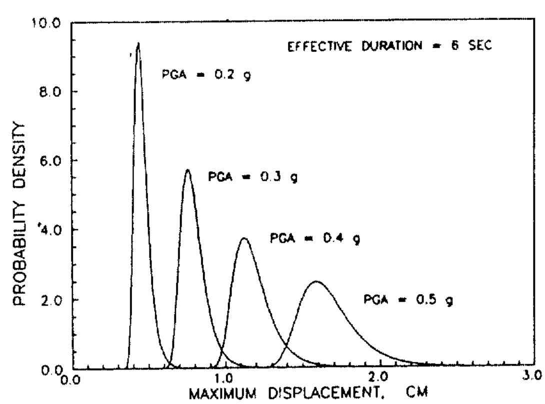

Fig. 13. Distribution type of maximum displacement

duration of loadings.

Transient RMS response of system varies as the effective duration and magnitude of earth-

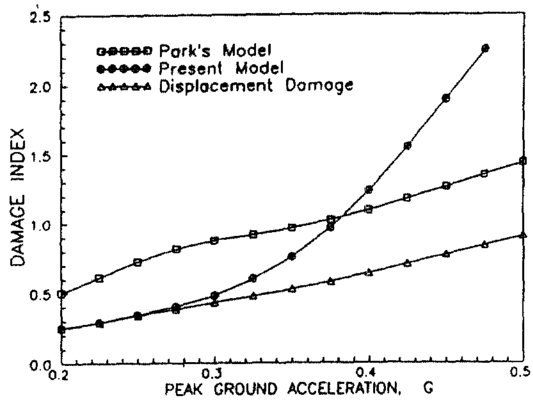

Fig. 14. Damage index for the present study and Park's

quake increase.

Fig. 13 shows the distribution type of maximum displacement for $\mu_d=6$ sec. From this nonlinearity and variability are known to be larger as PGA level is higher.

And the damage index for $\mu_d=6$ sec is shown in Fig. 14, compared with Park's. In low level of PGA, deformation damage is dominant and as the PGA increase the strength damage is the predominant damage. The trend for the result of present study is different from Park's result and this may be caused from the different accumulating method of damage and use of the senstive prediction formula of strength deterioration.

And then extensive study and sensitivity analysis are required.

7. Conclusion

The purpose of the present study is to propose a probabilistic accumulation method of the damage for the component of RC structures subjected to strong earthquake excitations. To this end, the seismic responses and the characteristics of stochastic responses under random seismic loadings are analyzed. The damage during earthquake motions is accumulated by the use of the distribution of maximum seismic responses and the damage index is then obtained.

The damage index of structural component is found to be sensitive to the excitation levels. This arises from the fact that the strength deterioration beyond yielding varies sensitively with the response levels.

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